Normal Distribution

Normal distn is a conti. distn. It is derived as the limiting form of the Binomial distn for large values of n & p & q are not very small nearly close to ½.

The normal distn. is given by the eqn.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where $-\infty < x < \infty$ & μ & σ are called the parameters of the distribution & $f(x)$ is called pdf of the normal distribution.

Basic Properties:-

The pdf of the normal distn. is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

(i) $f(x) \geq 0$ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

The total area under the normal curve above the x-axis is 1.

(iii) The normal distn is symmetrical about its mean.

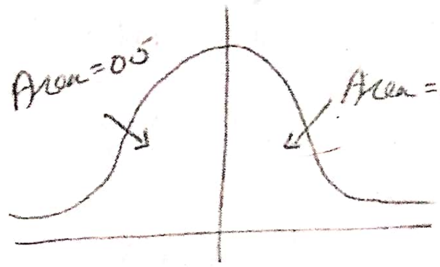
(iv) The mean, mode & median of this distn coincide

Standard form of the Normal $dist^n$

If X is a normal random variable with mean μ and standard deviation σ, then the random variable $Z = \frac{X-\mu}{\sigma}$ has the normal $dist^n$ with mean 0 and S.D 1. The random variable Z is called the Standardized normal random variable.

The Probability density function for the normal $dist^n$ in standard form is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

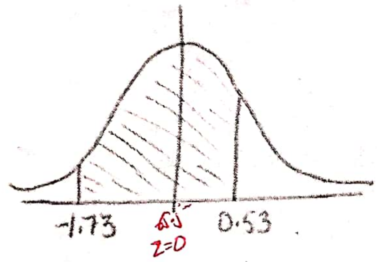


Quesn A large number of measurement is normally distributed with a mean 65.5" and S.D of 6.2". find the percentage of measurements that fall b/w 54.8" and 68.8"

Sol

$\mu = 65.5$, $S.D = 6.2$

$$Z_1 = \frac{X_1-\mu}{\sigma} = \frac{54.8-65.5}{6.2} = -1.73$$



$$Z_2 = \frac{X_2-\mu}{\sigma} = \frac{68.8-65.5}{6.2} = 0.53$$

$$P(-1.73 \le Z \le 0.53) = P(-1.73 < Z < 0) + P(0 \le Z \le 0.53)$$

$$= P(0 \le Z < 1.73) + P(0 \le Z \le 0.53)$$

$$= 0.4582 + 0.2019 = 0.6601$$

$\therefore$ Reqd Percentage of Measurements = 66.01%

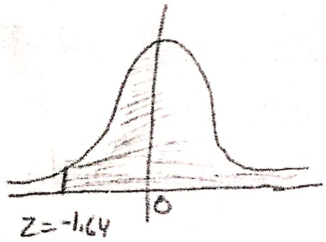
Quesr The income of a group of 10,000 persons ~~on~~ was found to be normally distributed with mean RS 750 p.m. and S.D of RS 50 Show that, of this group about 95%. had income ~~Exceed~~ Exceeding RS 668 and only 5%. had income Exceeding Rs 832. Also find the lowest income ~~abov~~ among the richest 100.

Sol $\mu = 750$, $S.D = 50$ (given)

Standard normal variable $Z = \dfrac{X-\mu}{\sigma}$

(i) if $x_1 = 668$, $Z_1 = \dfrac{668-750}{50} = -1.64$

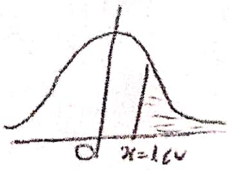
$$P(x_1 > 668) = P(Z > -1.64)$$
$$= 0.5 + P(-1.64 \le Z \le 0)$$
$$= 0.5 + P(0 \le Z \le 1.64)$$
$$= 0.5 + 0.4495$$
$$= 0.9495$$



∴ Required % of persons having income Exceeding Rs 668 = 94.95%
= 95%. (approx)

(ii) if $X_2 = 832$, $Z_2 = \dfrac{X_2-\mu}{\sigma} = \dfrac{832-750}{50} = 1.64$

$$P(X_2 > 832) = P(Z_2 > 1.64)$$
$$= 0.5 - P(0 \le Z_2 \le 1.64)$$
$$= 0.5 - 0.4495$$
$$= 0.0505$$



