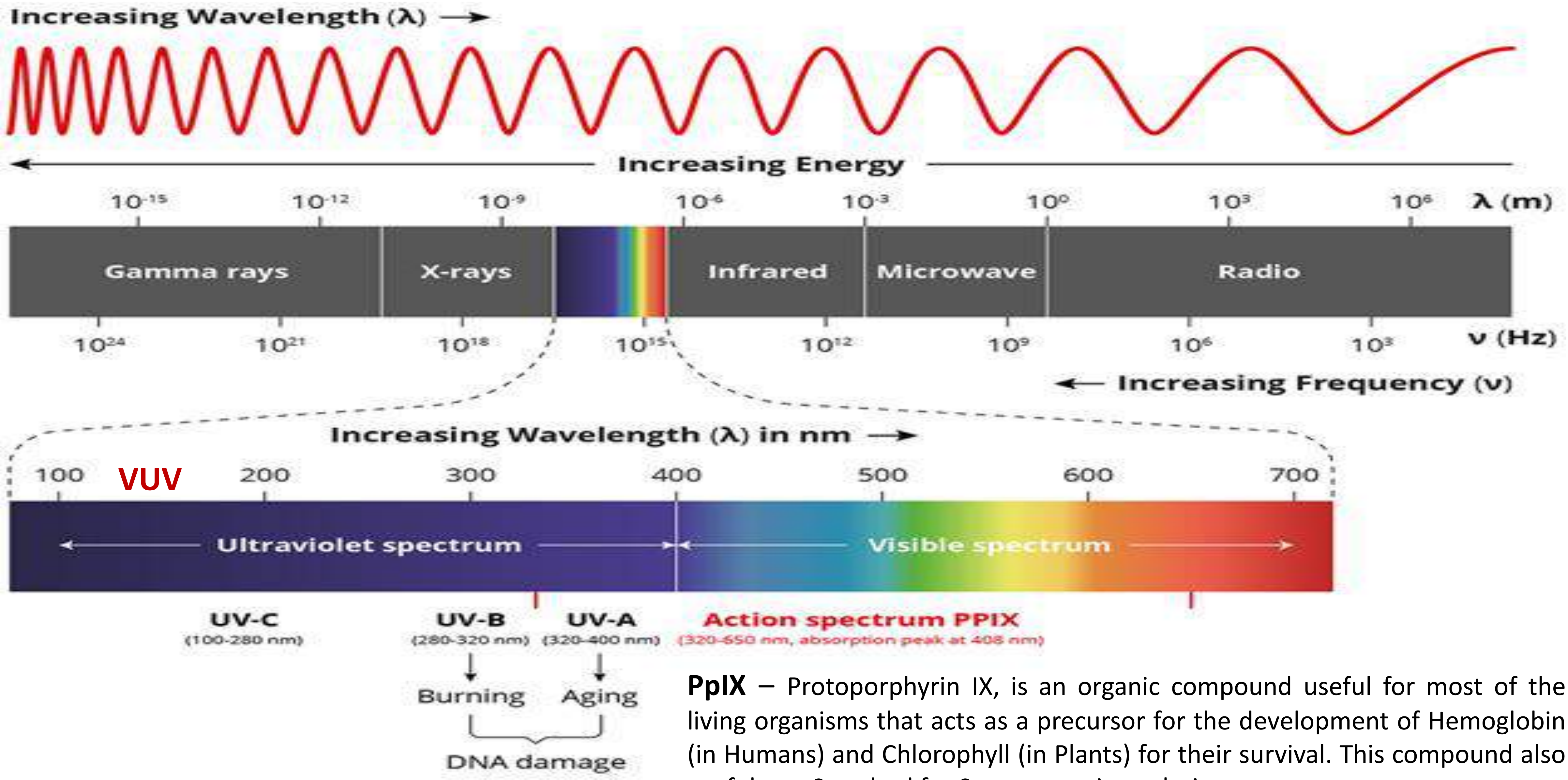
Interference

Suggested Reading:

1) Halliday, Resnic and Walker, “Fundamentals of Physics”, 9th Ed., John Wiley, 2011.
2) R. K. Gaur and S. L. Gupta, “Engineering Physics”, Dhanpat Rai Publications, 2003.
3) Ajoy Ghatak, Optics, 5th Ed., Tata McGraw Hill, 2012.
4) Raghuvanshi, “Engineering Physics”
5) D. N. Vasudeva, “A Text Book of Light”
6) D. P. Khandelwal, “Optics and Atomic Physics”

Electromagnetic Spectrum



PpIX – Protoporphyrin IX, is an organic compound useful for most of the living organisms that acts as a precursor for the development of Hemoglobin (in Humans) and Chlorophyll (in Plants) for their survival. This compound also useful as a Standard for Spectroscopic analysis.

Principle of Superposition (PoS):

The resultant displacement of a particle in a medium acted upon by 2 or more waves simultaneously = algebraic sum of the disp of the same particle due to individual waves, in the absence of the other waves.

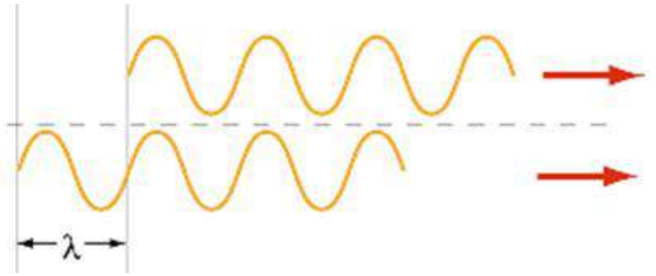
If y_1 and y_2 are two individual displacements of a particle in a given direction by two waves, then according to PoS, the resultant 'y' is given by:

$y = y_1 + y_2$ (if displacements are in ***same*** direction)

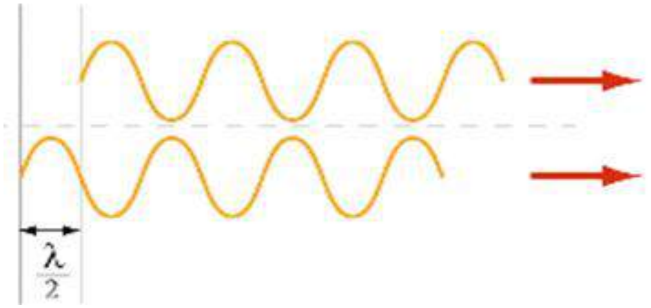
$y = y_1 \sim y_2$ (if displacements are in ***opposite*** direction)

Then what about Light Waves?

In case of light waves, the modifications in the intensity distribution in the region of superposition is called **Interference.**



If 'y' is the sum of amplitudes, it's called ***constructive*** Interference.



If 'y' is the difference of amplitudes, it's called ***destructive*** Interference

If a_1& a_2 = amplitudes of waves from 2 sources

δ = phase diff between 2 waves reaching a point

y_1 & y_2 = displacement of waves then,

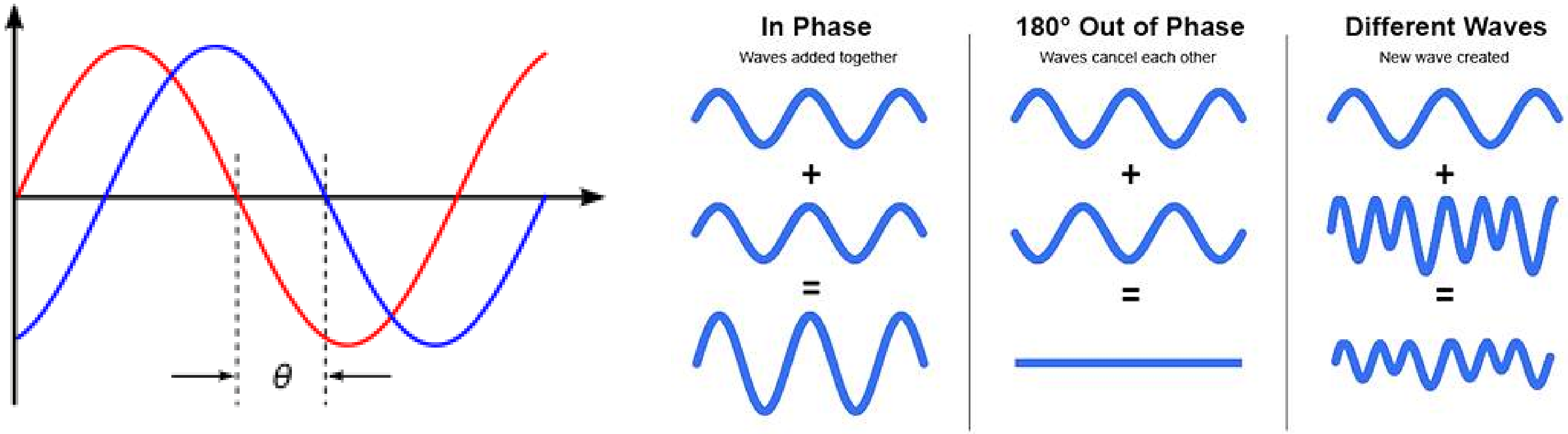
$$\boxed{\text{Phase difference, } \delta = \frac{2\pi}{\lambda} \text{(path difference)}}$$

What do you understand by ‘Phase’?

The phase of an oscillating particle at any instant defines the state of the particle as regards to

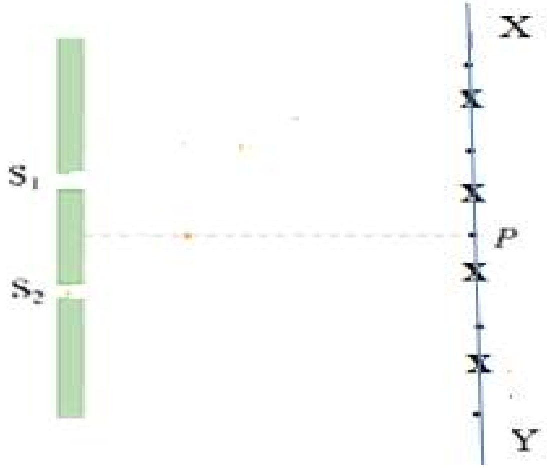
- It’s ***position***
- It’s ***direction*** of motion at that instant

Phase can be measured in ***distance, time, or degrees***.



If the peaks/troughs of two signals with the same frequency are in exact alignment at the same time, they are said to be in phase.

Coherence & Coherent Sources:



Let S_1 & S_2 = sources emitting light of Wavelength λ

XY = screen placed at a distance from S_1 & S_2

Let the '.' represents pts for which the dist from S_1 & S_2 differ by $n\lambda$ (where 'n' is an whole number, n = 0,1,2,3,…)

The 'X' represents pts for which corresponding path diff is $(n + \frac{1}{2})\,\lambda$ or $(2n + 1)\,\frac{\lambda}{2}$

Now we consider the following cases:

Case 1: The waves starting from S_1 & S_2 are in ***same phase.***

Implies, all '.' = bright

'X' = dark all the time and

A stationary interference is seen on the screen.

Case 2: The waves starting from S_1 and S_2 ***differ in phase by π***

Implies, all '**.**' = dark

'X' = bright all the time and

A stationary interference pattern is seen on the screen

Case 3: The waves starting from S_1 and S_2 ***differ by a const phase***

Implies, in this case also we get stationary interference pattern but the pts of bright and dark intensities are not optimum.

Case 4: The waves staring from S_1 and S_2 ***differ in phase*** which is changing at ***random*** with time.

Implies, the interference pattern will not be stationary. One can observe only an average uniform intensity every where on the screen. Therefore, no interference pattern will be observed.

Therefore, the two sources that maintain zero or constant phase diff between them are known as ***coherent sources***. This phenomena is called ***coherence***. However, if the phase diff changes with time, the two sources are called ***incoherent sources***.

Production of coherent light:

In case of mechanical vibrations or electric oscillations, we can control 2 independent sources and maintain a const phase relation between them.

But the emission of light from any lamp comes from billions of atoms, and hence, the emission from each lamp would be random. Hence, 2 independent sources of light can never be coherent.

Implies, in other words, interference of light can ***never be observed*** with 2 independent sources.

Then how to produce coherent sources of light?

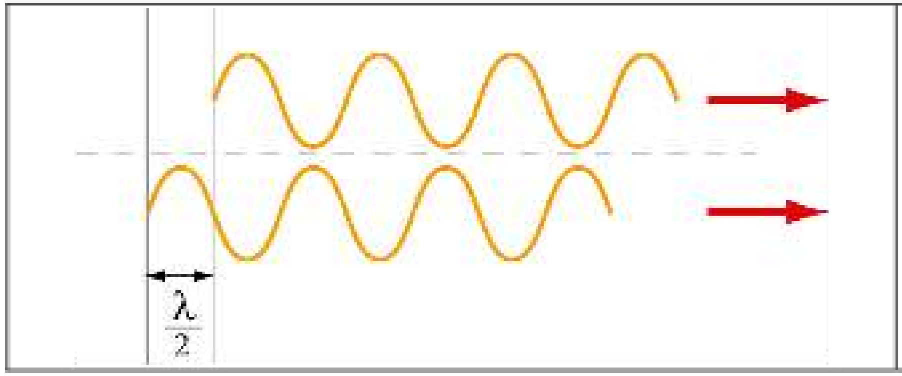
If both sources of light are originated from a single parent point source by some means, then all the random phase changes occurring in the parent source are repeated in the secondary ones also, so that relative phase remains constant. Then these are called coherent sources of light.

Note:

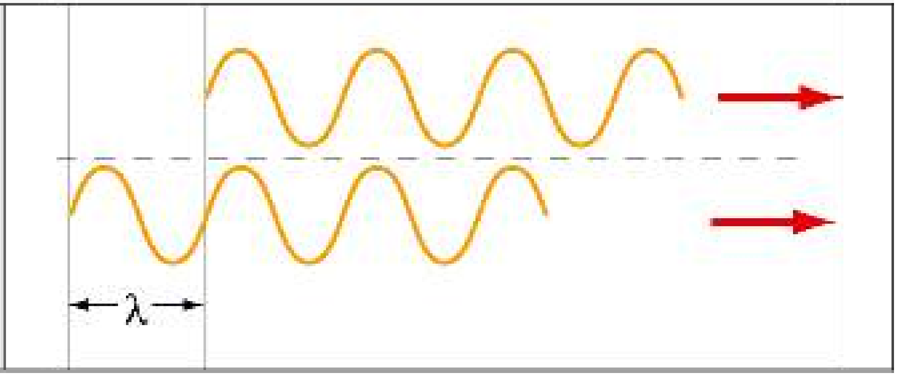
If we choose both the sources of light from a single extended source then also interference of light cannot be observed as there will be a random phase relation between them.

Interference between coherent waves

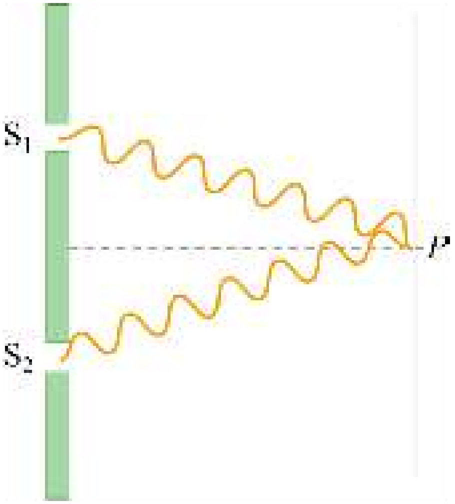
Interference can only occur between waves that have zero or constant phase diff between them.



