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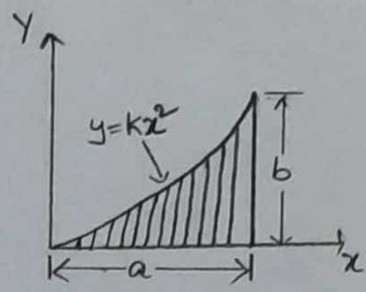
200102071, T8
ECE

TUTORIAL - 3 [ME 101]

1) Given,

Diagram is as follows:-



Let's find area using integration.

Here x has 0, a as limits.

Area under the curve $= \int_0^a y\,dx$

$$= \int_0^a kx^2\,dx$$

$$= \frac{kx^3}{3}\Big|_0^a = \frac{ka^3}{3} - 0$$

$$= \frac{a(ka^2)}{3}$$

$$= \frac{ab}{3}.$$ ($\because b = ka^2$ (a,b) is point on curve)

$\boxed{\therefore \text{Area under curve} = \frac{ab}{3} \text{ sq.units.}}$

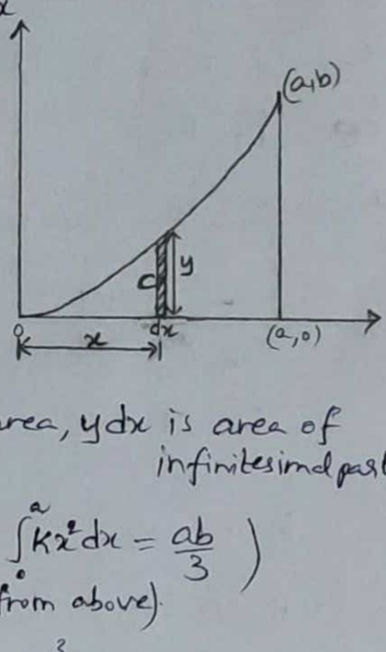
To find co-ordinates of Centroid:-

let us take a small crosssection (infinitesimal part) "dx"

at 'x' units distance from origin.

Position of Centroid of small part $C = (x, y/2)$

Here $y = kx^2$.



Now, Centroid be $(\bar{x}, \bar{y})$

$$\Rightarrow \bar{x} = \frac{\int_0^a (y\,dx).x}{A}$$ (where A is total area, $y\,dx$ is area of infinitesimal part).

$$= \frac{\int_0^a kx^2.x\,dx}{\frac{ab}{3}}$$ $\left(A = \int y\,dx = \int_0^a kx^2\,dx = \frac{ab}{3}\right)$ ($\because$ from above)

$$= \frac{3k}{ab}\int_0^a x^3\,dx = \frac{3k}{ab}\left(\frac{x^4}{4}\right)\Big|_0^a = \frac{3k\cancel{a^4}^{\,a^3}}{4\cancel{a}b} - 0 = \frac{3a.(ka^2)}{4b}$$

$$\bar{x} = \frac{3ab}{4b} = \frac{3a}{4}$$

Now, $\bar{y} = \dfrac{\int_0^a y\,dx \cdot y/2}{ab/3} = \dfrac{3}{2ab} \times \int_0^a y^2 dx$

$$= \frac{3}{2ab} \int_0^a k^2 x^4 dx$$

$$= \frac{3k^2}{2ab} \left(\frac{x^5}{5}\right)\Big|_0^a = \frac{3k^2}{10ab} \times (a^5 - 0)$$

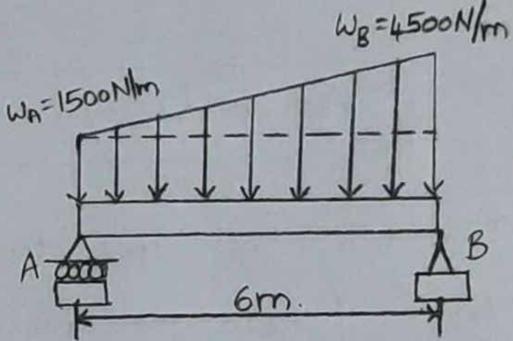
$$= \frac{3k^2 a^4}{10\cancel{a}b} = \frac{3(ka^2)^2}{10(ka^2)}$$

$$= \frac{3b}{10}.$$

$$\boxed{\therefore \text{Co-ordinates of Centroid} = (\bar{x}, \bar{y}) = \left(\frac{3a}{4}, \frac{3b}{10}\right)}$$

2) Given,

Diagram is as follows:-



Here given distributed force can be converted into concentrated load form by using area concept.