Required % of persons having income Exceeding Rs 832 = 5.05%

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(iii) Let x be the lowest income among the richest 100 persons. (17)

i.e. @ 1% of 10000

The area b/w 0 and Z = 0.49 by normal distⁿ table,

$$\frac{1}{10000}\times 100 = 0.01$$

$$0.50 - 0.01 = 0.49$$

$$Z = 2.33$$

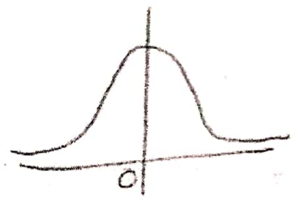
$$\frac{x-\mu}{\sigma} = 2.33$$

$$\frac{x-750}{50} = 2.33 \Rightarrow x = 866.5$$

Quesr if the height of 300 students are normally distributed with mean 64.5 inches and S.D 3.3 inches find the height below which 99% of the students lie.

Sol Mean $\mu = 64.5$ inches

$\sigma = 3.3$



$$\cancel{z = \frac{x-\mu}{\sigma}}$$

Area b/w 0 and $Z = \dfrac{x-\mu}{\sigma}$

Area b/w 0 and $\dfrac{x-64.5}{3.3} = 0.99 - 0.5$

$= 0.49$

from the table for the area 0.49, Z = 2.327

$$\frac{x-64.5}{3.3} = 2.327$$

$$x = 7.68 + 64.5 = 72.18 \text{ inches}$$

hence 99% students are of height less than 6 ft 0.18 inches

Quesr In a normal distⁿ, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D of the distⁿ

~~It is given that if $f(t) = \frac{1}{\sqrt{2\pi}}\int_0^t e^{-\frac{1}{2}x^2}\,dx$ then $f(0.5) = 0.19$ and $f(1.4) = 0.42$~~

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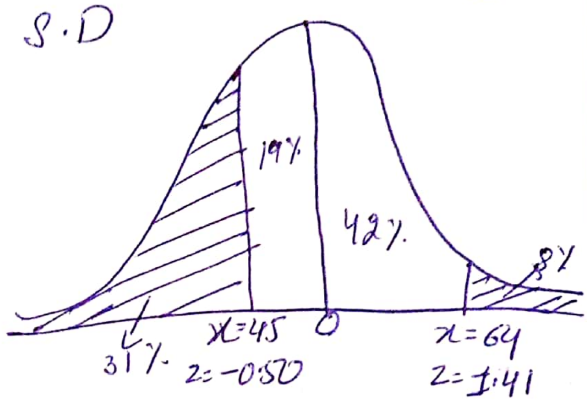
(12)

Sol^n Let μ be the mean & σ be S.D



If $x=45$, $z=\frac{45-\mu}{\sigma}$

If $x=64$, $z=\frac{64-\mu}{\sigma}$

Area b/w $z=0$ & $\frac{45-\mu}{\sigma} = 0.5 - 0.31 = 0.19$

Area b/w $z=0$ & $\frac{64-\mu}{\sigma} = 0.5 - 0.08 = 0.42$

From the table, for the area 0.19, $z=-0.50$

" " " , for the area 0.42, $z = 1.41$

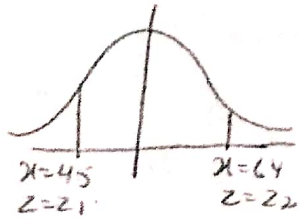
$\Rightarrow \quad -0.50 = \frac{45-\mu}{\sigma}$ —— (1)

$1.41 = \frac{64-\mu}{\sigma}$ —— (2)

Solving (1) & (2), we get $\mu = 50$, $\sigma = 10$

Let μ and S.D respectively 31%. of the items are under 45
Area to the left of the ordinate $x = 45$ is 0.31. (18)

$$P(z_1 < z < 0) = 0.5 - 0.31$$
$$= 0.19$$



when $x = 64$ Let $z = z_2$

$$P(0 < z < z_2) = 0.5 - 0.08$$
$$= 0.42$$

from ~~table~~ table to this area is 1.4.