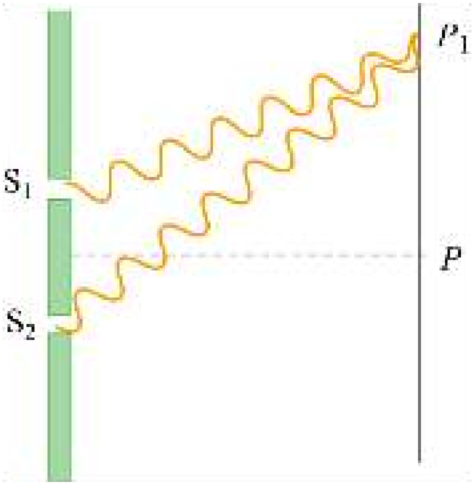
Destructive interference



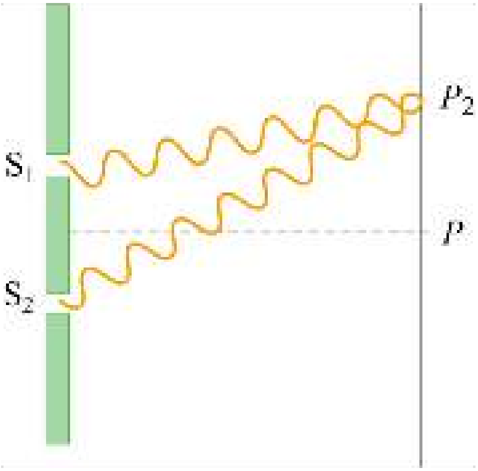
Constructive interference



Constructive interference



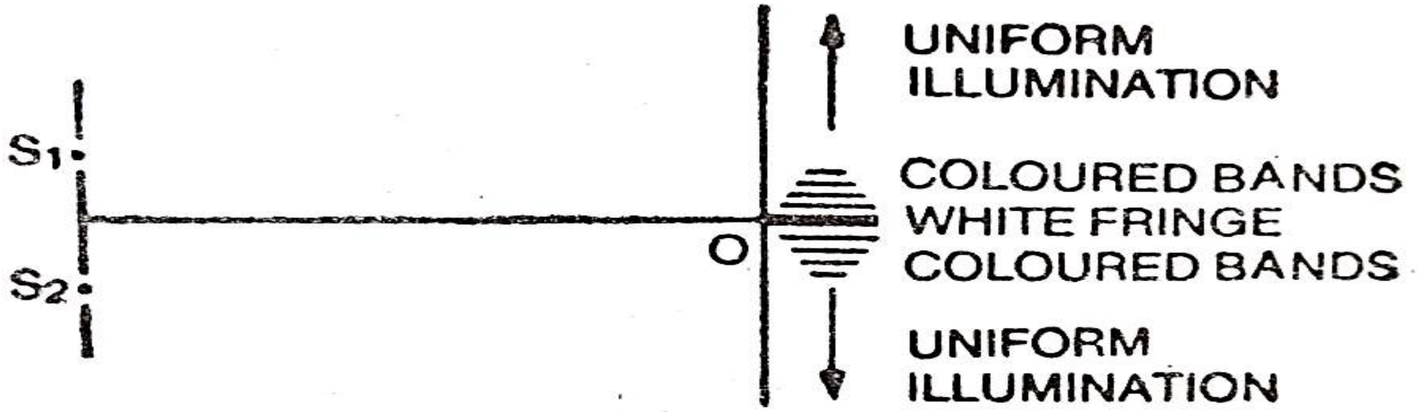
Constructive interference



Destructive interference

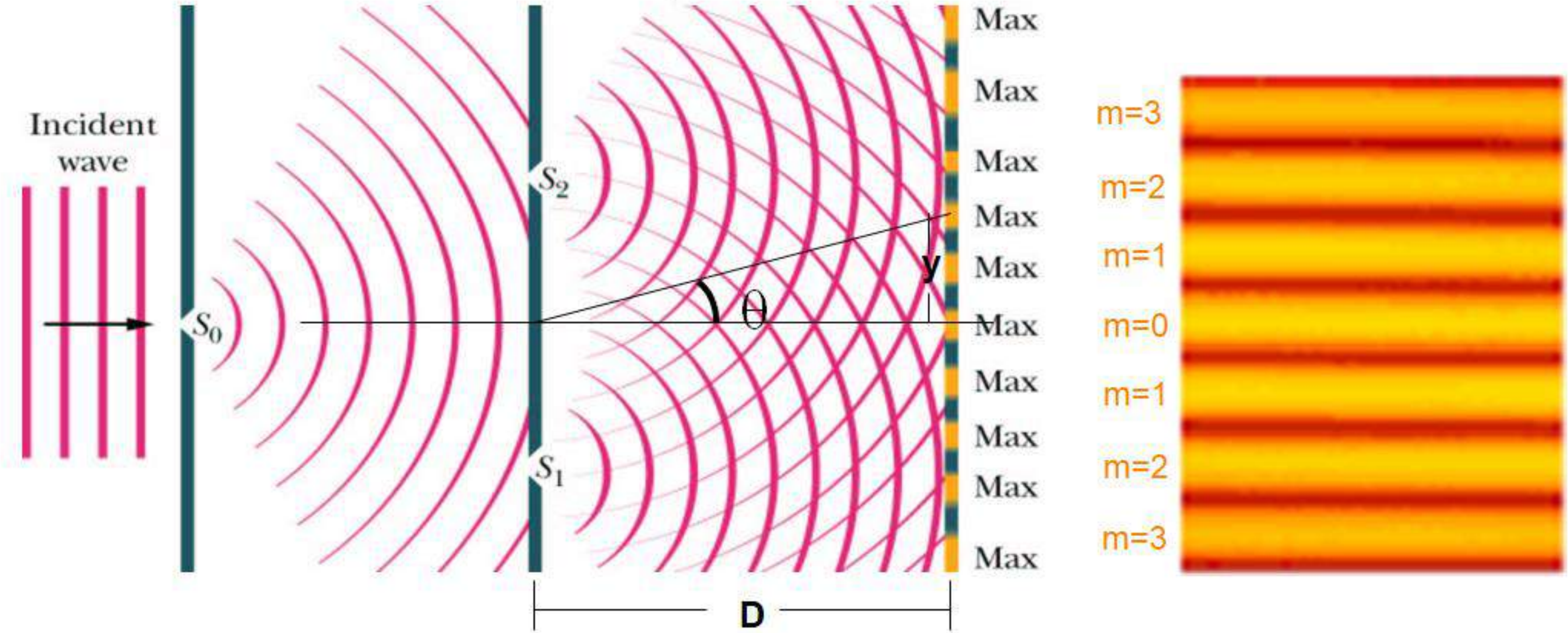
Young’s double slit experiment (1802 AD)

- First experiment to demonstrate interference of light
- Used sunlight as a source
- Sufficiently close pin holes and an opaque screen
- Observed few colored bright and dark bands on screen

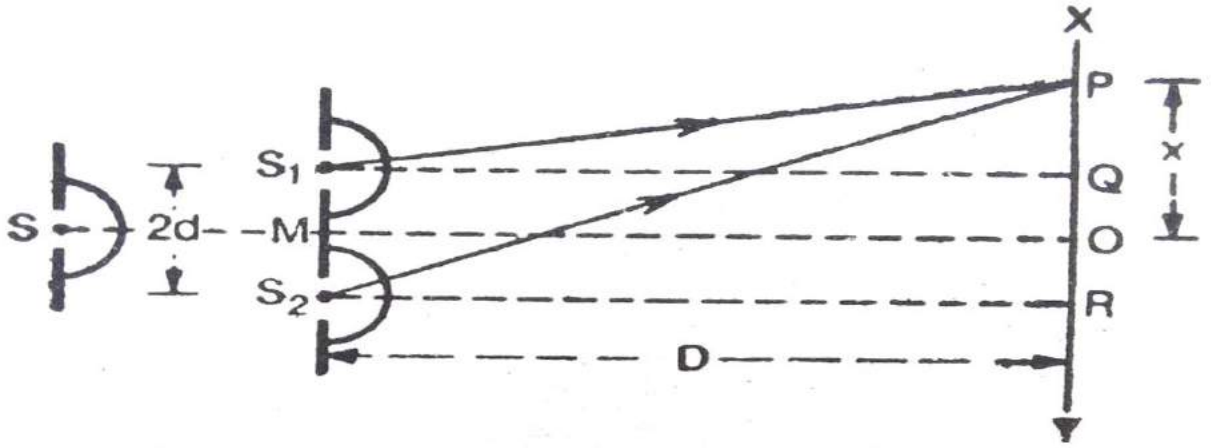


Later on,

- ➢ To ***increase brightness*** – pin holes were replaced by narrow slits
- ➢ To ***increase*** the number of ***fringes*** – sunlight was replaced by monochromatic light


Incident wave
S_0
S_2
S_1
θ
y
D
Max
Max
Max
Max
Max
Max
Max
Max
Max
Max
m=3
m=2
m=1
m=0
m=1
m=2
m=3

Intensity at a point in a plane:



S = monoch. source of light
S_1 & S_2 = 2 narrow pin holes equidistant from S
→ waves reaching at S_1 and S_2 will always be in 'phase'.

Now let us determine the res intensity at a unknown pt 'P' on the screen due to interference.

Let a_1 & a_2 = amp of waves from S_1 & S_2

δ = phase difference between 2 waves reaching pt 'P'

ω = angular frequency

y_1 & y_2 = displacement of the waves due to S_1 & S_2 then,

$$y_1 = a_1 \sin \omega t \qquad (1)$$

$$y_2 = a_2 \sin(\omega t + \delta) \qquad (2)$$

According to principle of superposition,

$$y = y_1 + y_2 = a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$$= \sin\omega t\,(a_1 + a_2\cos\delta) + \cos\omega t\,(a_2\sin\delta) \qquad (3)$$

Consider, $(a_1 + a_2\cos\delta) = R\cos\theta \qquad (4)$

$(a_2\sin\delta) = R\sin\theta$ (where R & θ are const.) $\qquad (5)$

Substituting in the above eq (3) we get,

$$y = \sin\omega t\, R\cos\theta + \cos\omega t\, R\sin\theta$$

$$\boxed{y = R\sin(\omega t + \theta)} \qquad (6)$$

This eq represents a ***SHM*** with amplitude R.

Therefore, squaring and adding eq (4) & (5) we get,

$$R^2\cos^2\theta + R^2\sin^2\theta = (a_1 + a_2\cos\delta)^2 + (a_2\sin\delta)^2$$

$$R^2 = a_1^2 + a_2^2\cos^2\delta + 2a_1a_2\cos\delta + a_2^2\sin^2\delta$$

$$\boxed{I = R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta} \qquad (7)$$

The phase difference between 2 waves reaching at P is given by

$$\delta = \frac{2\pi}{\lambda}\,(\text{path difference})$$

$$\delta = \frac{2\pi}{\lambda}\,(S_2P - S_1P) \qquad (8)$$

<u>Condition for I_{max} :</u> We know, $I = R^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos\delta$

Intensity will be max if $\cos\delta = +1$

Implies, phase diff $\delta = 2n\pi$, where n = 0, 1, 2, 3…..

Or $\delta = 0, 2\pi, 4\pi,$ …..

Therefore, path diff, $(S_2P - S_1P) = n\lambda$ (9)

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$$

$$I_{max} = (a_1 + a_2)^2 \qquad (10)$$

Result: Intensity is ***greater than*** the sum of two individual intensities i.e., $a_1^2 + a_2^2$

Condition for I_{min} : We know, $I = R^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \delta$

Intensity will be min, if $\cos \delta = -1$

phase diff $\delta = (2n + 1)\,\pi$, where $n = 0,1,2,\ldots.$

Or $\delta = 1,3, 5,$

Therefore, path diff $(S_2P - S_1P) = (2n+1)\frac{\lambda}{2}$ (11)

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$$

$$I_{min} = (a_1 - a_2)^2 \quad (12)$$

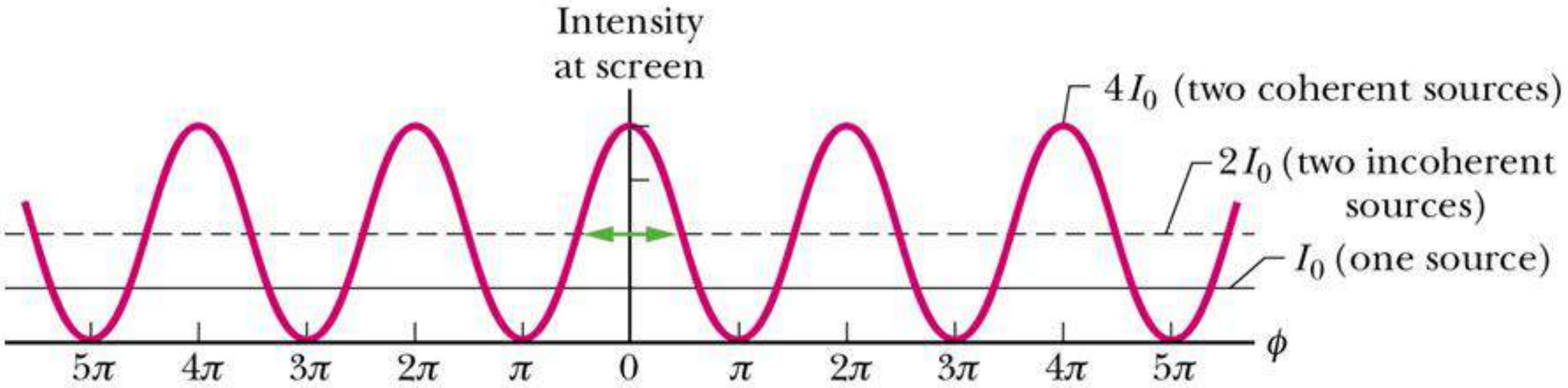
Result: Intensity is ***less than*** the sum of two individual intensities i.e., $a_1^2 + a_2^2$

Special case: when $a_1 = a_2 = a$ (say) then

$I_{max} = (a + a)^2 = 4a^2$ (constructive interference)

$I_{min} = (a - a)^2 = 0$ (destructive interference)

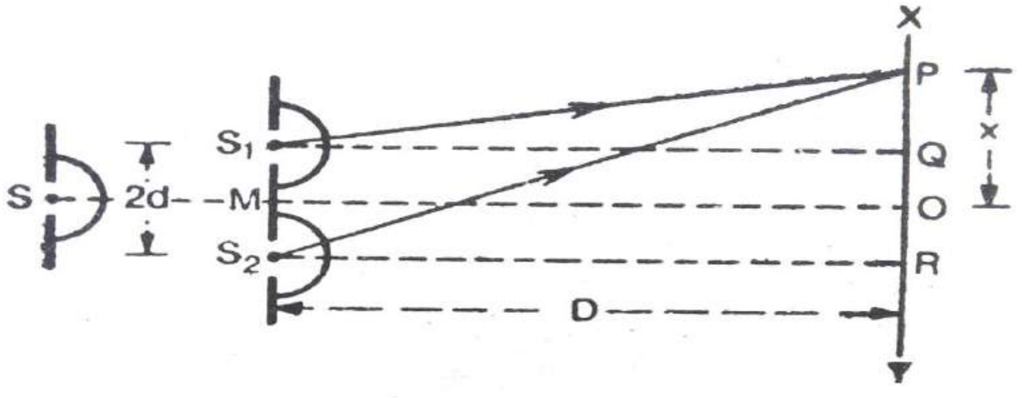
What about the intensity of light along the screen?



Note:

- The formation of interference fringes is in accordance with the 'Law of Conservation of Energy'
- There is no destruction of E as a result of interference
- I_{max} increases at the expense of I_{min}
- No violation of the Law of conservation of E in the phenomenon of interference.

Conditions for bright and dark fringes:



2d=dist between 2 sources S_1 and S_2

D=dist between sources and the screen

x = dist of pt 'P' from 'O'

Pt 'O' is equidistant from S_1 and S_2 and hence, the path diff = 0.