Here it is a combination of two concentrated forces. Let them be R_1, R_2.

Here R_1 = Area of rectangle

$= W_A \times L$

$= 1500 \times 6$

$R_1 = 9000$ N

R_2 = Area of triangle

$= (W_B - W_A) \times L \times \frac{1}{2}$

$= 3000 \times 6 \times \frac{1}{2}$

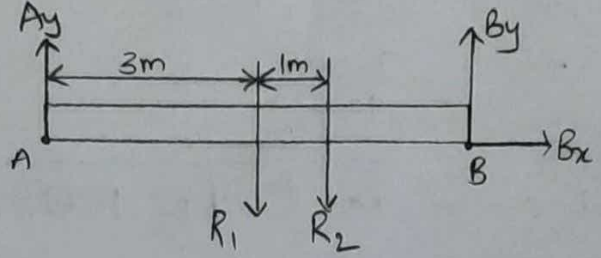
$R_2 = 9000$ N

W.K.T Their lines of action are through centroids of shapes.

$\Rightarrow$ R_1 is perpendicular to rod and at 3m from A.

R_2 is perpendicular to rod and at 4m from A

So, final FBD of system will be :-



Here A_y is reaction at support A on load

B_x, B_y are reactions at support B on Load.

Now,

About B, $(\Sigma M)_B = 0$ (∵ rod is in equilibrium)

$\Rightarrow R_2(2) + R_1(3) - A_y(6) = 0$

$\Rightarrow 6A_y = 9000(2+3)$ (∵ $R_1 = R_2 = 9000N$)

$\Rightarrow A_y = \frac{\cancel{9000}^{1500} \times 5}{\cancel{6}} = 7500N$

on total load,

$(\Sigma F_x) = 0 \Rightarrow B_x = 0$

$(\Sigma F_y) = 0 \Rightarrow A_y + B_y - R_1 - R_2 = 0$

$\Rightarrow B_y = 9000 + 9000 - 7500$

$= 10500N$

∴ The equivalent concentrated loads are

9000N at 3m from A and 9000N at 4m from A.

Reaction at A = 7500N

Reaction at B = 10500N

So, resultant concentrated Load $= R_1 + R_2 = 18000N = 18kN$.

Now, in order to find the position of it's line of action,

Let's take A as reference and x be distance from A.

By equating moments,

$$R \cdot x = R_1 x_1 + R_2 x_2$$

$$(18000)\,x = 9000\,(3) + 9000\,(4)$$

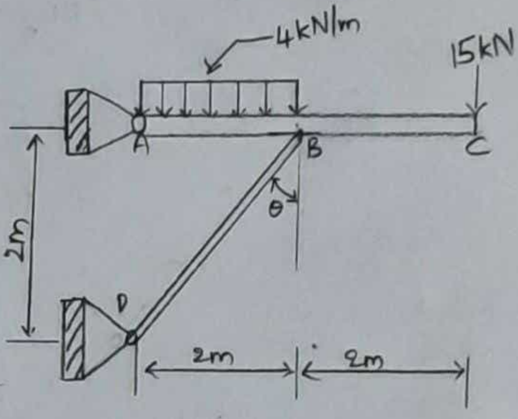
$$x = \frac{9000 \times 7}{18000} = 3.5\,m$$

∴ The equivalent concentrated load is 18kN and it acts at 3.5m distance from A.

Reaction at support A = 7.5kN.

Reaction at support B = 10.5 kN

3) Given, diagram is as follows:-



a) Sign convention for

i) shear force

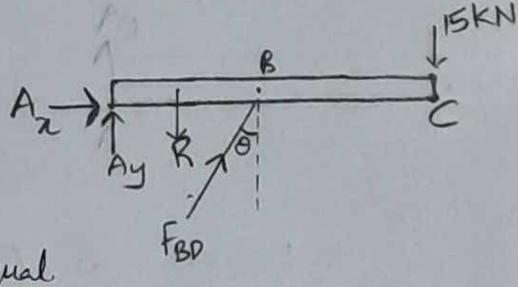
↑▭↓ positive.

ii) bending moment

+M (▭) +M (positive).

