Ques:- The first four moments about mean of a freq. distn. are 0, 60, -50 & 8020 resp. Discuss kurtosis of the distn.

Soln Here $\mu_1 = 0$, $\mu_2 = 60$, $\mu_3 = -50$, $\mu_4 = 8020$

$$\therefore \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{8020}{(60)^2} = 2.2278 < 3$$

$\therefore$ The distn is platykurtic.

Ques The first four moments about mean of a freq. distn are 0, 100, -7 & 35000 resp. Discuss kurtosis of the distn.

Soln $\mu_1 = 0$, $\mu_2 = 100$, $\mu_3 = -7$, $\mu_4 = 35000$

$$\therefore \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{35000}{(100)^2} = 3.5 > 3$$

$\therefore$ The distn is leptokurtic.

Ques The S.D. of a symmetric distn is 5. what should be the value of the fourth moment about mean in order the distn is (i) leptokurtic (ii) mesokurtic (iii) platykurtic?

Soln Given $\sigma = 5$ $\therefore \mu_2 = \sigma^2 = 25$

$$\therefore \beta_2 = \frac{\mu_4}{\mu_2^2} \Rightarrow \mu_4 = \beta_2 \mu_2^2 =$$

$$\Rightarrow \boxed{\mu_4 = 625\,\beta_2}.$$

(i) For the distn to the leptokurtic, $\beta_2 > 3$

$\Rightarrow 625\beta_2 > 1875 \Rightarrow \boxed{\mu_4 > 1875}$

So, for the distn to be leptokurtic, $\mu_4 > 1875$

(ii) For the distn to be mesokurtic, $\beta_2 = 3$

$\therefore \mu_4 = 625 \times 3 = 1875$

So, for the distn to be mesokurtic, $\boxed{\mu_4 = 1875}$

(iii) For the distn. to be platykurtic, $\beta_2 < 3$

$\therefore \mu_4\ 625\beta_2 < 1875 \Rightarrow \boxed{\mu_4 < 1875}$

So, the distn to be platykurtic, $\mu_4 < 1875$

Ques Calculate the first four moments about the mean for the following data, hence find kurtosis

Sol

Variate (x):	0	1	2	3	4	5	6	7	8	Total.
Frequency (f):	1	8	28	56	70	56	28	8	1	256 (Σf)
fx:	0	8	56	168	280	280	168	56	8	1024 (Σfx)

Mean fr. $M = \frac{\Sigma fx}{\Sigma f} = \frac{1024}{256} = 4$.

$\mu_1 = 0$ $\quad [\because \mu_r = \frac{1}{N}\Sigma f(x-M)^r]$

$\mu_2 = \frac{1}{N}\Sigma f(x-M)^2$

$\mu_3 = \frac{1}{N}\Sigma f(x-M)^3$

$\mu_4 = \frac{1}{N}\Sigma f(x-M)^4$.

X	$d = X - M$ $= X - 4$	f	fd^2	fd^3	fd^4
0	-4	1	16	-64	256
1	-3	8	72	-216	64
2	-2	28	112	-224	448
3	-1	56	56	-56	56
4	0	70	0	0	0
5	1	56	56	56	56
6	2	28	112	224	448
7	3	8	72	216	64
8	4	1	16	64	256
Total		256	512	0	1648

$$\mu_1 = 0$$

$$\mu_2 = \frac{1}{N}\Sigma fd^2 = \frac{512}{256} = 2$$

$$\mu_3 = \frac{1}{N}\Sigma fd^3 = \frac{0}{256} = 0$$

$$\mu_4 = \frac{1}{N}\Sigma fd^4 = \frac{1648}{256} = 6.438$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{6.438}{(2)^2} = \frac{6.438}{4} = 1.61$$

$$\Rightarrow \boxed{\beta_2 = 1.61}$$

Ques The prob. of a man hitting a target is $1/4$

(a) If he fire 7 times, what is the prob. of his hitting the target atleast twice

(b) How many times must he fire so that the prob. of his hitting the target atleast once is greater than $2/3$

Soln (a) $n = 7, \quad p = \frac{1}{4}, \quad q = \frac{3}{4}$

$$\text{Req. Prob.} = 1 - [P(X=0) + P(X=1)]$$

$$= 1 - {}^7C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^7 - {}^7C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^6$$

$$= 0.5551$$

(b) $p = \frac{1}{4}, \quad q = \frac{3}{4}$

Let n be the no. of times he fires to hit the target atleast once.