$$z_2 = 1.4$$

$$z = \frac{x-\mu}{\sigma} = 1.4 = \frac{64-\mu}{\sigma} \longrightarrow (1)$$

$$-0.5 = \frac{45-\mu}{\sigma} \longrightarrow (ii)$$

Solving eq $\Rightarrow$ $\mu = 50$

Quest Pipes for tobacco are being packed in fancy plastic boxes. The length of the pipe is normal distributed with $\mu = 5''$ and $\sigma = 0.1''$. The internal length of the boxes is $5.2''$ what is the probability that the box would be small for the pipe.

(ii) The life of army shoe is normally distributed with mean 8 months and Standard deviation 2 months. If 5000 pairs are insured. how many pairs would be expected to need replacement after 12 month. [given that $P(z \geq 2) = 0.0228$]

Sol (i) $\mu = 5''$, $\sigma = 0.1''$, $x = 5.2''$

$$z = \frac{x-\mu}{\sigma} = \frac{5.2 - 5}{0.1} = 2$$

(19)

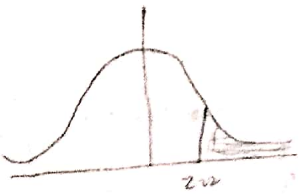
The box will be small if the length of the pipe is more than 5.2" ($\because z=2$)

$P(z>2) = P(0<z<\infty) - P(0<z<2)$

$= 0.5 - 0.4772$

$= 0.0228$

Hence Probability is 0.0228.



(ii) Mean $\mu = 8$, $\sigma = 2$.

Number of pairs of shoes $= 5000$

Total month $(x) = 12$

$$z = \frac{x-\mu}{\sigma} = \frac{12-8}{2} = 2.$$

Area $(z>2) = 0.0228$

Number of pairs whose life is more than 12 month $= 5000 \times 0.0228$
$= 114$

Pair of shoes needing replacement after 12 month $= 5000 - 114$
$= 4886$

Quest The mean inside diameter of a sample of 200 washers produced by a machine is 0.502 cm. and the S.D is 0.005 cm. The purpose for which these washers are intended allows a minimum tolerance in the diameter of 0.496 to 0.508 cm. Otherwise the washers are considered defective. Determine the % of defective washers produced by the machine assume the diameters are normally distributed.

Sol given $\mu = 0.502$

$\sigma = 0.005$

$x_1 = 0.496$, $x_2 = 0.508$

Now $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{0.496 - 0.502}{0.005} = -1.2$

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{0.508 - 0.502}{0.005} = 1.2$

Area for non defective washers $= P(-1.2 \le z \le 1.2)$

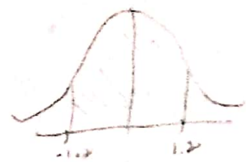
$= P(-1.2 \le z \le 0) + P(0 \le z \le 1.2)$

$= P(0 \le z \le 1.2) + P(0 \le z \le 1.2)$

$= 0.3849 + 0.3849$

$= 0.7698$

$= 76.98\%$



$\therefore$ Percentage of defective washers $= 100 - 76.98$

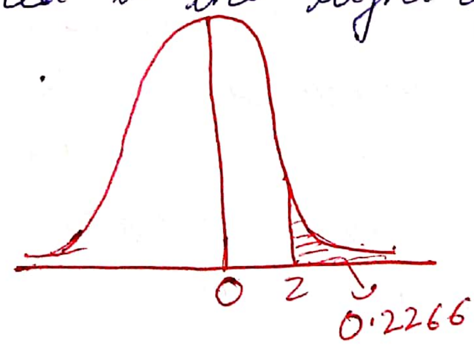
$= 23.02\%$

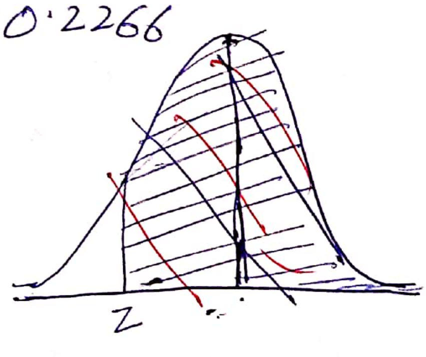
Ques Find the value of z s.t

(a) Area to the right of z is 0.2266

(b) Area to the left of z is 0.0314

Sol^n (a) Area to the right of z is 0.2266





Since the area is less than 0.5.