Now, Let's find forces in system.

FBD is as follows:-



Here $\theta = 45^\circ \rightarrow$ As opposite side and adjacent side to angle are equal.

For body, $\Sigma F_x = 0 \Rightarrow A_x + F_{BD}\sin 45^\circ = 0 \Rightarrow \boxed{A_x = \frac{-F_{BD}}{\sqrt{2}}}$

Now, based on area of distributed force $\rightarrow R = 4 \times 2 = 8\,kN$ and it acts at 1m distance from A (centre of rectangle).

About B, $(\Sigma M)_B = 0 \Rightarrow R \times 1 - 15 \times 10^3 \times 2 - A_y \times 2 = 0$

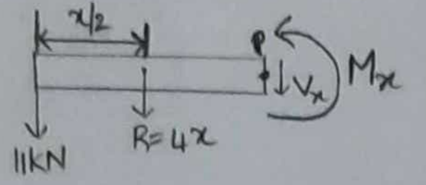
$\Rightarrow 2A_y = (8 - 30)\,kN \Rightarrow \boxed{A_y = -11\,kN}$

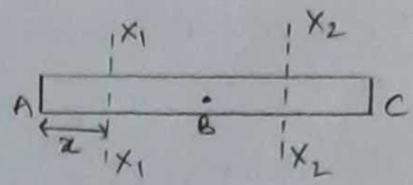
Now, By taking $(\Sigma F)_y = 0$ for complete system, we get $-15-R+A_y+\frac{F_{BD}}{\sqrt{2}}=0$

$$-15-8+(-11)+\frac{F_{BD}}{\sqrt{2}}=0$$

$$\Rightarrow \frac{F_{BD}}{\sqrt{2}} = 34 \Rightarrow \boxed{F_{BD}=34\sqrt{2}\,kN} \Rightarrow \boxed{A_x=-34\,kN}$$

b) Now, Let us consider sections for calculating shear force.





Here $R=4x$ would vary only for $0<x<2$.

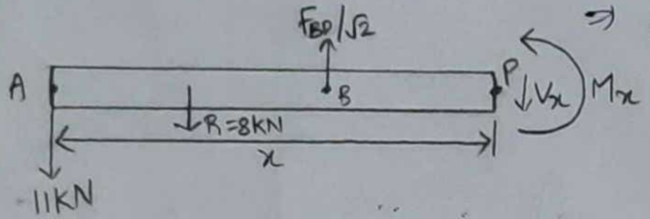
$\Rightarrow$ for $0<x<2$, $V_x+4x+11=0$

$$\Rightarrow V_x=-11-4x.$$

Now, about P,

$$(\Sigma M)_P=0 \Rightarrow M_x+(4x)\frac{x}{2}+(11)(x)=0$$

$$\Rightarrow M_x=-11x-2x^2 \text{ (kNm) for } 0<x<2.$$



About P, shear force be "V_x".

Here, this is applicable to $x \geq 2$, as $R=8kN$ only from $x=2$ and $x>2$

$\Rightarrow$ for $2 \leq x$

$$(\Sigma F_y)=0 \Rightarrow V_x+8+11-\frac{F_{BD}}{\sqrt{2}}=0$$

$$V_x=\frac{34\sqrt{2}}{\sqrt{2}}-19=15\,kN.$$

$$\therefore V_x=15\,kN$$

About P, $M_x+8(x-1)+11(x)-\frac{F_{BD}}{\sqrt{2}}(x-2)=0 \Rightarrow M_x+19x-8-34x+68=0$

$$M_x = (15x - 60)\, kNm$$

c) Using above formulae, we can draw shear force and bending moment diagrams.