Therefore, the pt 'O' is the pt of I_{max}

Now, we will determine the conditions for bright and dark fringes at 'P'

From ΔS_1QP, $(S_1P)^2 = (S_1Q)^2 + (QP)^2$

$= D^2 + (x - d)^2$

From ΔS_2PR, $(S_2P)^2 = D^2 + (x + d)^2$

Therefore, $(S_2P)^2 - (S_1P)^2 = (x + d)^2 - (x - d)^2 = 4xd$

$(S_2P - S_1P)(S_2P + S_1P) = 4xd$

In Young's expt D >> 2d or x hence, $(S_2P + S_1P) \approx 2D$

Therefore, $(S_2P - S_1P)\, 2D = 4xd$

That implies, $(S_2P - S_1P) = \frac{2xd}{D}$ (1)

Conditions for bright fringes: Pt ‘P’ will be bright if path diff = nλ (n=0,1,2,3,…..)

$$(S_2P - S_1P) = n\lambda$$

$$n\lambda = \frac{2xd}{D}$$

$$\boxed{x = \frac{n\lambda D}{2d}} \qquad (2)$$

This give distance of bright fringes from pt ‘O’

At pt ‘O’, path diff = 0 that implies, central bright fringe

For n=1, $x_1 = \frac{nD}{2d}$ 1st order

And n=2, $x_2 = \frac{2\lambda D}{2d}$ 2nd order and so on

Therefore, the distance 2 consecutive bright fringes is

$$x_2 - x_1 = \frac{2\lambda D}{2d} - \frac{\lambda D}{2d}$$

$$\boxed{x_2 - x_1 = \frac{\lambda D}{2d} \text{ (for bright)}} \qquad (3)$$

Condition for dark fringes: The pt P will be dark if path diff = $(2n+1)\frac{\lambda}{2}$ (n = 0, 1, 2,….)

$$(S_2P - S_1P) = (2n+1)\frac{\lambda}{2}$$

$$(2n+1)\frac{\lambda}{2} = \frac{2xd}{D}$$

$$\boxed{x = \frac{(2n+1)\lambda D}{4d}} \qquad (4)$$

This equation gives the distance of dark fringes from pt 'O'

For n=0, $x_0 = \frac{\lambda D}{4d}$

For n=1, $x_1 = \frac{3\lambda D}{4d}$ and so on

Therefore, distance between 2 consecutive dark fringes will be

$$\boxed{x_2 - x_1 = \frac{\lambda D}{2d} \text{ (for dark)}} \qquad (5)$$

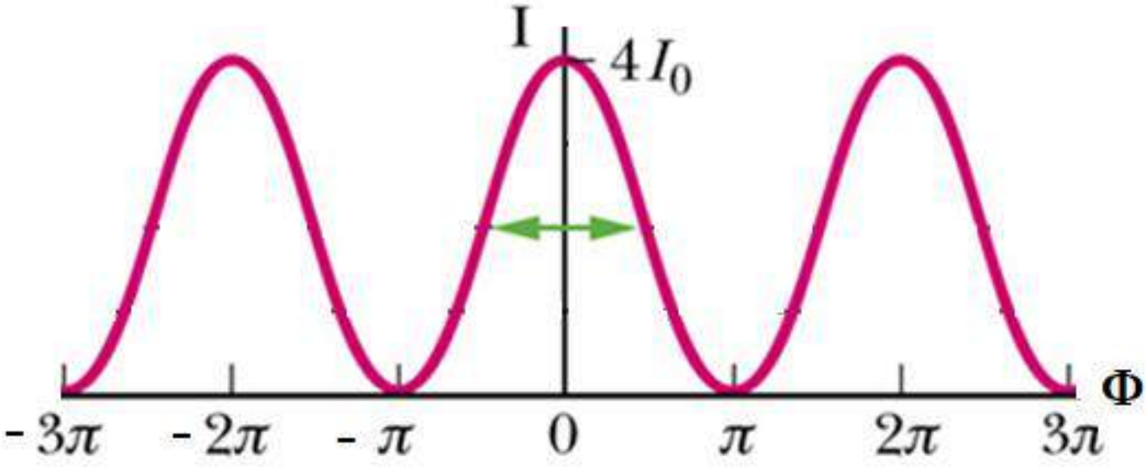
From eq (3) and eq (5), it can be concluded that the spacing between any 2 consecutive "bright" or "dark" fringes is same and = $\frac{\lambda D}{2d}$

This is usually called "fringe width (β)"

$$\boxed{\beta = \frac{\lambda D}{2d}}$$

Fringe width ∝ λ and D

∝ 2d



What happens to the fringe width if the YDS experiment is carried out under water (having ref index, μ = 1.5) instead of air?

Conditions for interference of light:

The conditions for stationary interference pattern are classified into 3 parts.

1. <u>Condition for sustained interference</u>

- The 2 sources must be ***coherent*** (i.e., they should vibrate with same phase or constant phase difference between them)
- The 2 sources must emit ***continuous waves*** with same λ and time period.

2. <u>Condition for observation of fringes</u>

- The ***separation*** between 2 sources should be small.
- The ***distance*** between sources and screen should be large
- The ***background*** should be dark.

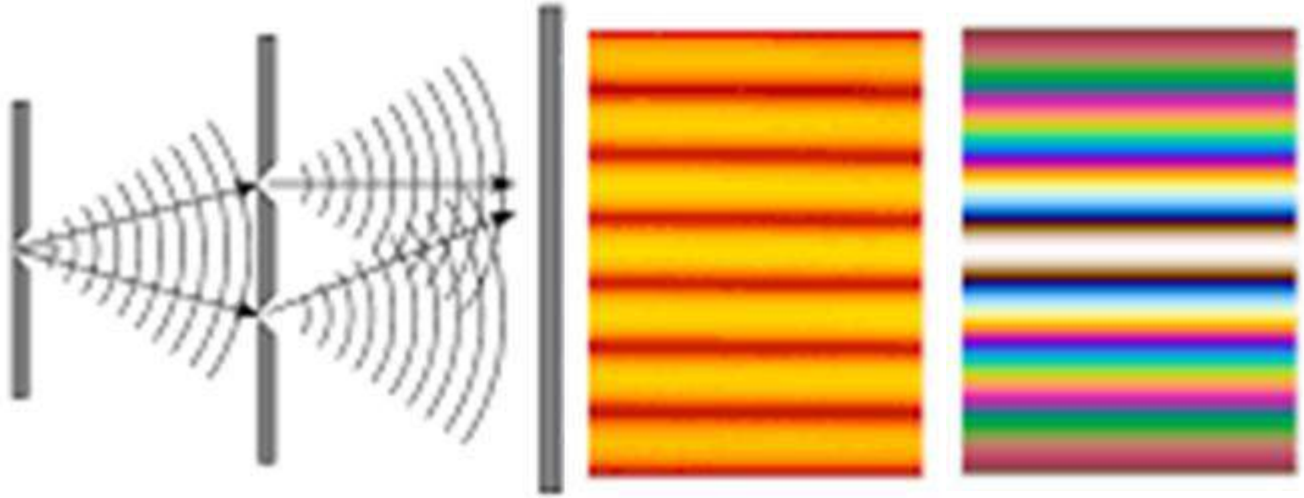
3. **<u>Condition for good contrast between maxima and minima</u>**

- The ***amplitudes*** of interfering waves should be equal
- The sources must be extremely ***small***/narrow.
- The sources must be ***monochromatic***.

Instead of monochromatic light, if a ***white light is used***, the central fringe is ‘white’. There exists a few colored fringes on both sides of central fringe.

Blue color – nearer to central fringe

Red color – farther from central fringe



How to determine the location of zero order fringe in case of slit illuminated by a monochromatic light?

Classes of Interference

Division of Wavefront

The incident **wavefront** is divided into 2-parts by utilizing the phenomenon of reflection, refraction or diffraction. These 2-parts travel **unequal distances and reunite** at some angle to produce interference bands.

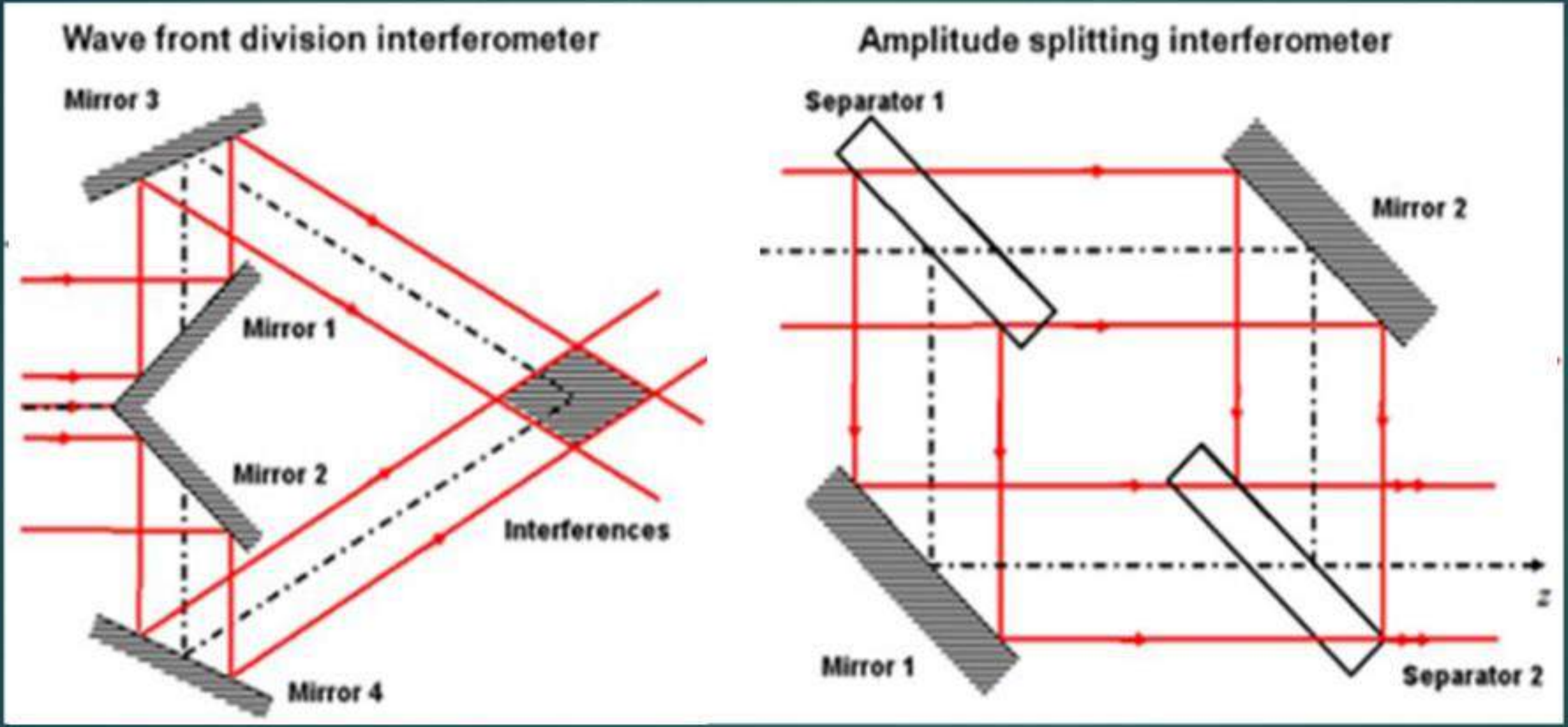
Exs: Lloyd's Mirror
Fresnel's Biprism

Division of Amplitude

The **amplitude** of the incoming beam is divided into 2-parts either by parallel reflection or refraction. These 2-parts travel **different paths and reunite** to produce interference. It's not essential to use a point or a narrow source but a broad light source could be used to produce bright bands.

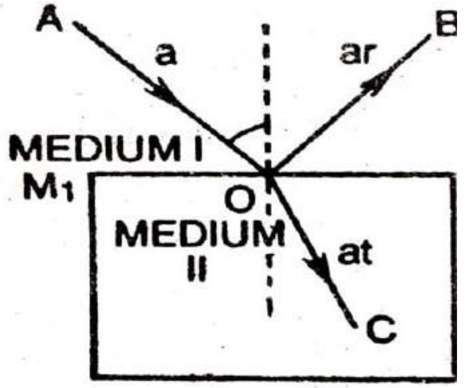
Exs: Thin-film interference
Newton's rings
Michelson's interferometer
Fabry-Perot interferometer

Division of Wavefront & Division of Amplitude:

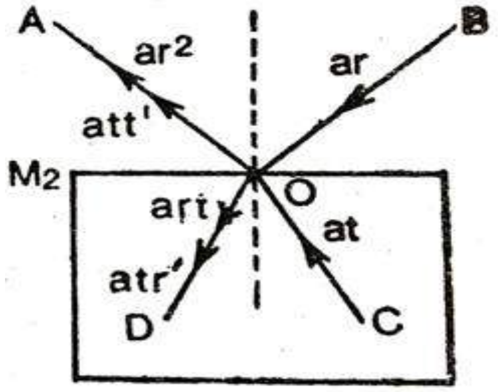


Phase change on reflection:

According to Stoke's law, when a light ray is reflected at the surface of an optically denser medium it suffers a phase change of **π (or a path diff of $\lambda/2$)**



Here, a = amplitude of the light wave
r = reflection coefficient (rarer to denser)
r' = reflection coefficient (denser to rarer)
t = transmission coefficient (rarer to denser) and
t' = transmission coefficient (denser to rarer)
Therefore, amplitude of reflected wave is 'ar' and that of transmitted wave is 'at'.



Now suppose that the direction of reflected and transmitted rays are **reversed** (as shown in 2nd figure).

Then the amplitude of OA will become $ar^2 + att'$ which is nothing but 'a'

Therefore, $ar^2 + att' = a$ (1)

Similarly, $art + atr' = 0$ (2)

As there was no wave originally present along OD

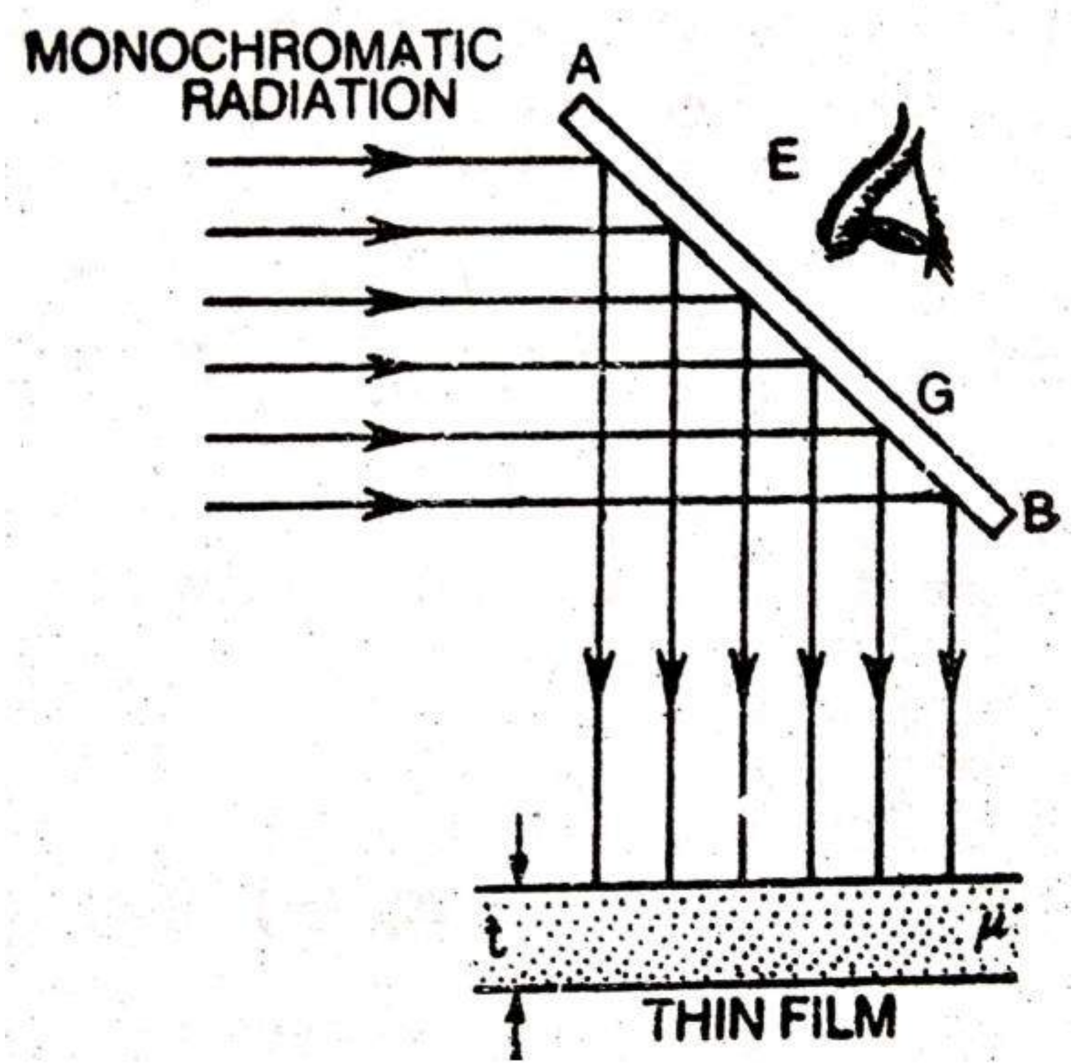
From eq(2), $r' = -r$ (3)

From eq(1), $tt' = 1-r^2$ (4)

Note: The –ve sign in eq(3) indicates the phase change of π upon reflection from denser to rarer medium.

Interference by Division of Amplitude

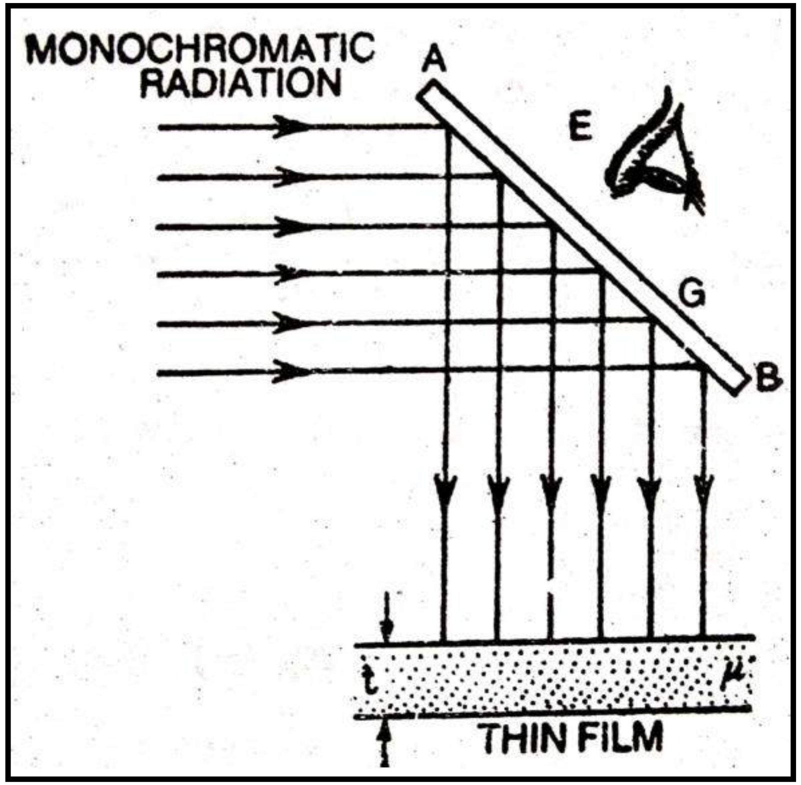
Ex. Thin film interference (Normal incidence):



Thin film interference (Normal incidence):

The plane wavefront is partly reflected at the upper surface of the transparent film and partly transmitted, and again reflected at the bottom surface. Both waves interfere and produce interference pattern observed with eye.