$\therefore$ Area b/w 0 & z $= 0.5 - 0.2266 = 0.2734$

The value of z corresponding to area $= 0.2734$ is 0.75

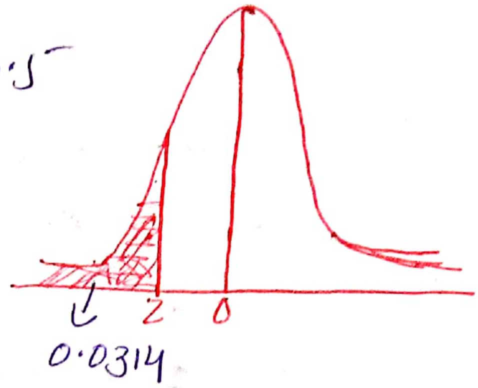
$\therefore$ $\boxed{z = 0.75}$

(b) Area to the left of Z is 0·0314

Since area is less than 0·5

∴ Area b/w 0 & Z is given

by $0.5 - 0.0314 = 0.4686$



The value of Z corresponding to area 0·4686 is

ie $\boxed{Z = 1.86}$

Ques: X is normally distributed & the mean on X is 12 & S.D is 4. Find out the prob., of the following:

(a) (i) $X \geq 20$ (ii) $X \leq 20$ (iii) $0 \leq X \leq 12$

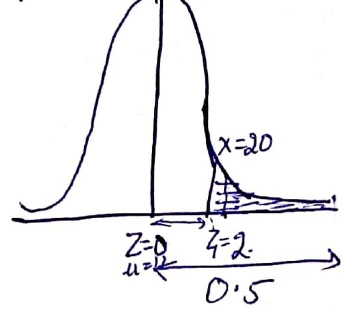
(b) Find a, when $P(X > a) = 0.24$

(c) Find b & c, when $P(b < X < c) = 0.50$

& $P(X > c) = 0.25$

Sol^n Mean ie. $\mu = 12$, S.D ie. $\sigma^2 = 4$

(a) (i) $Z_1 = \frac{X - \mu}{\sigma} = \frac{20-12}{4} = 2$

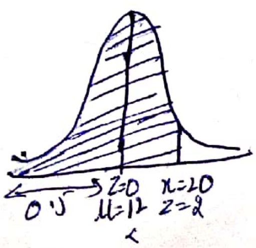


$P(X \geq 20) = 0.5 -$ Area lies b/w 0 & 2

$= 0.5 - 0.4772 = 0.0228$

(ii) $P(X \leq 20) = 0.5 +$ Area lies b/w 0 & 2

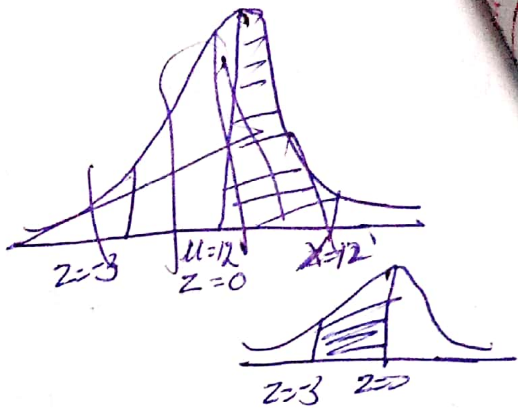
$= 0.5 + 0.4772 = 0.9772.$



(iii) $P(0 \le X \le 12) = ?$

$Z = \frac{X-\mu}{\sigma} = \frac{12-12}{4} = 0$ | $Z_2 = \frac{0-12}{4} = -3$

$\Rightarrow \cancel{P(0 \le X \le 12) = 0}$



$P(-3 \le Z \le 0) = P(0 \le Z \le 3)$

$= 0.5 - 0.49865$

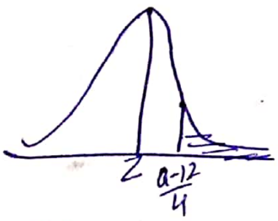
$= 0.0135$

$\Rightarrow P(0 \le X \le 12) = 0.0135$

(b) Find a, when $P(X > a) = 0.24$

when $X = a$, $Z = \frac{a-\mu}{\sigma} \Rightarrow Z = \frac{a-12}{4}$

$\Rightarrow P(X > a) = P\left(Z > \frac{a-12}{4}\right) = 0.24$