SHEAR FORCE DIAGRAM (SFD):-

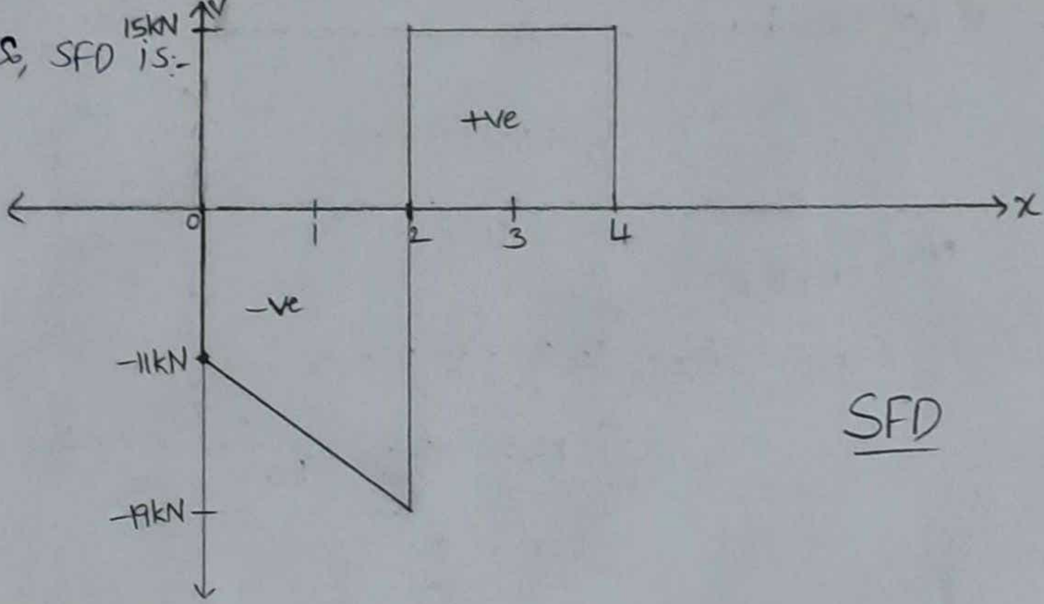
At point A → $x = 0 \Rightarrow V_A = -11\, kN$

Between A to B → $0 < x < 2 \Rightarrow V_x = -11 - 4x$

At B → $x = 2 \Rightarrow V_B = 15 kN$

Between B to C and At C → $2 < x \leq 4 \rightarrow V = 15 kN$.

So, SFD is:-



SFD

BENDING MOMENT DIAGRAM (BMD):-

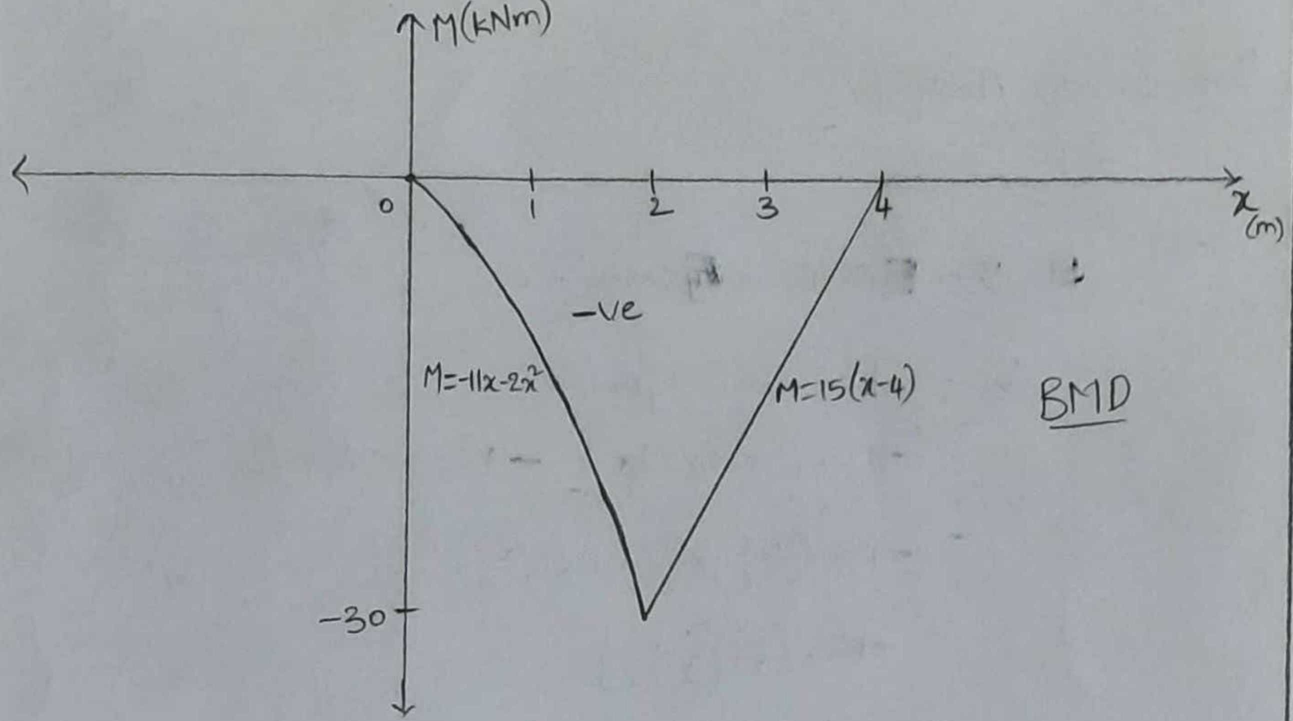
At point A → $x = 0 \rightarrow M_A = 0$