The wave reflected from lower surface of the film traverses an additional path of μt+μt (i.e. upper to lower surface and return).



When film is placed in air, the wavefront reflected from upper surface undergoes a phase change of π (because of reflection from surface of denser medium)

Note: There will NOT be phase change taking place at the lower surface because reflection takes place at surface of denser to rarer medium. Thus, the conditions of max and min are ***changed*** (***reversed***).

Therefore, $2\mu t = n\lambda$ for destructive interference

$2\mu t = (2n+1)\lambda/2$ for constructive interference

(Where n = 0, 1, 2, 3, …..)

Thus,

film appears Bright when the path difference

$$2\mu t = (2n+1)\lambda/2$$

film appears Dark when the path difference

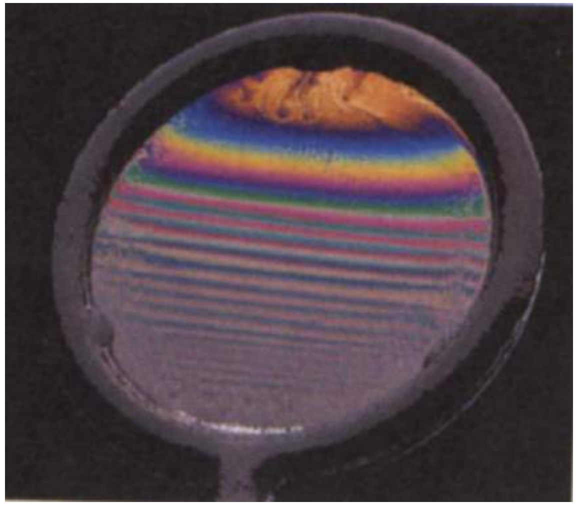
$$2\mu t = n\lambda$$

(Where n = 0, 1, 2, 3, …..)

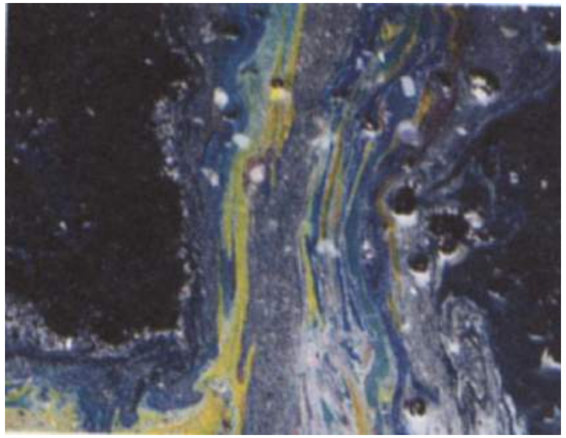
Interference in thin films / Colors in thin films

- Thin film interference patterns seen in

Thin film of soapy water



Seashell



A thin layer of oil on the Water of a street puddle

Colors in thin films

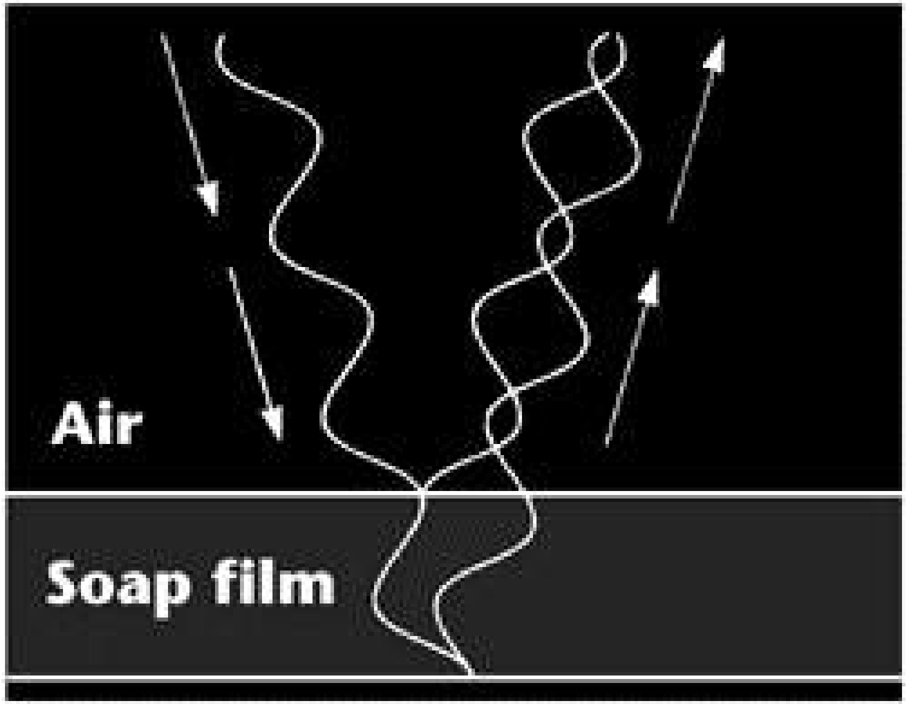
It's known that when a thin film is ***exposed to white light*** source beautiful colors are observed. The incident light was split up by reflection at the top and bottom of the film, the interference of these rays is responsible for colors.

The appearance of dark or bright depends on μ, t and r.
In case of white light, t and r are constants and μ varies with λ.

At a particular point of the film and for a particular position of eye, the interfering rays of ***only certain colors*** will have a path difference satisfying the condition of bright fringe. Hence, only such wavelength will be seen over there. Other neighboring λs will be present with diminished intensity. In other words, for these other colors, condition of minima is satisfied.

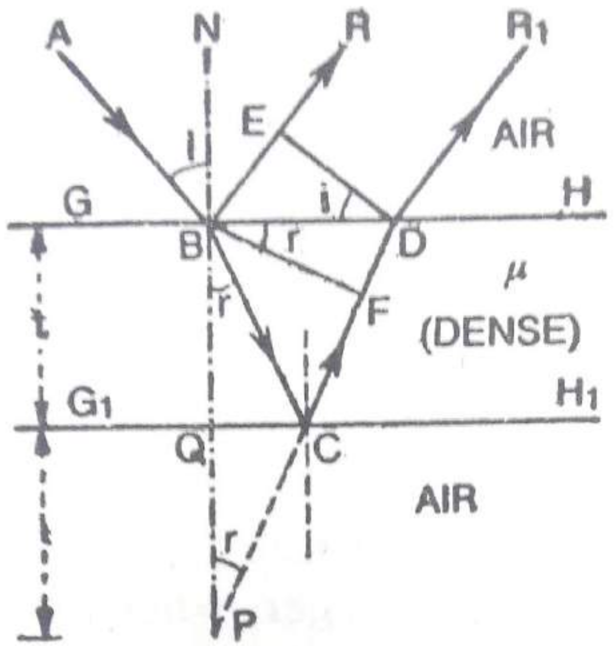
Similarly, if the same point of the film is observed with eye in different positions, a different set of colors is observed each time.

We have already discussed that the conditions of maxima and minima in transmitted lights are ***reversed*** that of reflected light. Hence, the colors suppressed or absent in the reflected light appears as intense colors in transmitted light.



Finally, it could be concluded that '***Colors of reflected and transmitted lights are complementary***'.

Interference due to reflected light: The Cosine Law



Let AB be the incident monochromatic ray, which is partly reflected along BR and refracted along BC into the 'dense' medium having ref index μ.

After one internal reflection at C, we get CD ray, which after refraction at D finally emerges out along DR_1 in 'air' medium.

Our aim is to find out the optical path diff (Δ) between the reflected rays BR and DR_1 given by:

Δ = Path (BC + CD) in film – Path BE in air

= μ(BC + CD) – BE (1)

From Snell's law, we know $\mu = \frac{\sin\ i}{\sin\ r}$

$= \frac{BE/BD}{FD/BD}$ = BE/FD

That implies, BE = μ FD (2)

Substituting eq (2) in eq (1), we get,

Δ = μ(BC + CD) – μ FD

= μ(BC + CF + FD) – μ FD

= μ(BC + CF)

= μ(PF) (since BC = PC)

= μ(2tcos r) (From the triangle BPF, PF = 2t cos r)

Therefore, Δ = 2μtcos r

This optical path difference is generally known as the ***Cosine Law***.

It should be remembered that a ray reflected at a surface backed by a denser medium suffers an abrupt phase change of **π**, which is equivalent to a path diff of ***λ*/2**

Thus, the effective optical path diff between the two reflected rays is = $2\mu t\cos r \pm \lambda/2$

We know that, the maxima occurs when the effective path diff,

$$\Delta = n\lambda$$

Therefore, $2\mu t\cos r \pm \lambda/2 = n\lambda$

That implies, $\boxed{2\mu t\cos r = (2n \pm \lambda/2)}$

If this condition is fulfilled, the film appears ***bright*** in the reflected light.

We know that, the minima occurs when the effective path diff,

$$\Delta = 2n \pm \lambda/2$$

Therefore, $2\mu t\cos r \pm \lambda/2 = 2n \pm \lambda/2$

That implies, $\boxed{2\mu t\cos r = n\lambda}$

If this condition is fulfilled, the film appears ***dark*** in the reflected light.

Note that in all the above cases, n = 0, 1, 2, 3, …. etc.

Application: Blooming of Lenses

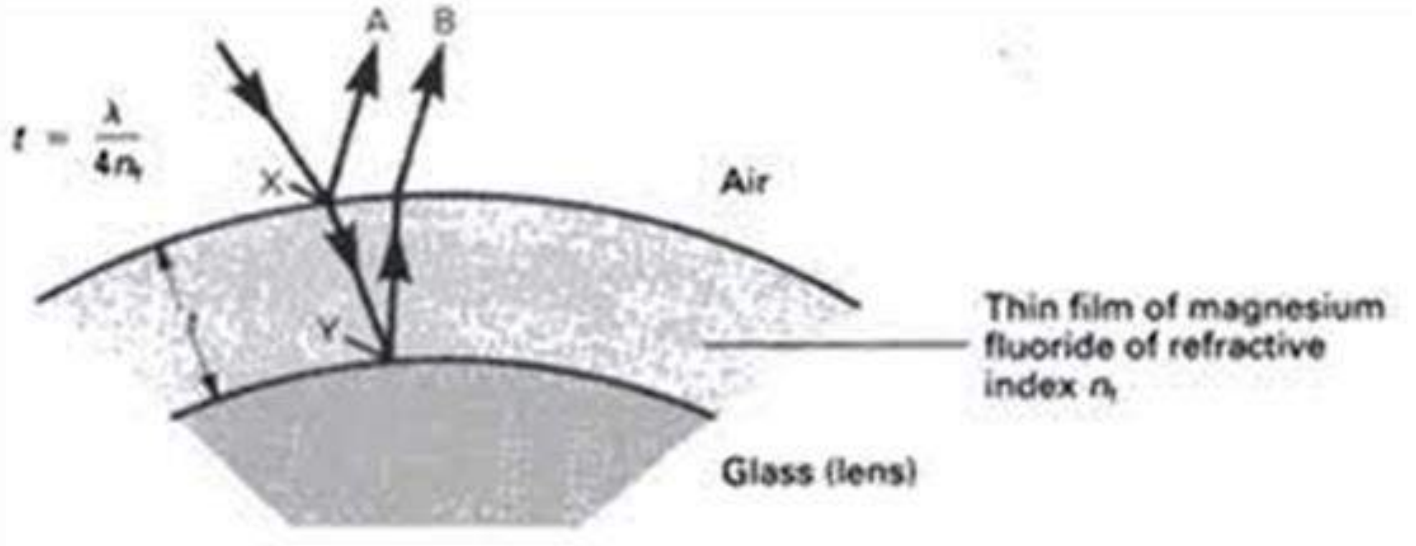
- The process of coating a non-reflecting film on the lens is called **blooming**.
- A very thin coating on the lens surface can reduce reflections of light considerably.
- Usually the camera lens are made up of MgF_2 glass with a protective plastic sheet over it.

http://users.erols.com/renau/thinfilm.html

- For a typical sample of MgF2 the refractive index at 632.8 nm is **1.38288**

https://www.shutterbug.com/content/why-lens-coatings-are-so-important-these-lovely-layers-are-more-just-pretty-face

- The amount of reflection of light at a boundary depends on the **difference in refractive index** between the two materials.

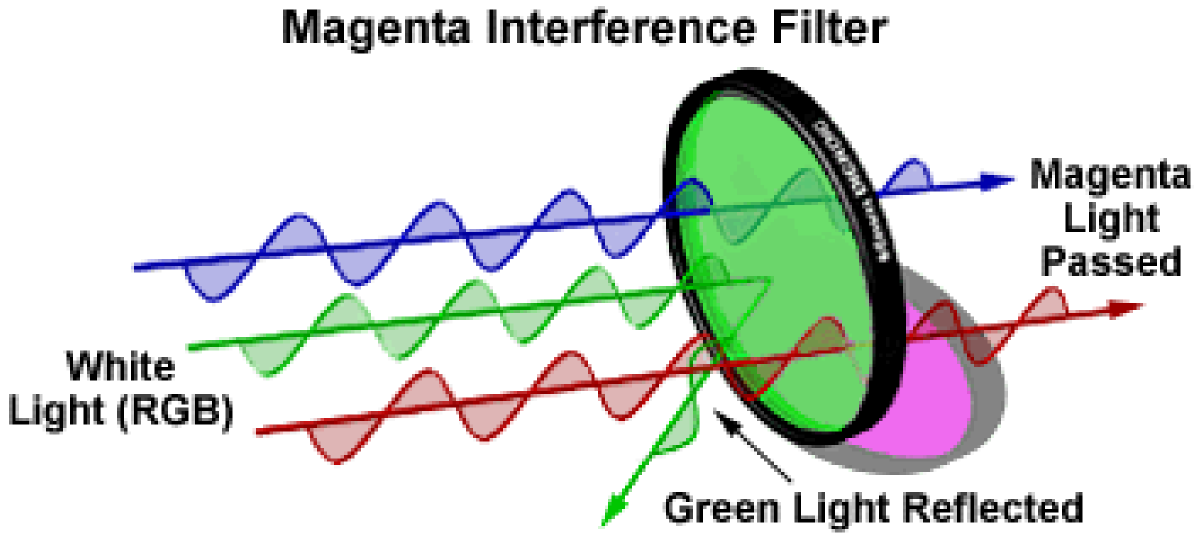


- Ideally, the coating material should have **lower refractive index** than the lens over which it was coated. Then destructive interference can occur nearly completely for one particular wavelength.

http://www3.ltu.edu/~s_schneider/physlets/main/thinfilm.shtml

- The thickness of the film is chosen so that light reflecting from the front and rear surfaces of the film destructively interferes.
- For cancellation of reflected light,

$$2t = \frac{1}{2}\left(\frac{\lambda_o}{\mu}\right)$$



http://www3.ltu.edu/~s_schneider/physlets/main/thinfilm.shtml

LASER INTERFEROMETER

Laser Interferometry

Young's expt gave quite accurate measurement of λ of light. With a more carefully designed equip., the optical interferometry has become the most accurate measurement technique in Physics.

<u>Ex</u>: With a strain sensitivity $\left(\frac{\delta l}{l}\right)$ of 10^{-22} achieved by LIGO, interferometry has now become the most sensitive measurement method in Physics.

In this section, we shall discuss a few interferometers based on interference of two separate waves and their applications using lasing sources.

Optical Devices: Laser Interferometer

- An instrument used for high precision measurements (Ex: distances, angles etc.)
- It uses interferometry as the basis for measurement.
- It uses a very small, stable and accurately defined λ of laser as a unit of measure.

<u>Components of Laser Interferometry</u>:

i. Two freq laser source
ii. Optical elements (beam splitters, beam benders, retro reflectors)
iii. Measurement receiver
iv. Measurement display

2-freq laser source:

It's generally a He-Ne laser that generates stable, coherent light beam of 2-frequencies.