ie. $0.5 -$ Area lying b/w $Z=0$ to $Z = \frac{a-12}{4} = 0.24$

$\Rightarrow$ Area lying b/w $Z=0$ to $Z = \frac{a-12}{4} = 0.5 - 0.24 = 0.26$

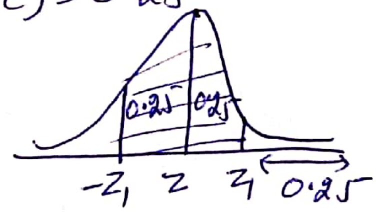
From table, The value of Z corresponding to area 0.26 is 0.71

$\Rightarrow 0.71 = \frac{a-12}{4} \Rightarrow \boxed{a = 14.84}$

(c) Find b & c, $P(b < X < c) = 0.50$ & $P(X > c) = 0.25$

when $X = b$, $-Z_1 = \frac{b-12}{4}$ — (1)

$X = c$, $Z_1 = \frac{c-12}{4}$ — (2)



from (2) & $P(X > c) = 0.25 \Rightarrow P(Z > Z_1) = 0.25$

ie. $0.5 -$ Area lying b/w $Z=0$ to $Z=Z_1 = 0.25 \Rightarrow$ Area $= 0.5 - 0.25 = 0.25$

$\Rightarrow$ The value of Z_1 corresponding to area 0.25 is 0.67

$\Rightarrow 0.67 = \frac{c-12}{14} \Rightarrow \boxed{c = 14.68}$

$P(-Z_1 < Z < Z_1) = P(-Z_1 < Z < 0) + P(0 < Z < Z_1)$

$\Rightarrow P(0 < Z < Z_1) + P(0 < Z < Z_1) = 2P(0 < Z < Z_1)$

from (1), we get

$-0.67 = \frac{b-12}{4} \Rightarrow \boxed{b = 9.32}$

$\therefore b = 9.23$, $c = 14.68$ s.t $P(9.23 < X < 14.68) = 0.50$

Ques In a sample of 1000 cases, the mean of a certain test is 14 & S.D is 2.5. Assuming the distⁿ to be normal, find

(a) how many scores b/w 12 & 15?

(b) " " " above 18?

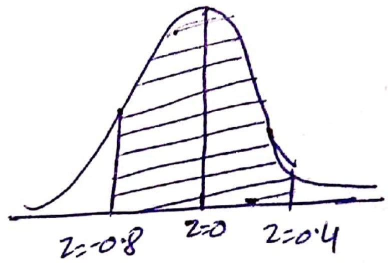
(c) " " " below 8?

(d) " " " 16?

Solⁿ $n = 1000$, $\mu = 14$, $\sigma = 2.5$

(i) $z_1 = \frac{x-\mu}{\sigma} = \frac{12-14}{2.5} = -0.8$

$z_2 = \frac{15-14}{2.5} = 0.4$



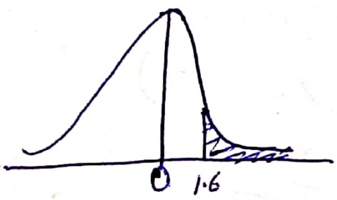
Area lying b/w -0.8 to 0.4 = Area 0 to 0.8 + Area 0 to 0.4