Between 0 to 2 → $x \in (0, 2) \Rightarrow M_x = -2x^2 - 11x$ kNm
(A) (B)

At $x = 2$ (B) → $M_B = -30$ kNm

Between B and C, at C → $M_x = 15(x-4)$ kNm, $M_C = 0$.

So, we can draw BMD as follows:-

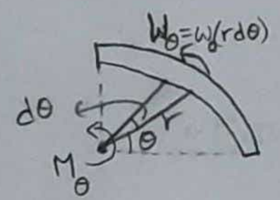


Bending Moment Diagram.

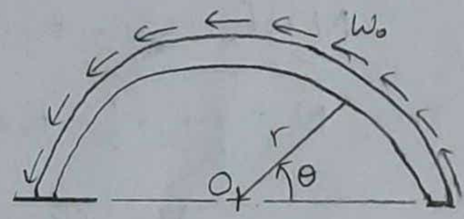
4) Given,

Diagram is as follows:-

Consider small part of semicircle.



($\because W_\theta$ = force)



Now, resultant force on small part is along tangential direction

Resolving this force into components. and Moment about centre is

$$R_x = W_\theta \sin\theta = r w_0 \sin\theta \, d\theta \qquad M_\theta = (w_0 r d\theta) \cdot r = r^2 w_0 \, d\theta$$

$$R_y = W_\theta \cos\theta = r w_0 \cos\theta \, d\theta$$

Now for complete ring of angle 'θ' will have forces as

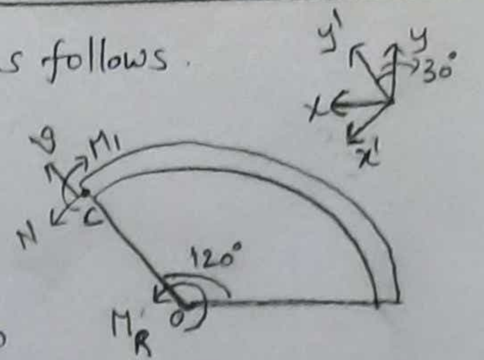
$$F_x = \int_0^\theta R_x = \int_0^\theta r w_0 \sin\theta \, d\theta = r w_0 (1 - \cos\theta)$$

$$F_y = \int_0^\theta R_y = \int_0^\theta r w_0 \cos\theta \, d\theta = r w_0 \sin\theta$$

& Resultant Moment as $\int_0^\theta M_\theta$

$$M_R = r^2 w_0 \theta.$$

Consider section of $120°$. It's FBD is as follows.



Here consider About C,

$$\left(\Sigma F_{y'}\right) = 0$$

$$\Rightarrow \quad V + F_x \cos 60° + F_y \cos 30° = 0$$

$$V = -F_x \cos 60° - F_y \cos 30°$$

$$= -r\omega_0 (1-\cos 120°) \times \frac{1}{2} - r\omega_0 \sin 120° \times \frac{\sqrt{3}}{2}$$

$$= -r\omega_0 \left(1+\frac{1}{2}\right) \times \frac{1}{2} - r\omega_0 \left(\frac{\sqrt{3}}{2}\right) \frac{\sqrt{3}}{2}$$