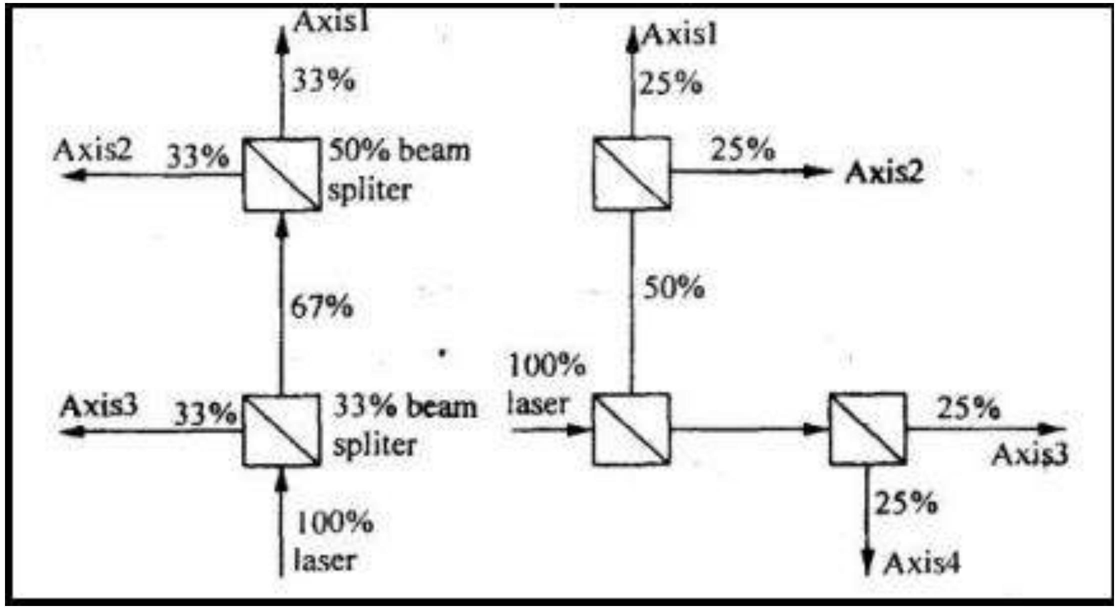
- One polarised vertically
- Another polarised horizontally relative to the plane of mounting.

The laser oscillates at two-freq (slightly different) by a cylindrical permanent magnet around the cavity.

The two freq components are distinguishable by their opp circular polarisation.

Beam splitter:

The component that divides the laser output along different axes (or) the component that divides the laser beam into separate beams is called "***Beam Splitter***"



Why do we need a Beam splitter?

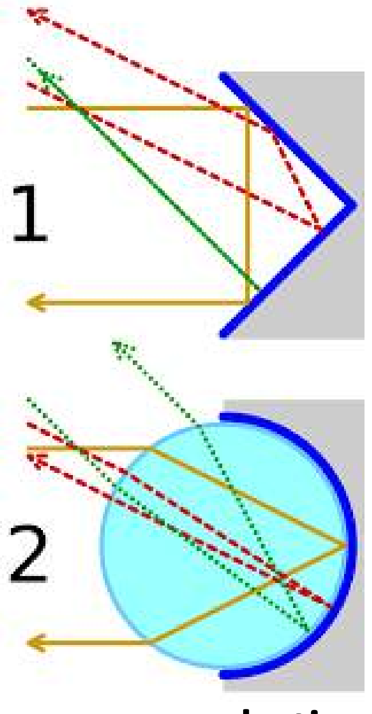
To avoid attenuation, it's essential that the beam splitters must be oriented such that the reflected beam is **perpendicular** to transmitted beam. By doing so, these 2-beams are **coplanar** with one of the polarisation vectors of the input laser.

Beam bender:

- These are used to deflect the light beam around the corners on its path from the laser to each axis.
- These are actually flat mirrors having very high reflectivity.
- These are designed such that they are meant for 90^o beam deflections to avoid disturbing the polarising vectors.

Retro reflector:

- These could be plane mirrors, roof prism or cube corner reflectors.
- Cube corners are 2-mutually perpendicular plane mirrors and the reflected beam is ***always parallel*** to the input laser.



www.maxbotix.com

Measurement receiver:

- To detect reflected beam and
- Doppler shifted frequency component.

Measurement display:

- A microcomputer to computer and display results.
- Calculations and analysis could also be done.

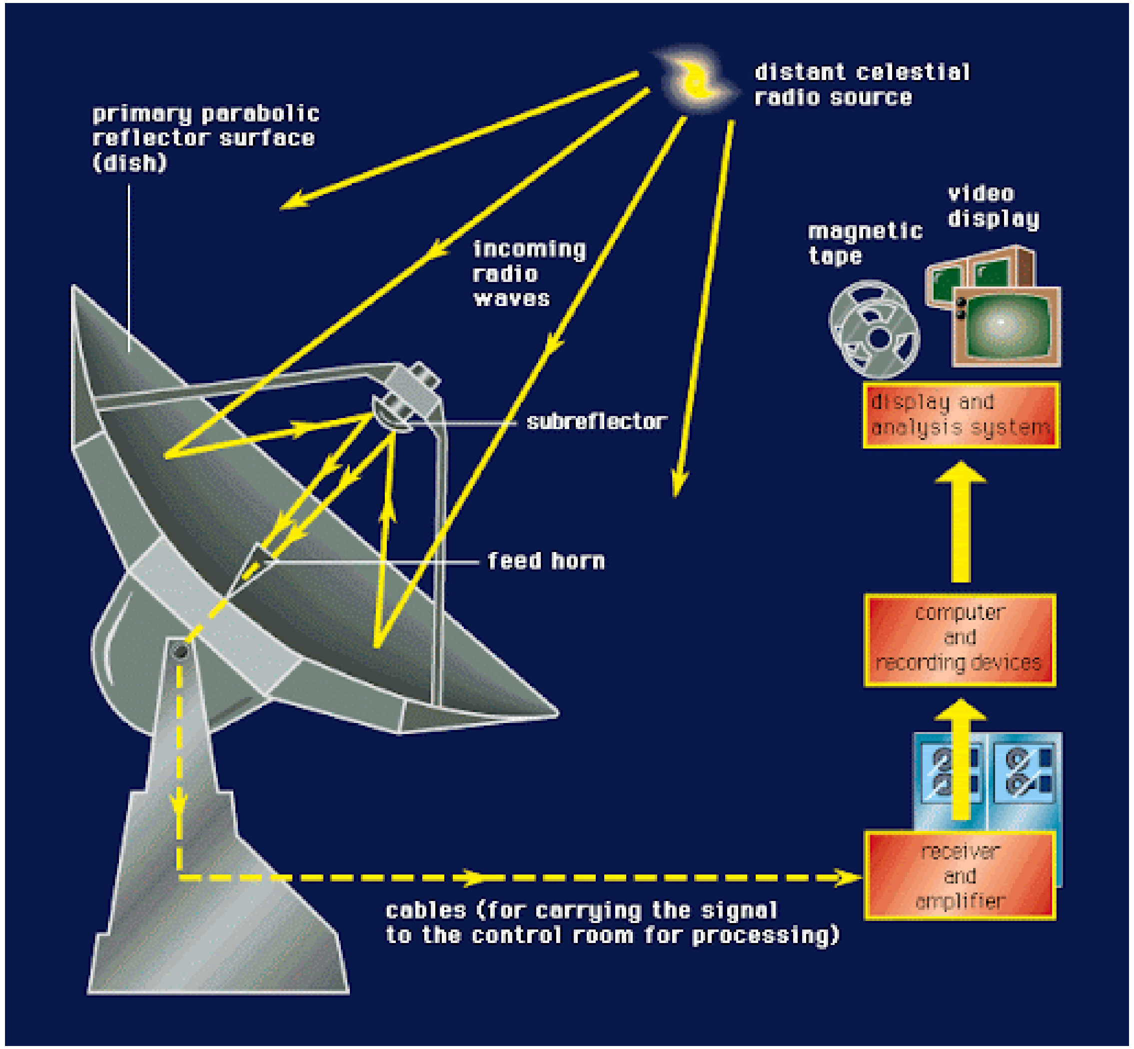


Image courtesy: https://abyss.uoregon.edu

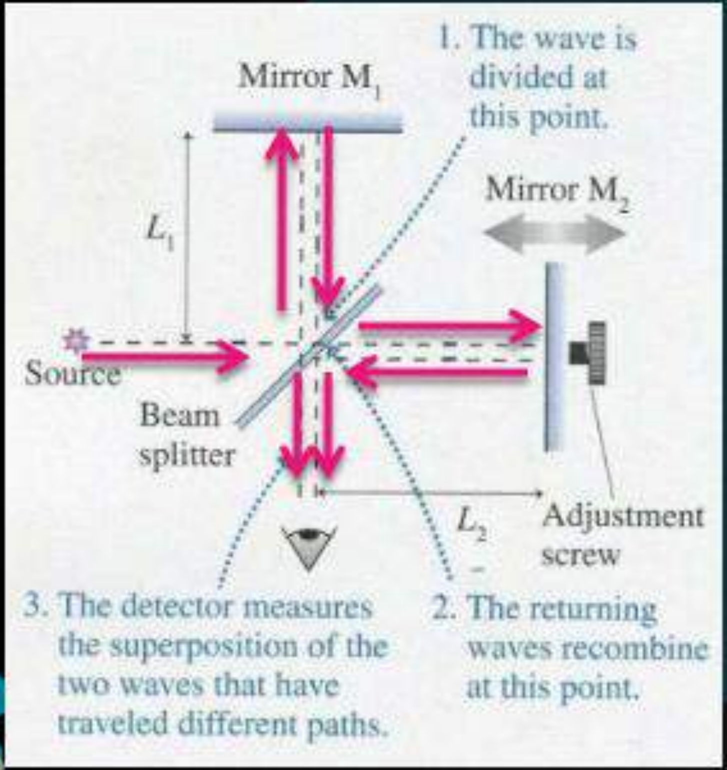
Principle of Michelson Interferometer

- Albert Michelson (1852~1931)
  - the first American scientist to receive a Nobel prize, invented the optical interferometer.
  - The Michelson interferometer has been widely used for over a century to make precise measurements of wavelengths and distances.

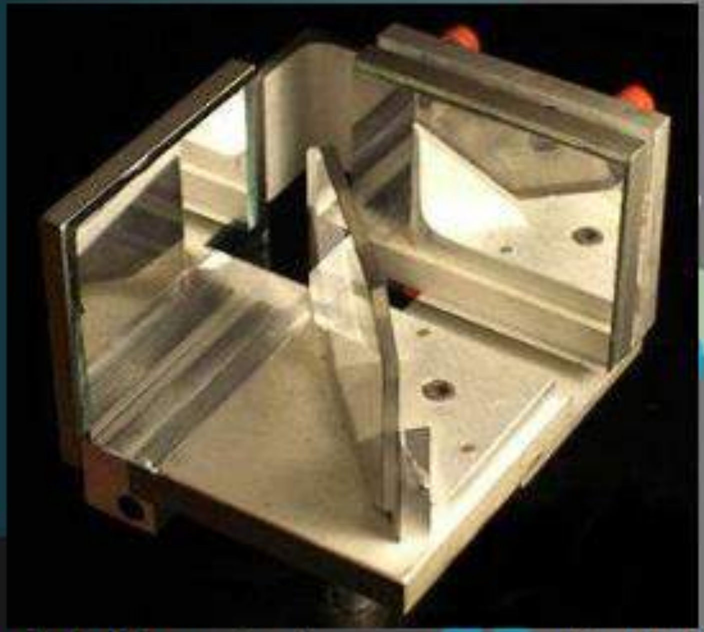
Albert Michelson

Principle of Michelson Interferometer

Michelson Interferometer



1) Separation
2) Recombination
3) Interference



A Michelson Interferometer for use on an optical table

Principle of Michelson Interferometer

- Analyzing Michelson Interferometer
  - The central spot in the fringe pattern alternates between bright and dark when Mirror M2 moves.
  - If we can know the spacing distance of M2 between two sequent central bright spots and the number of central bright spots appeared, then we can calculate how long M2 moved.

Photograph of the interference fringes produced by a Michelson interferometer.

Michelson Interferometer (Qualitative)

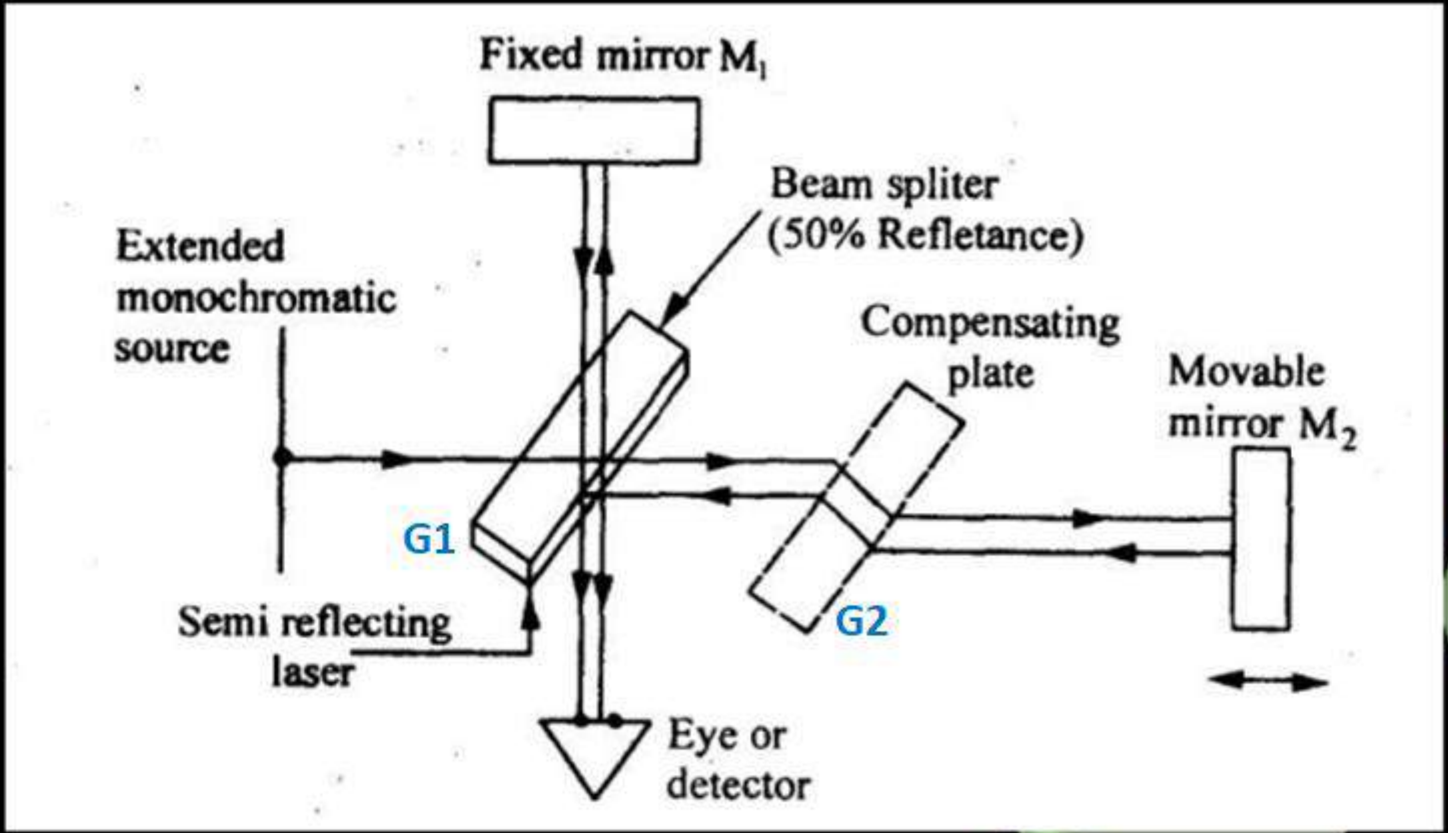
Measurement of wavelength: $$\lambda = \frac{2d}{N}$$

The distance moved by mirror M2 = *d* (say 6.5 µm)

No. of fringes appearing or disappearing = *N* (say 20)

We know that for one fringe to appear or disappear (within marked place), the mirror must be moved through a distance = $\lambda/2$

Knowing this, we can write, $$d = \frac{N\lambda}{2} \Longrightarrow \lambda = \frac{2d}{N}$$

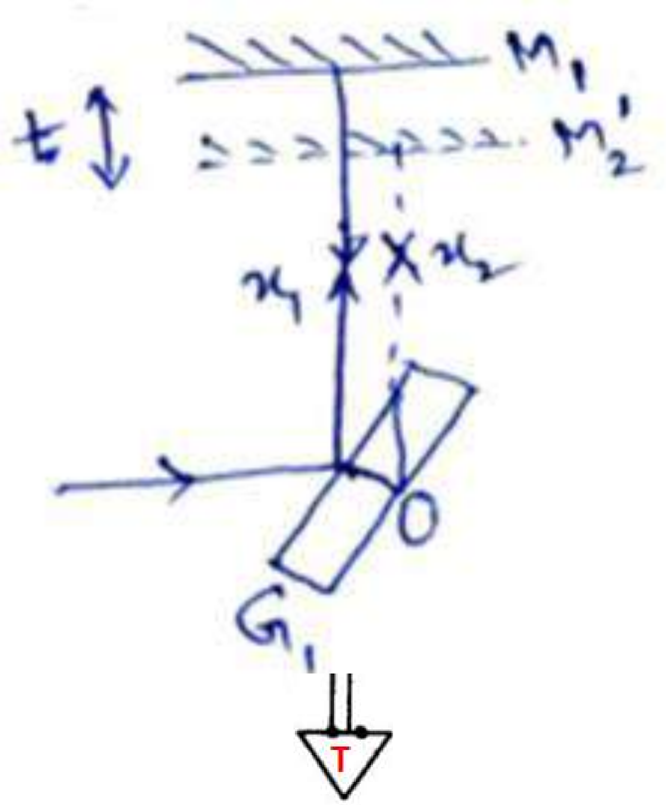


MICHELSON INTERFEROMETER

This instrument can produce both types of interference fringes i.e., *circular fringes of equal inclination at infinity* and *localized fringes of equal thickness*

G_2 is called a "***compensating plate***" because ray-1 passes through G_1 thrice whereas ray-2 only once, before reaching the detector.