A.T.Q

$$\frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} + \cdots + \text{n times} > 2/3$$

$$\frac{1}{4}\left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \cdots + \left(\frac{3}{4}\right)^n\right] > 2/3$$

$$\frac{1}{4}\left[\frac{1 - \left(\frac{3}{4}\right)^n}{1 - 3/4}\right] > 2/3 \Rightarrow \frac{1}{4}\left[\frac{1 - (3/4)^n}{1/4}\right] > 2/3$$

$$\Rightarrow 1 - 2/3 > \left(\frac{3}{4}\right)^n \Rightarrow \left(\frac{1}{3}\right) > \left(\frac{3}{4}\right)^n$$

Taking log B.S, we get:

$$\log 1 - \log 3 > n[\log 3 - \log 4] \Rightarrow \log 3 < n[\log 4 - \log 3]$$

$$\Rightarrow n > \frac{\log 3}{\log 4 - \log 3} \Rightarrow \boxed{n > 3.81}$$

$$\Rightarrow \boxed{n = 4}$$

Extra Questions

Q1) When the 1st proof of 392 pages of 1200 pages were read, the distn of printing mistakes were found to be as follows:

No. of mistakes in a page (x)	0	1	2	3	4	5	6
No. of pages (f)	275	72	30	7	5	2	1

Fit a poisson distn to the above data & Calculate theoretical freq^n

Sol^n

$$\frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{72+60+21+20+10+6}{392} = \frac{189}{392} = 0.482$$

$\therefore \lambda = 0.482$.

$\therefore$ Reqd. Poisson distn is $\frac{N \cdot \lambda^x e^{-\lambda}}{x!}$, $N = \Sigma f_i$

$$P(x) = \frac{392 \cdot (0.482)^x e^{-0.482}}{x!} = \frac{392 \times 0.61755 \times (0.482)^x}{x!}$$

$$\boxed{P(x) = \frac{(242.078)(0.482)^x}{x!}}$$

x	$P(x)$	Theoretical frequency
0	242.078	242
1	$(242.078)(0.482) = 116.68$	117
2		
3		
4		
5		
6		
	Total	392

Ques Three unbiased coins are tossed simul. Find prob. of getting (i) at least 2 heads (ii) At most 2 heads

Sol: $S = \{$ $\}$

(i) $P = \frac{4}{8} = \frac{1}{2}$

(ii) 2 heads $= \frac{7}{8}$

Ques A card is drawn from a well shuffled pack of playing cards. Find the prob. that it is either a diamond or a king.

Soln Let A: drawing a diamond
B: " " king.

Then $A \cap B$: drawing card which is both diamond & King.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Ques: A bag contains 8 black & 2 red pens if a pen is drawn at random. what is probability that it is black pen or red pen)

Sol: Let A: getting a black pen $\Rightarrow P(A) = \frac{8}{10} = \frac{4}{5}$
B: " " red " $\Rightarrow P(B) = \frac{2}{10} = \frac{1}{5}$

P (getting a black pen or red pen) $\Rightarrow P(A \cup B) = P(A) + P(B)$
$= \frac{4}{5} + \frac{1}{5}$
$= 1.$

[$\because$ A & B are mutually exclusive events)

Ques For a given prob. fn $f(x) = x^{-2}$, if $x \geq 1$, find $P(X \leq 2)$

Soln :- The interval given is $[1, 2]$

$$P(X \leq 2) = \int_1^2 x^{-2}\,dx = \left[\frac{-1}{x}\right]_1^2 = \frac{-1}{2} + 1 = 0.5$$

Ques: Find the prob. of finding a R.V. b/w $[34, 88]$

Soln To find the prob; the formula will be $\frac{1}{b-a}$

$b = 88$, $a = 34$

$$\text{Req. Prob} = \frac{1}{88-34} = \frac{1}{54}$$

Ques Find $P(2 \leq P \leq 4)$ for prob. fn $f(x) = 2x^{-3}$ for $x \geq 1$

Soln The given interval $[2, 4]$.

$$P(2 \leq P \leq 4) = \int_2^4 2x^{-3}\,dx$$

$$= \left[\frac{2x^{-3+1}}{-3+1}\right]_2^4 = -\left[\frac{1}{16} - \frac{1}{4}\right] = \frac{3}{16}.$$

Ques A Coin is tossed twice so that $S = \{HH, HT, TH, TT\}$.
Let $X \to$ No. of heads that can came up. Find the prob fn corresponding to R.V. 'X'. Assuming that coin is fair.

Soln $P(HH) = P(HT) = P(TH) = P(TT) = \frac{1}{4}$.

Then $P(X = 0) = P(TT) = \frac{1}{4}$

$P(X = 1) = P(HT \cup TH) = P(HT) + P(TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(X = 2) = P(HH) = \frac{1}{4}$.