$$= -r\omega_0 \left(\frac{3}{2}\right)\left[\frac{1}{2}+\frac{1}{2}\right]$$

$$= -1.5 r\omega_0$$

$$\boxed{\therefore V = -1.5 r\omega_0} \rightarrow \text{Shear force.}$$

$$\left(\Sigma F_{x'}\right) = 0$$

$$N + F_x \cos 30° - F_y \cos 60° = 0$$

$$N = F_y \times \frac{1}{2} - F_x \times \frac{\sqrt{3}}{2}$$

$$= r\omega_0 \sin 120° \times \frac{1}{2} - r\omega_0 (1-\cos 120°) \frac{\sqrt{3}}{2}$$

$$= \frac{r\omega_0}{2}\left(\frac{\sqrt{3}}{2}\right) - r\omega_0 \left(\frac{3}{2}\right)\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3} r\omega_0}{4}(-3+1) = -\frac{\sqrt{3} r\omega_0}{2} = -0.866 r\omega_0.$$

$$\boxed{\therefore N = -0.866 r\omega_0} \rightarrow \text{Normal reaction}$$

About O,

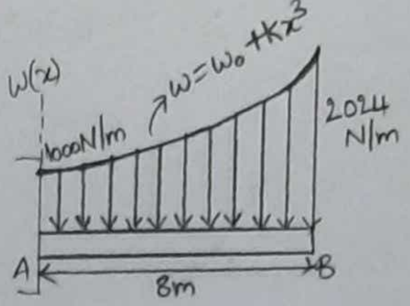
$$(\Sigma M) = 0 \Rightarrow \quad M_1 - M_R - r \times N = 0$$

$$M_1 - r^2\omega_0 \times \frac{2\pi}{3} - r.(-0.866 r\omega_0) = 0$$

$$\therefore \boxed{M_1 = r^2\omega_0 \left(\frac{2\pi}{3} - 0.866\right) = r^2\omega_0 (1.23)} \text{ Bending Moment.}$$

5) Given,

Diagram is as follows:-



At $x=0 \to W=1000=W_0+k(0)^3$

$\Rightarrow W_0=1000\ N/m.$

At $x=8 \to W=2024=1000+k(8)^3$

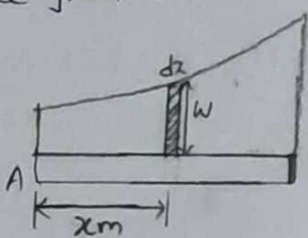
$1024=k\times 2^9$

$\Rightarrow \boxed{k=2}$

$\Rightarrow W(x)=1000+2x^3.$

Let us find centre of gravity w.r.t A as origin.

Assume a small cross-section (dx) at x distance from A



To get equivalent concentrated Load,

$R=\int W_x.\qquad (\because W_x=Wdx)$

$=\int W.dx$

$=\int_0^8 (1000+2x^3)\,dx$

$=1000(8)+\frac{2}{4}(8)^4=8000+\frac{8^4}{2}=10048$

$\therefore R=10048N.$

To find COG,

$R\bar{X}=\int_0^8 (Wdx).x$ where $\bar{X}$ is distance of Centre of gravity from A.

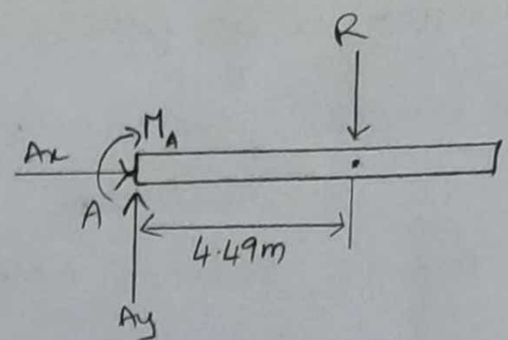
$\Rightarrow \bar{X}=\frac{\int_0^8 (W_0+kx^3)x\,dx}{R}$

$=\frac{\int_0^8 (1000x+2x^4)dx}{10048}$

$=\frac{500(64)+\frac{2}{5}(8)^5}{10048}=\frac{45107.2}{10048}\simeq 4.49m$

$$\boxed{\therefore\ \bar{x} = 4.49\,m}$$

Let's draw FBD of beam,



$\Sigma F_y = 0$ on body.