Implies, in the absence of G_2, the two paths OM_1 and OM_2 are not equal. Therefore, to equalize these two paths, G_2 has been introduced.

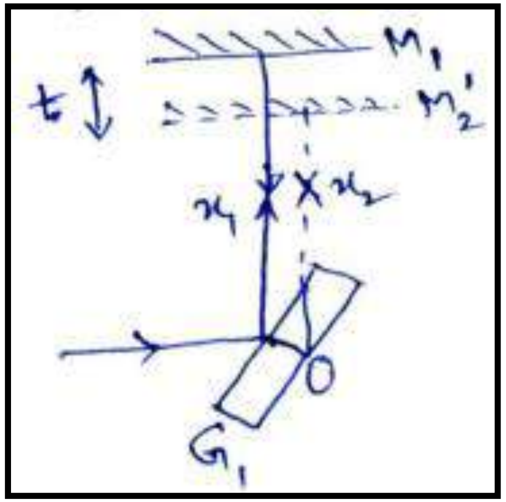


If we observe through telescope 'T', one observes M_1 and virtual image of M_2 (say M'_2) formed in G_1. Let the distance between M_1 and M'_2 be 't' the distance $OM_1 = x_1$ and $OM'_2 = x_2$.

Therefore, path difference $= 2(x_1 \sim x_2) \pm \frac{\lambda}{2}$

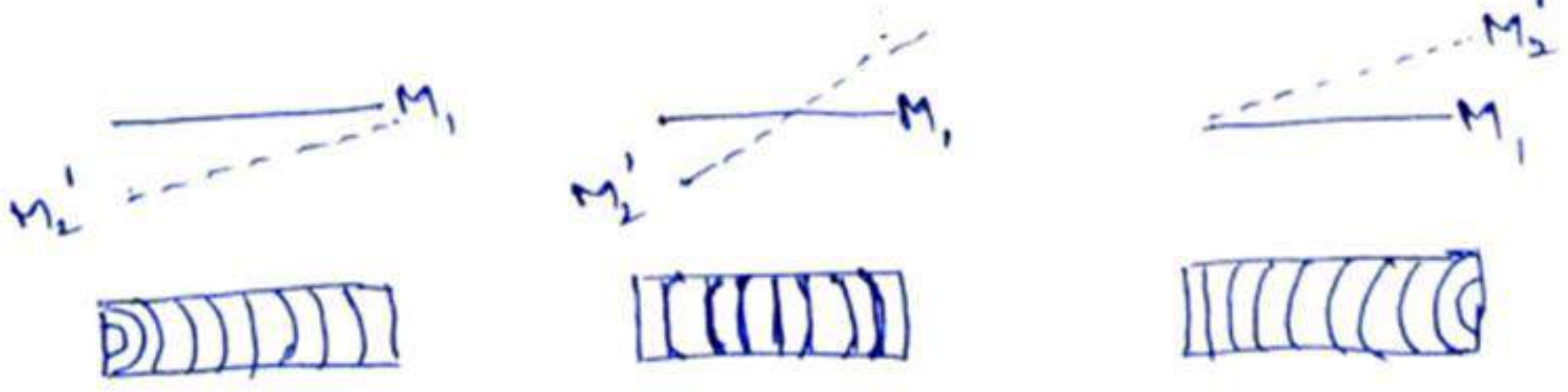
The interference fringes formed here could be ***straight, circular, parabolic etc.*** depending upon path difference and the angle between M_1 and M'_2.

Types of fringes:



1. **Circular fringes:** When M_1 and M_2 are ***perfectly perpendicular*** to each other (or) M_1 and M'_2 are parallel to each other forming an air film of constant thickness 't' between them then we get circular fringes.

2. **Localized fringes:** When M_2 is ***not perpendicular*** to M_1 (or) M_1 and M'_2 are inclined and forms an air film of wedge – shaped then we get the localized fringes. The shape of the fringes ***depends on*** the thickness of the air film and the angle of incidence.



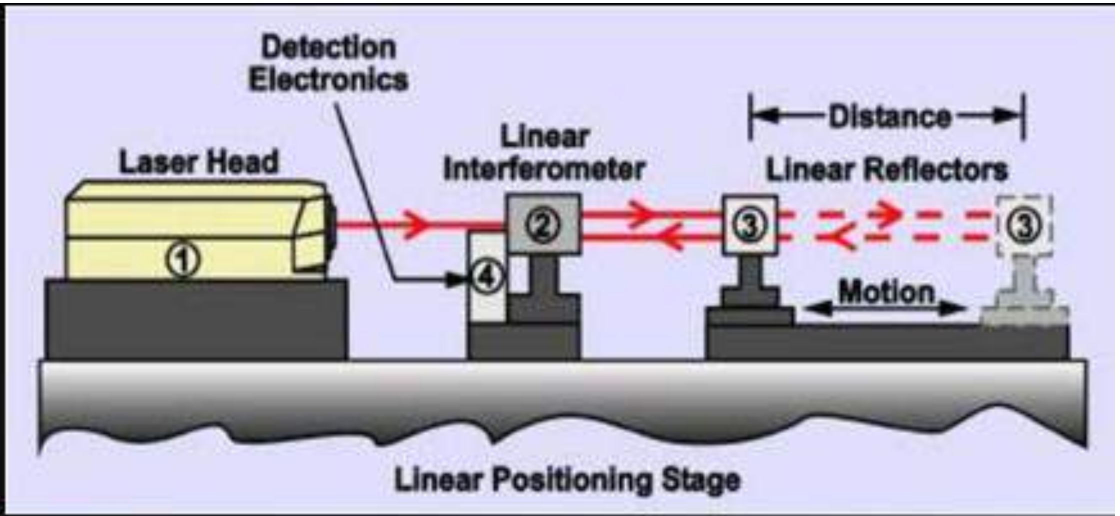
3.<u>Localized white light fringes</u>: When the monochromatic source is ***replaced by*** a white light source, then coloured and curved localized fringes are obtained.

Note that the air film should have ***very small thickness***. The fringes at ***zero thickness*** will be ***dark and straight***. Other fringes are coloured due to overlapping of various colours.

If the air film is ***thick***, uniform illumination is observed with relatively ***no fringes***.

White light fringes are ***useful*** for the determination of zero path difference, especially in the ***standardization*** of “meter”.

Applications

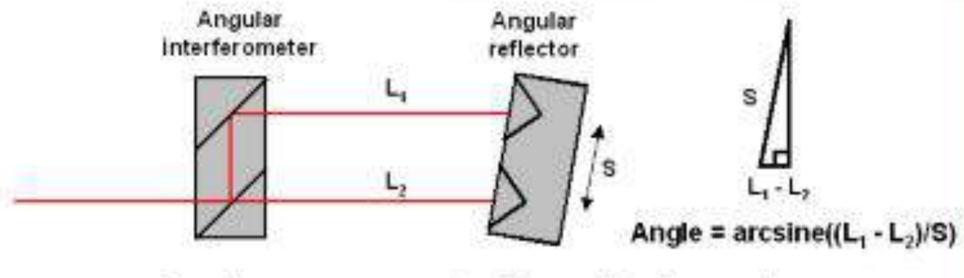


Aerotech's LZR3000 Series Laser Interferometer System

- Measurement of Distance
  - 1) frequency stabilized He-Ne laser tube
  - 2) combination of beam-splitter and retroreflector
  - 3) a moving retroreflector
  - 4) detection electronics`

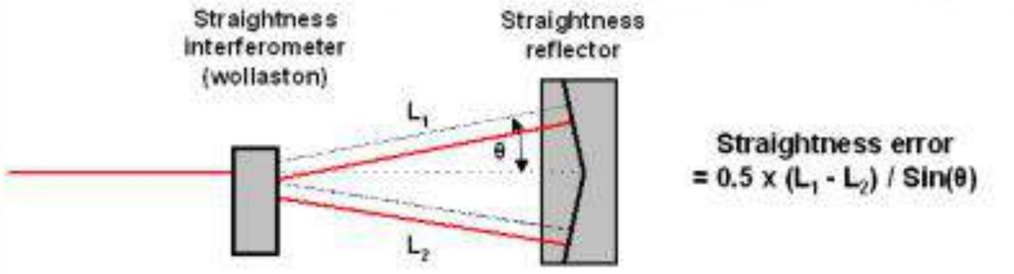
Applications

- Other Applications
  - Measure angles, flatness, straightness, velocity and vibrations, etc.



Angular measurement with an interferometer.

Rearrangements of the light paths



Straightness measurement with an interferometer.

Resolution

- XL-80 Laser Measurement System

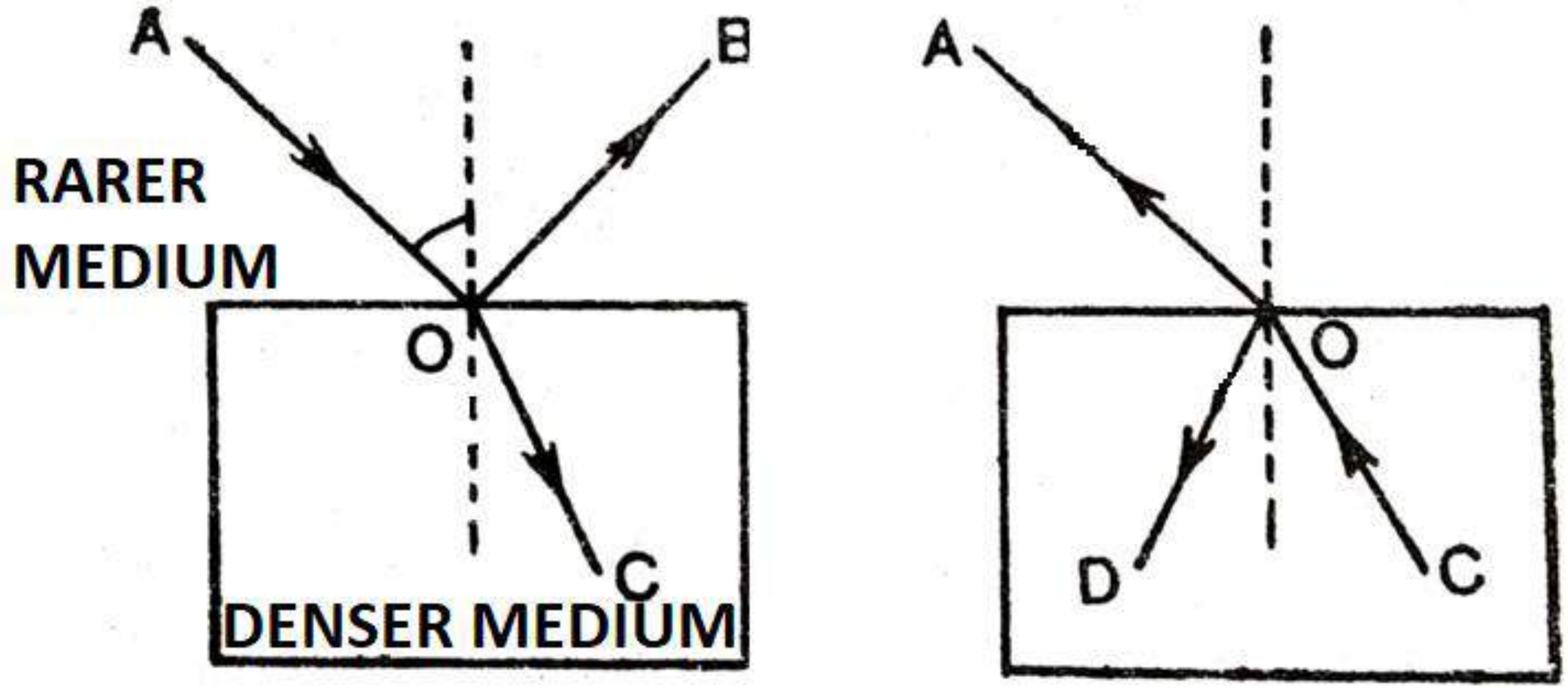
RENISHAW
apply innovation

System performance		
Linear measurement range	80 m	
Linear measurement accuracy	±0.5 ppm	
Laser frequency accuracy	±0.05 ppm	
Resolution	1 nm	
Maximum travel velocity	4 m/s*	
Dynamic capture rate	10 Hz - 50 kHz**	
Preheat time	<6 minutes	
Specified accuracy range	0 °C - 40 °C	
Environmental senors		
	Range	Accuracy
Material temperature	0 °C - 55 °C	±0.1 °C
Air temperature	0 °C - 40 °C	±0.2 °C
Air pressure	650 mbar - 1150 mbar	±1 mbar
Relative humidity (%)	0% - 95% non-condensing	±6% RH

* 1.6 m/s (80 nm quadrature); 0.2 m/s (10 nm quadrature)
** 20 MHz in quadrature mode

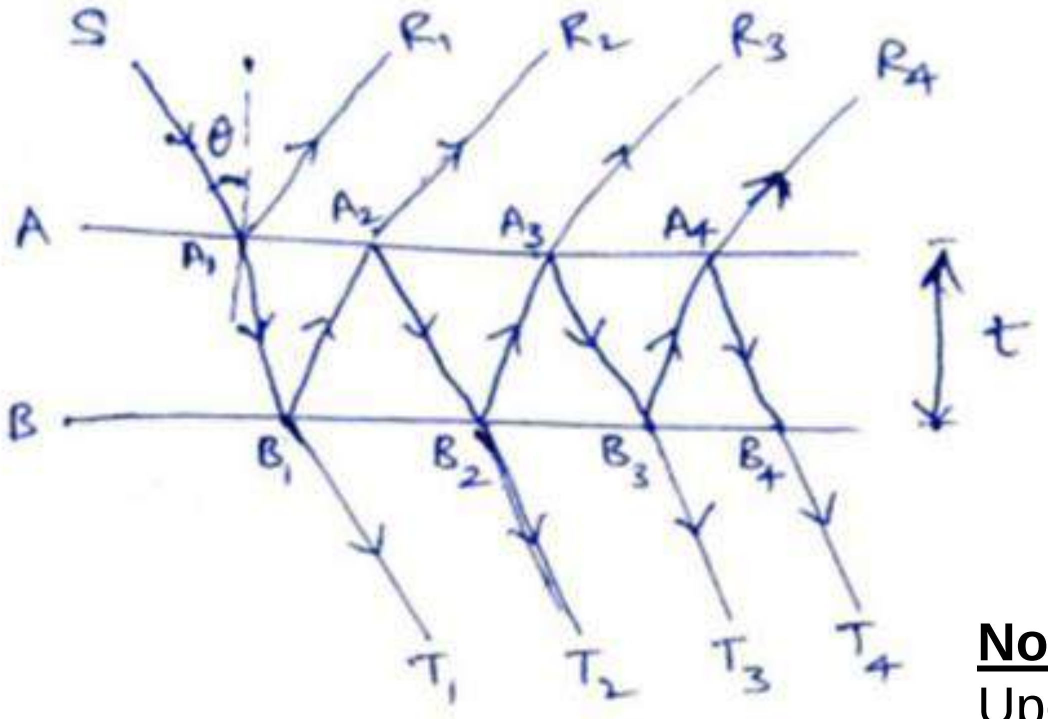
Phase change on Reflection:

According to Stoke's law, when a light ray is reflected at the surface of an optically denser medium it suffers a phase change of **π (or a path diff of λ/2)**



Multiple Beam Interferometry: (interference by multiple reflections)

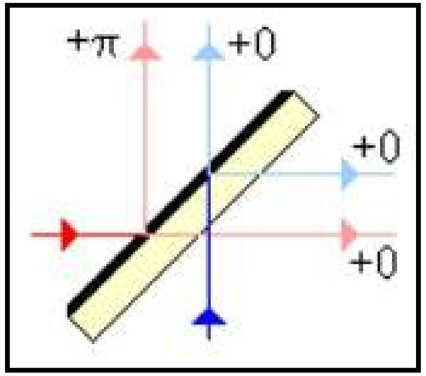
So far we have been discussing the interference between two beams. Now we consider a more general case of interference between parallel beams obtained by multiple reflections between 2 parallel and partially reflecting surfaces shown below:



A & B – surfaces are partly silvered from inside enclosing an air film or a slab of transparent material having thickness 't'.