$= 0.2881 + 0.1554 = 0.4435$

∴ Req. no. of students $= 1000 \times 0.4435 = 443.5$

$\cong 444$

(ii) $z = \frac{18-14}{2.5} = 1.6$



Area right to 1.6 = 0.5 − Area b/w 0 & 1.6

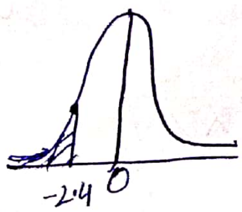
$= 0.5 - 0.4452 = 0.0548$

∴ Req. no. of students $= 1000 \times 0.0548 = 54.8 \cong 55$

(iii) $z = \frac{8-14}{2.5} = -2.4$

Area left to -2.4 = 0.5 − Area b/w 0 & 2.4

$= 0.5 - 0.4918 = 0.0082$



∴ Req. no. of students $= 1000 \times 0.0082 = 8.2 \cong 8$.

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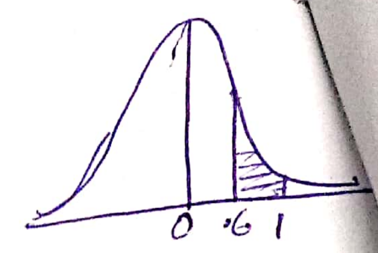
(iv) Between 15.5 & 16.5



$z_1 = \frac{15.5-14}{2.5} = 0.6$ & $z_2 = \frac{16.5-14}{2.5} = 1$

Area b/w 0.6 & 1 = 0.3413 − 0.2257 = 0.1156

∴ Req. no. of students = 0.1156 × 1000
= ~~0~~ 115.6 ≅ 116

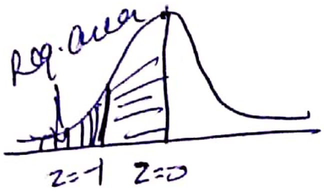
Ques An aptitude test was conducted on 900 employes of the metro tyres limited, in which the mean score was found to be 50 & S.D was 20. On the basis of this information, you are req. to answer this questions:-

(a) what was the no. of employees whose mean score was less than 30?
" " " " " " " exceeded 70?

(b) " " " " " " " " " " were b/w 30 & 70?

(c) " " " " " " "

Soln $\mu = 50$, $\sigma = 20$



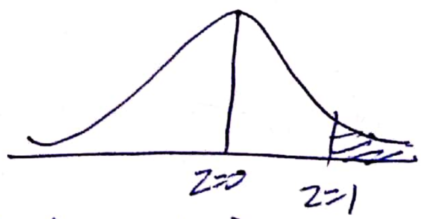
(a) $P(X \le 30) = ?$

$z = \frac{X-\mu}{\sigma} = \frac{30-50}{20} = -1$

$P(X \le 30) = P(Z \le -1) = 0.5 -$ Area lying b/w z=0 to z=−1
$= 0.5 - 0.3413 = 0.1587$

∴ No. of employees whose mean score less than 30 = 900 × 0.1587
= 142.83 ≅ 143.

(b) $P(X \ge 70) = ?$



$z = \frac{70-50}{20} = 1$

$P(X \ge 70) = P(Z \ge 1) = 0.5 -$ Area lying b/w z=0 to z=1
$= 0.5 - 0.3413 = 0.1587$

Req. result is 143 employees

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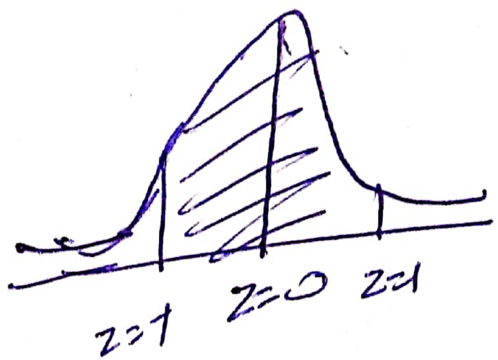
(c) $P(30 \leq X \leq 70)$



$= P(-1 \leq Z \leq 1)$

$= P(-1 \leq Z \leq 0) + P(0 \leq Z \leq 1)$

$= 2P(0 \leq Z \leq 1)$

$= 2 \times 0.1587 = 0.3174$

$\therefore$ Req. result is $0.3174 \times 900 = 285.66$

$\simeq 286$ employees