$$\Rightarrow A_y - R = 0$$

$$A_y = R = 10048N$$

$$\boxed{\therefore A_y = 10048\,N}$$

$\Sigma F_x = 0 \Rightarrow \boxed{A_x = 0}$

$$\boxed{\therefore \text{Reaction at support } A = 10048N}$$

$(\Sigma M)_A = 0$ (about A) $\Rightarrow$ $M_A + R(4.49) = 0$

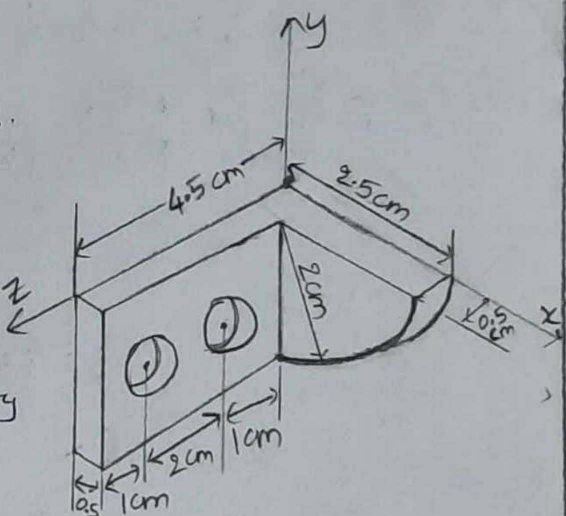
$$M_A = -(10048) \times 4.49$$

$$= -45107.2\ Nm.$$

$$\boxed{\therefore M_A = 45107.2\ Nm.}$$

6) Given,

Diagram is as follows:-



Let's break this figure into four parts.

They are → Cuboid (+ve)

Quarter cylinder (+ve)

2 Cylinders (−ve) → As they are holes.

Let's find co-ordinates of Centre of gravity of each of them

Cuboid $= (x_C, y_C, z_C) = \left(\frac{0.5}{2}, \frac{-2}{2}, \frac{4.5}{2}\right) = (0.25, -1, 2.25)$ cm, cm, cm

Quarter cylinder $= (x_Q, y_Q, z_Q) = \left(0.5 + \frac{4(2)}{3\pi}, \frac{-4(2)}{3\pi}, \frac{0.5}{2}\right) = \left(0.5 + \frac{8}{3\pi}, \frac{-8}{3\pi}, 0.25\right)$ cm, cm, cm

Hole 1 → $(x_1, y_1, z_1) = (0.25, -1, 1.5)$

Hole 2 → $(x_2, y_2, z_2) = (0.25, -1, 3.5)$

Here, quater circle C.G co-ordinates are found using Pappus – Guldinus Theorem.

$$\Rightarrow \quad (\cancel{2}\cancel{\pi}\bar{y}) \times \frac{\pi r^2}{4} = \frac{\cancel{2}}{3}\cancel{\pi} r^3$$

$$\Rightarrow \quad \boxed{\bar{y} = \frac{4r}{3\pi}}$$

Now, CG of body $= (\bar{x}, \bar{y}, \bar{z})$

Here $\bar{x} = \dfrac{\sum V_i x_i}{\sum V_i}$ (Similar expressions for $\bar{y}, \bar{z}$)

$$\Rightarrow \bar{x} = \frac{(4.5 \times 2 \times 0.5)(0.25) + \left(0.5 \times \pi \times \frac{2^2}{4}\right)\left(0.5 + \frac{8}{3\pi}\right) - 2 \times \left(\pi \times (0.5)^2 \times 0.5\right) \times 0.25}{(4.5 \times 2 \times 0.5) + \left(0.5 \times \pi \times \frac{2^2}{4}\right) - \left(2 \times \pi (0.5)^2 \times 0.5\right)}$$

$\simeq 0.58$ cm

$$\Rightarrow \bar{y} = \frac{(4.5 \times 2 \times 0.5)(-1) + \left(0.5 \times \pi \times \frac{2^2}{4}\right)\left(-\frac{8}{3\pi}\right) - 2 \times \left(\pi \times (0.5)^2 \times 0.5 \times (-1)\right)}{5.285}$$

$\simeq -0.96$ cm

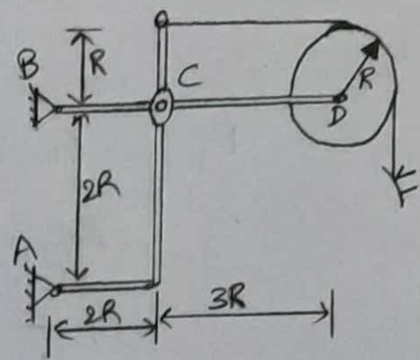
$$\Rightarrow \bar{z} = \frac{(4.5 \times 2 \times 0.5)(2.25) + \left(0.5 \times \pi \times \frac{2^2}{4}\right)(0.25) - (\pi \times (0.5)^2 \times 0.5)(1.5 + 3.5)}{5.285}$$