Note that at each reflection, there is also a partial refraction. In this way multiple beams are obtained in reflected as well as transmitted light.

Note: From rarer to denser medium only:
Upon Transmission – No phase shift
Upon Reflection – phase shift of π (180°)



Consider a plane wave originated from source 'S' with ***unit amplitude*** incident at an angle θ on the glass plate A. Multiple reflections occur between A & B and we obtain a set of

A_1R_1, A_2R_2, A_3R_3, ….. Parallel ***reflected*** rays

B_1T_1, B_2T_2, B_3T_3, …… Parallel ***transmitted*** rays

Let $\sqrt{R}$ & $\sqrt{T}$ be the reflection and transmission coefficients of amplitude from the surface.

Therefore, amplitude of incident wave SA_1 is unity.

Amplitude of A_1B_1 is $\sqrt{T}$

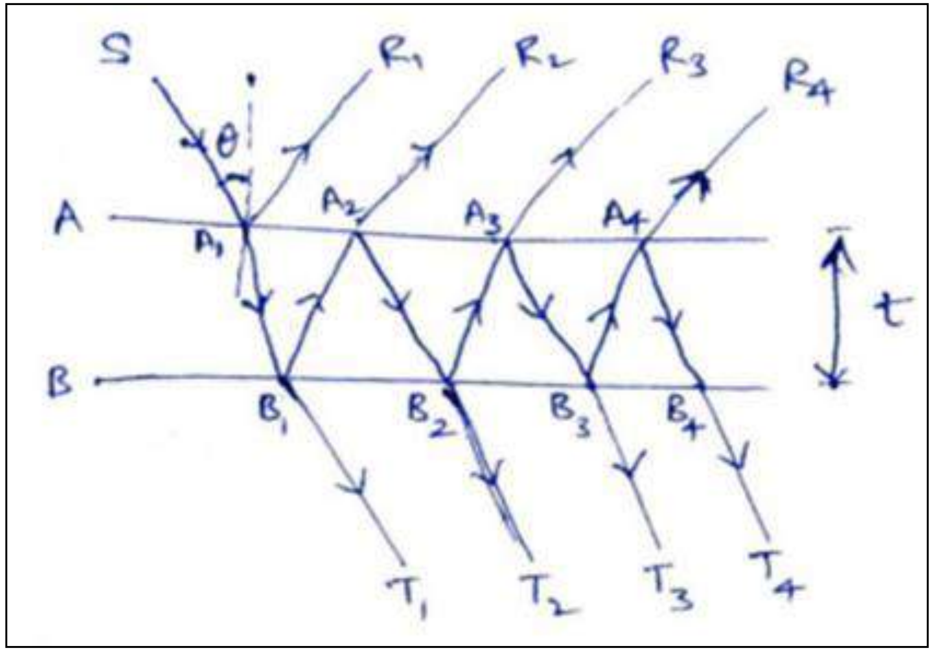
B_1A_2 is $\sqrt{T} * \sqrt{R} = \sqrt{(RT)}$

A_2B_2 is $\sqrt{R} * \sqrt{(RT)} = R\sqrt{T}$ and so on

Amplitude of B_1T_1 is $\sqrt{T}$

B_2T_2 is $\sqrt{(RT)} * \sqrt{(RT)} = RT$

B_3T_3 is $R\sqrt{T} * R\sqrt{T} = R^2T$ and so on



These transmitted waves are from some incident wave, hence they produce interference.

Condition for max intensity:

We know that I = = $\frac{T^2}{(1-R)^2\,[1+\frac{4R}{(1-R)^2}\sin^2\left(\frac{\delta}{2}\right)]}$

(Derivation NOT included)

I_{max} when $\sin^2 \delta/2 = 0$

Implies, $\delta/2 = n\pi$

$\delta = 2n\pi$ where n = 0,1,2,3,.....

Therefore, $\boxed{I_{max} = \frac{T^2}{(1-R)^2}}$

Condition for min intensity:

I_{min} when $\sin^2 \delta/2 = 1$

Implies, $\delta/2 = (2n + 1)\frac{\pi}{2}$

$\delta = (2n + 1)\pi$

Upon substitution, $\boxed{I_{min} = \frac{T^2}{(1+R)^2}}$

The conditions of maxima and minima in terms of path difference are

$$\boxed{\begin{aligned} 2\mu t \cos \Theta &= n\lambda \text{ (maxima)} \\ 2\mu t \cos \Theta &= (2n + 1)\frac{\lambda}{2} \text{ (minima)} \end{aligned}}$$

, where n = 0, 1, 2, ...

(μ=refractive index of air = 1)

Note that 'Θ' is constant for a particular value of 'n', 'λ' and 't'. Hence, the locus of all points having same value of Θ forms circles. Implies, ***circular fringes*** are formed.

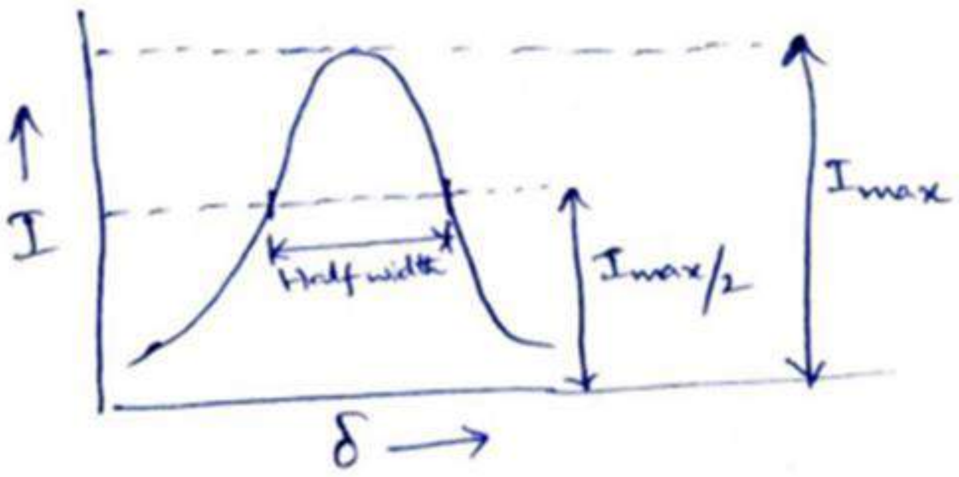
Visibility of fringes:

The visibility (v) of the fringes is given by $\boxed{V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}}$

Substituting the values of I_{max} and I_{min} we get $\boxed{V = \frac{2R}{1+R^2}}$ (12)

Note: Visibility of fringes is a function of reflection coefficient only.

Half width and sharpness of fringes:



Half width is defined as the width of fringes in terms of phase difference between 2-points on either side of maxima where the ***intensity falls to half*** its maximum value.

Implies, $$\frac{I}{I_{max}} = \frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\delta/2}$$ Let $\boxed{F = \frac{4R}{(1-R)^2}}$

$$\frac{I}{I_{max}} = \frac{1}{1+F\sin^2\delta/2} = \frac{1}{2}$$

When, $\frac{I}{I_{max}} = \frac{1}{2}$

implies, $\frac{1}{1+F\sin^2\delta/2} = \frac{1}{2}$

$$1 + F\sin^2\delta/2 = 2$$

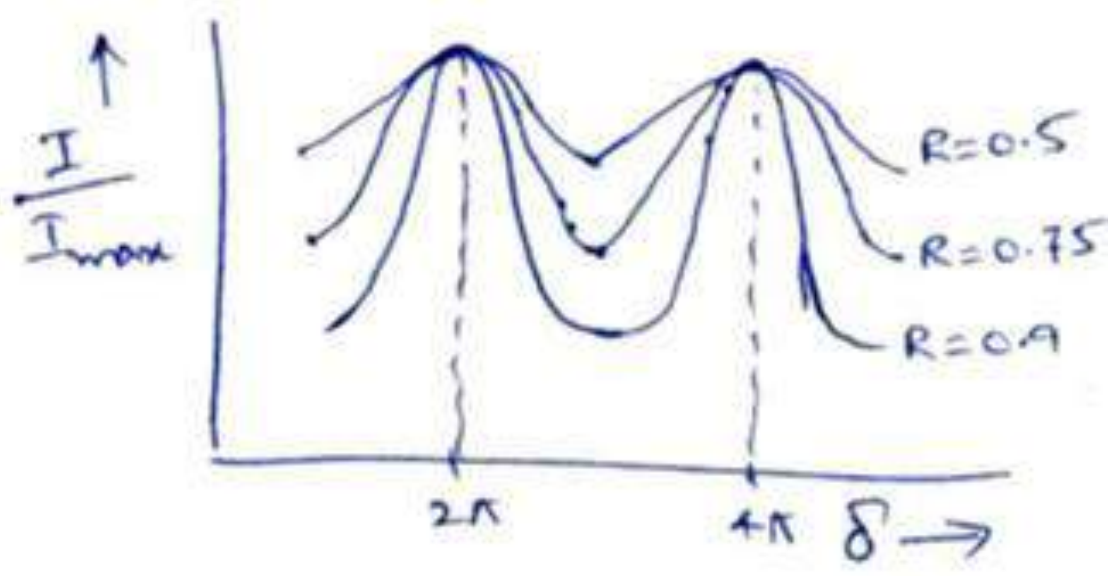
$$\text{Sin}\,(\delta/2) = \frac{1}{\sqrt{F}}$$

Implies, $\sin\left(\frac{\delta}{2}\right) = \frac{(1-R)}{2\sqrt{R}}$

Since, δ is very small $\sin\left(\frac{\delta}{2}\right) = \frac{\delta}{2}$

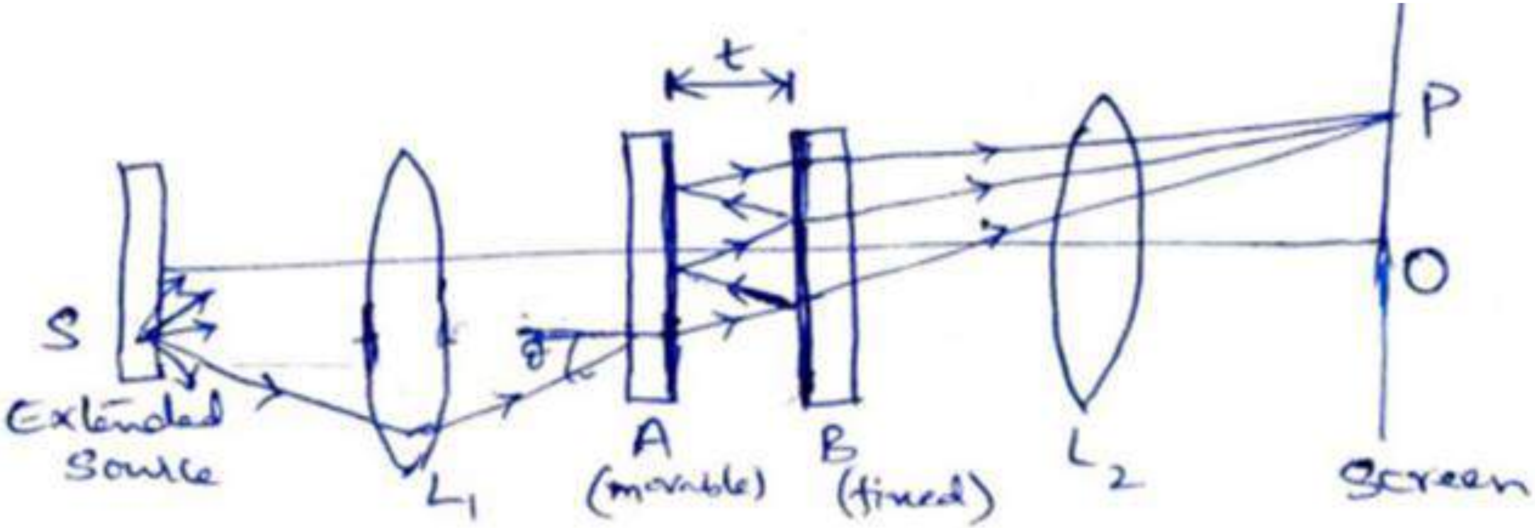
Therefore, $\frac{\delta}{2} = \frac{(1-R)}{2\sqrt{R}}$

Implies, $\boxed{\delta = \frac{(1-R)}{\sqrt{R}}}$



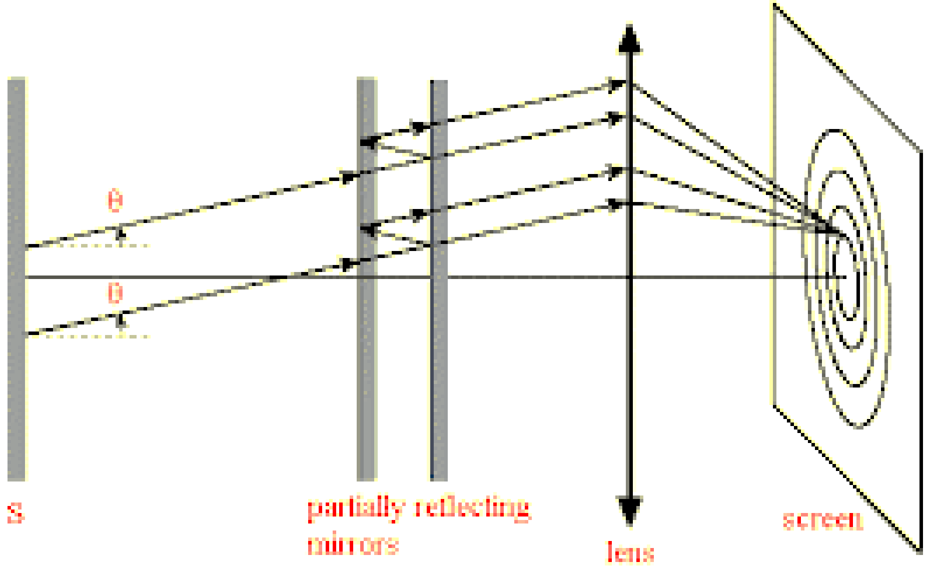
Note: The sharpness of maxima ***depends upon*** the half width. Smaller the half width, sharper is the maxima. It is clear from above Fig that greater the value of R, sharper is the maxima.

An Example for Interference due to multiple reflections:
Fabry-Perot Interferometer

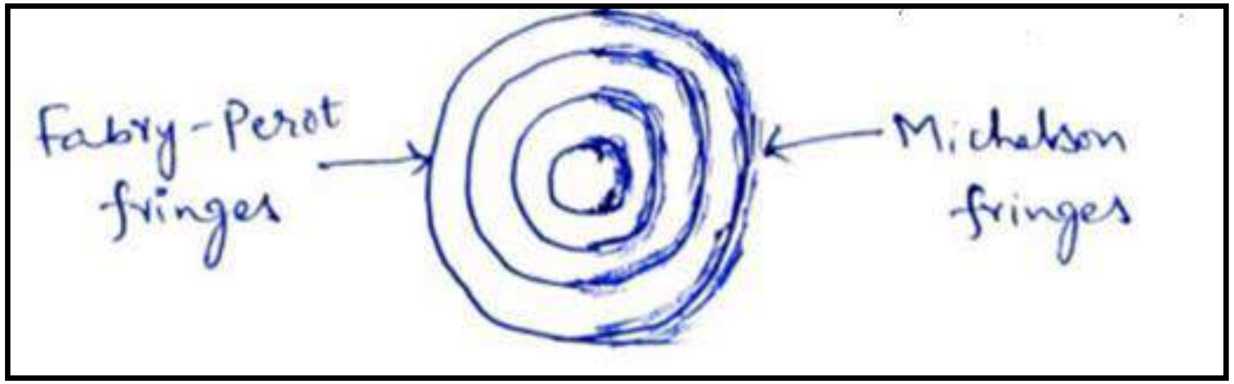


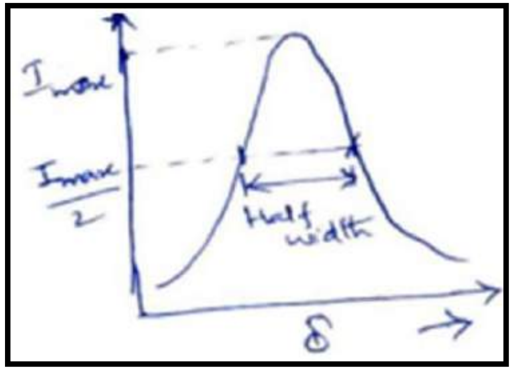
A & B are 2 glass plates that are silvered from inner side to reflect 80 – 90% of incident light. One of the plates (A) is attached to a fine screw and made movable.
If both A & B are fixed it is called “F-P etalon”
If one is movable – “F-P interferometer”

Usually forms ***circular fringes***.