$\simeq 1.62$cm

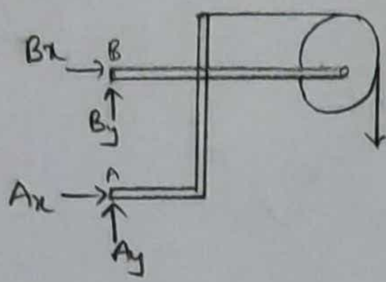
∴ Co-ordinates of Centre of gravity = (0.58 cm, −0.96cm, 1.62cm)

7) Given,

Diagram is as follows:-



FBD of total system is:-



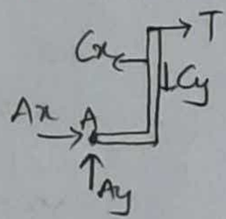
Now, about A → $(\Sigma M)_A = 0 \Rightarrow B_x \times 2R + F \times 5R = 0$

$\Rightarrow \boxed{B_x = -\frac{5F}{2}}$

$\Sigma F_x = 0$ for total system $\Rightarrow A_x + B_x = 0 \Rightarrow A_x = \frac{5F}{2}$

$\Sigma F_y = 0$ for system $\Rightarrow B_y + A_y = F \rightarrow (1)$

Considering each component's FBD

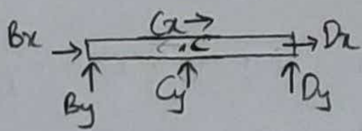


$\Sigma F_x = 0 \Rightarrow A_x - C_x + T = 0$

$\Rightarrow \boxed{C_x = A_x + T} \rightarrow (2)$

$\Sigma F_y = 0 \Rightarrow A_y - C_y = 0$

$\boxed{A_y = C_y} \rightarrow (3)$



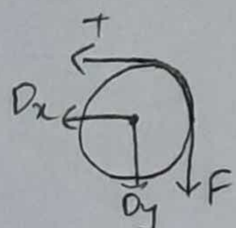
$\Sigma F_x = 0$

$\Rightarrow B_x + C_x + D_x = 0$

$C_x + D_x = -\left(-\frac{5F}{2}\right)$

$\boxed{C_x + D_x = \frac{5F}{2}} \rightarrow (4)$

$\Sigma F_y = \boxed{B_y + C_y + D_y = 0} \rightarrow (5)$



About Centre,

$\Sigma M = 0$

$\Rightarrow T \times R - F \times R = 0$

$\Rightarrow \boxed{T = F}$

$\Sigma F_x = 0$

$\Rightarrow D_x + T = 0$

$\boxed{D_x = -F}$

$\Sigma F_y = 0$

$D_y + F = 0$

$\boxed{D_y = -F}$

In Equation 4, $C_x = \frac{5F}{2} - D_x = \frac{5F}{2} + F$

$\Rightarrow \boxed{C_x = \frac{7F}{2}}$

In eq (2),

$A_x = \frac{7F}{2} - F = \frac{5F}{2}$.

In equation ⑤ → We need B_y, C_y.

So, consider moments w.r.t C for BD.

$$\Rightarrow \quad B_y(2R) - 3R(D_y) = 0$$

$$\Rightarrow \boxed{B_y = \frac{3D_y}{2}}$$

$$\Rightarrow \boxed{B_y = -\frac{3F}{2}}$$

Now,

$$C_y = -D_y - B_y \quad (\because \text{from eqn ⑤})$$

$$= +F + \frac{3F}{2}$$

$$\boxed{C_y = \frac{5F}{2}} \Rightarrow \boxed{A_y = \frac{5F}{2}}$$

$\therefore$ Reactions at A are $A_x = \frac{5F}{2}$, $A_y = \frac{5F}{2}$

at B are $B_x = \frac{5F}{2}$, $B_y = \frac{3F}{2}$

at C are $C_x = \frac{7F}{2}$, $C_y = \frac{5F}{2}$

at D are $D_x = F$, $D_y = F$

Tension in rope $T = F$.