This instrument finds enormous applications in the field of ***high resolution*** spectroscopy. The plates are adjusted such that the circular fringes with fine structures are formed.





If ‘t’ is the separation between the inner surface of A & B and ‘Θ’ is the angle of inclination of the incident ray on the plates, then optical path difference between two consecutive transverse rays will be = 2t cos Θ.

Condition for max intensity is **$2t\cos\Theta = n\lambda$** where n = 0, 1, 2, 3, …..

Condition for minimum intensity is **$2t\cos\Theta = (2n+1)\frac{\lambda}{2}$**

Here, interference pattern consists of ***bright concentric rings***.

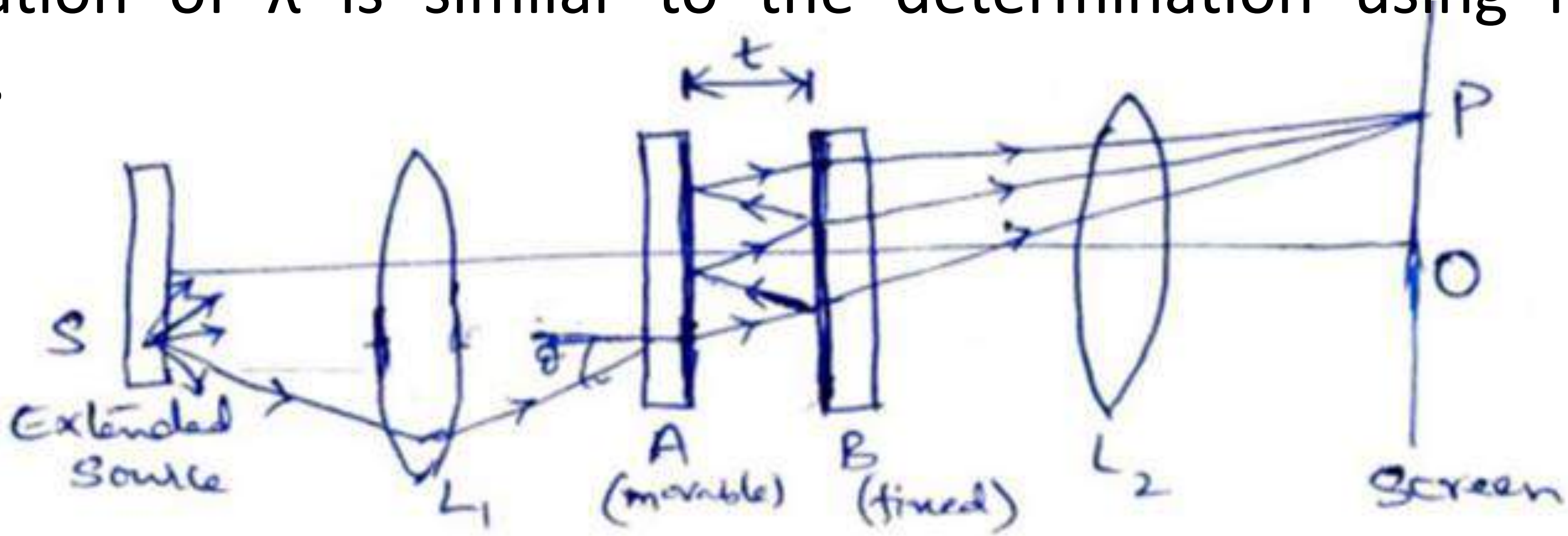
Each ring corresponds to a particular value of Θ.

Importance of F – P fringes:

If the incident beam consists of two λs (say λ_1 and λ_2), then two sets of fringes (rings) are formed. However, the bright fringes are so narrow that any such fringe due to λ_1 cannot completely cover a dark fringe due to λ_2. Hence, when the distance between plates is altered, the fringes ***will not*** disappear as in Newton's rings or Michelson interferometer. But we see new bright fringes lying between the original ones. This arrangement thus enables us to detect the presence of waves of different λ very precisely and is ***superior to*** Michelson interferometer in this aspect.

Determination of λ:

The determination of λ is similar to the determination using Michelson's interferometer.



If the movable plate A is moved such that N bright fringes have crossed at the centre and the position of the plate moved to t_2 from t_1 then,

$$N\frac{\lambda}{2} = t_2 - t_1$$

$$\boxed{\lambda = \frac{2(t_2 - t_1)}{N}}$$

Thus, λ could be calculated.

If the plate 'A' is rotated such that $\Theta = 0^o$ then, $\boxed{2t = N\lambda}$

Determination of $\Delta\lambda$:

As discussed earlier, if two λs are being emitted by the source, then two sets of fringes are observed.

Let the direction 'Θ', max of λ_1 and λ_2 coincides and 't_1' be the separation then,

$$2t_1 \cos\Theta = m\lambda_1 = n\lambda_2 \text{ where, m, n are orders.} \qquad (1)$$

Let the plate separation be increased in steps. If the max of one λ falls over the min of another λ then there will be perfect 'indistinctness" in the field of view. When the plate separation is further increased, the two sets of fringes resolve and again another field of max indistinctness appears.

Let the plate separation be 't_2'. At this position the order of wavelength λ_1 is increased by 'p' and that if λ_2 by (p+1).

Hence, $2t_2 \cos\Theta = (m + p)\lambda_1 = (n + p + 1)\lambda_2$ (2)

Subtracting eq (1) from (2) we get,

$$2(t_2 - t_1) \cos\Theta = p\lambda_1 = (p + 1)\lambda_2 \text{ (as } \Theta = 0, \cos\Theta = 1 \text{ at centre)}$$

$$2(t_2 - t_1) = p\lambda_1 = (p + 1)\lambda_2 \qquad (3)$$

$$p\lambda_1 = (p + 1)\lambda_2$$

$$\boxed{p = \frac{\lambda_2}{\lambda_1 - \lambda_2}} \qquad (4)$$

Substitute eq (4) in (3), we have,

$$2(t_2 - t_1) = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

$$(\lambda_1 - \lambda_2) = \frac{\lambda_1 \lambda_2}{2(t_2 - t_1)}$$

$$\boxed{\Delta\lambda = \frac{(\lambda_{max})^2}{2(t_2 - t_1)}} \qquad (5)$$

Where, λ_{max} is the mean of λ_1 and λ_2 and
$(t_2 - t_1)$ is the dist travelled by the movable plate between two consecutive positions of max indistinctness.

Resolving power of F-P interferometer:

Resolving power is the ability of any instrument to discriminate between 2 close λs. It is expressed as ***λ/Δλ***.

Let us consider two wavelengths, λ and (λ + Δλ), which are to be analysed using F-P interferometer.

We know that, $I_{trans} = \frac{I_0}{1+F\,sin^2\,\delta/2}$ (1)

where $F = \frac{4R}{(1-R)^2}$

and $\delta = \frac{4\pi}{\lambda} t \cos \Theta$ (2)

where δ = phase difference

Differentiating eq (2) we get

$\Delta\delta = \frac{4\pi}{\lambda} t \cos \Theta \, \Delta\lambda$ (3)

(neglecting the negative sign)

We have also deduced $\boxed{\Delta\delta = \frac{2(1-R)}{\sqrt{R}}}$ (4)

where, √R = reflective coefficient

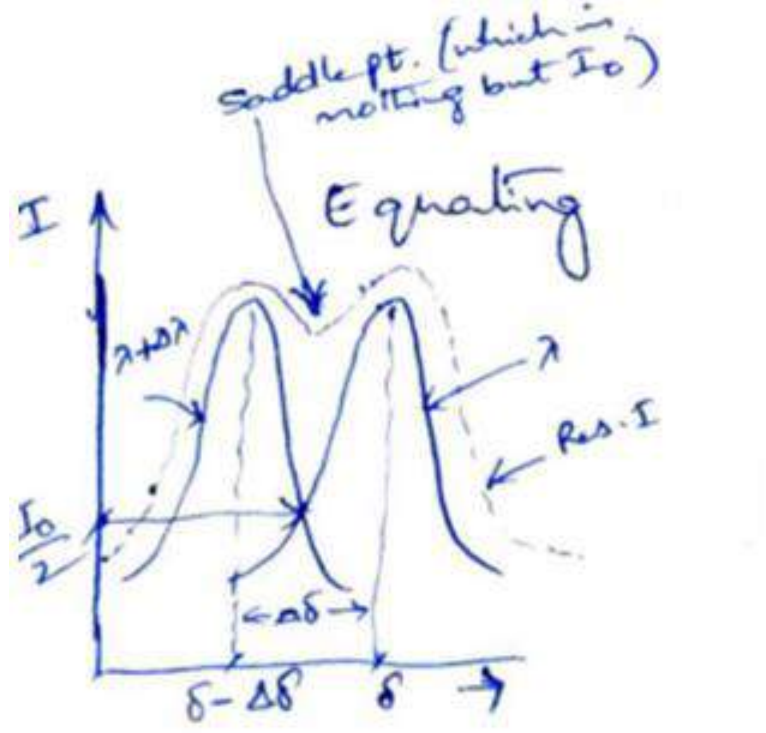
Eq (3) and (4) we get,

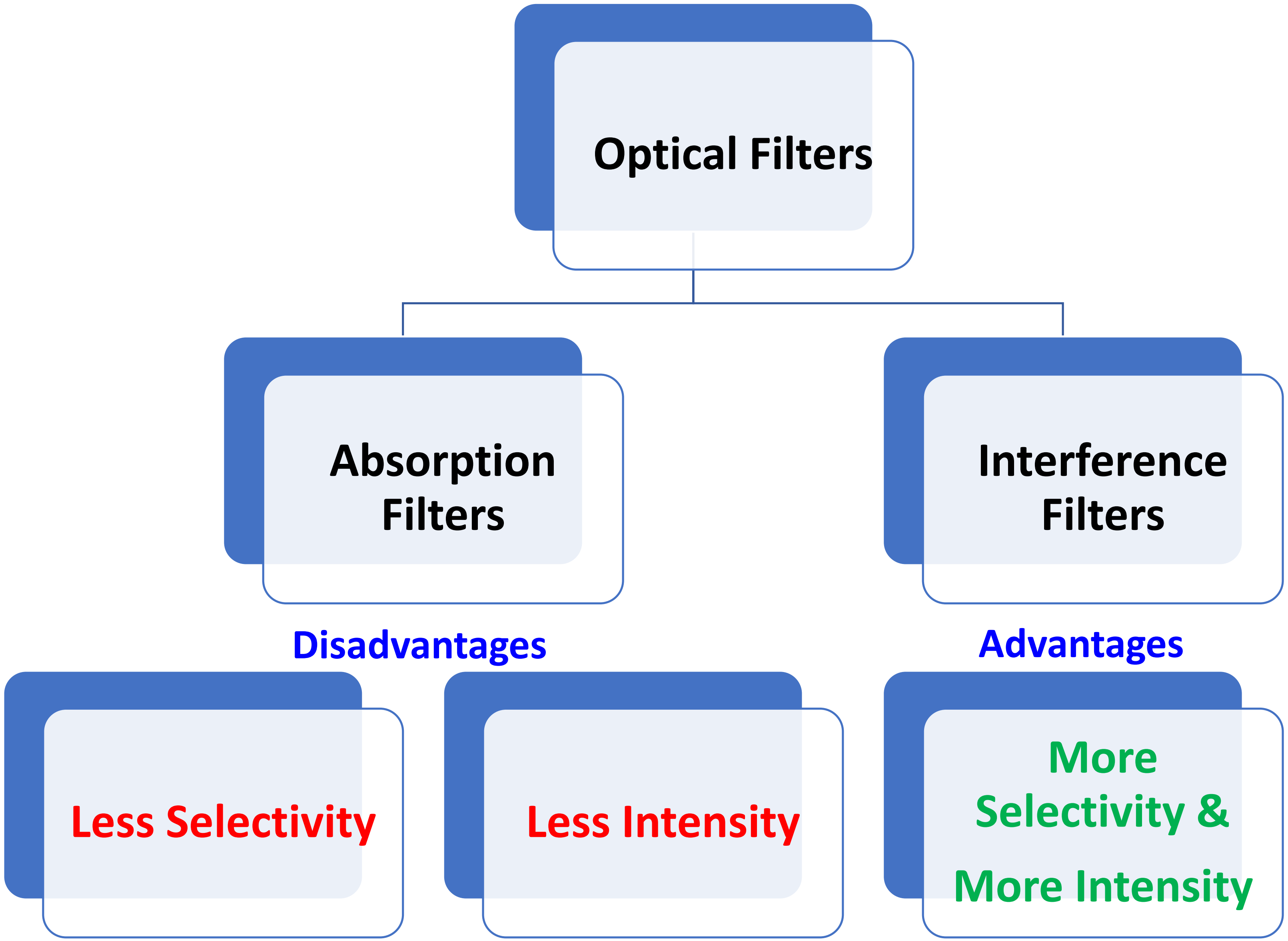
$$\frac{4\pi}{\lambda^2} t \cos\Theta \,\Delta\lambda = \frac{2(1-R)}{\sqrt{R}}$$

Implies,

$$\boxed{\frac{\lambda}{\Delta\lambda} = \frac{2\pi t}{\lambda}\left(\frac{\sqrt{R}}{1-R}\right)}$$

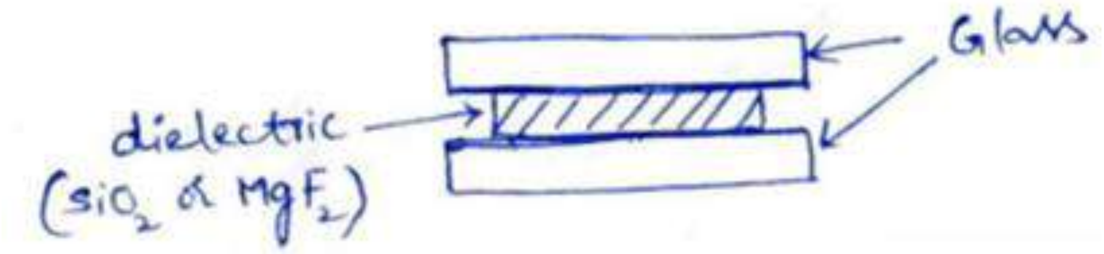
This is the expression for res power of F-P interferometer.




Optical Filters
Absorption Filters
Interference Filters
Disadvantages
Advantages
Less Selectivity
Less Intensity
More Selectivity &
More Intensity

Interference Filters: (Walter Gaffcken in 1939)

For certain expt we require a narrow band of λ centred on a chosen λ. Generally for this purpose, a coloured glass or dyed gelatins were used for many years W. Gaffcken in 1939 made the transmission type interference filters.



When a white light falls normally on this filter (i.e, ϴ=0)

Then **$2\mu t \cos \Theta = n\lambda$** $\mu=1$ for air & $\cos 0 = 1$

Implies, $\boxed{\lambda = \frac{2t}{n}}$ or $\boxed{n = \frac{2t}{\lambda}}$ where n = whole number

Inference: When ‘t’ is too large, large number of maxima are observed in the visible light

When ‘t’ is too small (compared to λ of incident light) only 1 or 2 maxima are observed in visible light.

An Example:

Let λ of 6000 Å is incident on the filter with the plate separation of $5000 \text{ X } 10^{-8}$ cm then,

$$n = \frac{2\, x\, 5000 \text{ x } 10^{-8}}{6000 \text{ x } 10-8} < 2 \text{ that means only 1 maxima.}$$

Implies, it is ***possible to filter out*** a particular λ from the incident white light. This is known as "***Interference Filter***".

When a beam of white light is incident perpendicular to filter, it transmits only a band of order 'n' at particular λ.

Note: Interference filters are used in spectroscopic work for studying spectra in a narrow range of λs.

Will there be overheating of optical instrument due to absorption of various wavelengths in interference filters?

Note: It is highly imp to note that interference filters are made in such a way that they practically absorb/retain no energy (or wavelength) as they are coupled to many heat sinks and hence, they are ***free from*** the issue of ***overheating*** in highly sophisticated optical instruments.