PROBABILITY — QUICK NOTES (2–3 Pages)

1. Introduction to Probability
Probability quantifies uncertainty. It measures how likely an event is to occur on a scale from 0 to 1.
0 = impossible event
1 = certain event
Example: Probability of getting heads on a fair coin = 1/2.

2. Basic Terminology
• **Experiment:** A process that leads to outcomes (e.g., rolling a die).
• **Sample Space (S):** Set of all possible outcomes. For a die: S = {1,2,3,4,5,6}.
• **Event (E):** A subset of sample space (e.g., even numbers).

3. Types of Events
• **Simple Event:** Contains only one outcome.
• **Compound Event:** Contains more than one outcome.
• **Mutually Exclusive Events:** Cannot occur together (e.g., odd and even in a die roll).
• **Exhaustive Events:** Events that cover the entire sample space.

4. Classical (Theoretical) Probability
If all outcomes are equally likely:
P(E) = (Number of favorable outcomes) / (Total outcomes)
Example: Probability of drawing an Ace from a deck = 4/52 = 1/13.

5. Complement of an Event
E' = Event that E does not occur.
P(E) + P(E') = 1
Example: If probability of rain = 0.3, no rain = 0.7.

6. Conditional Probability
Probability of event A given that B has occurred:
$P(A|B) = P(A \cap B) / P(B)$, where $P(B) \neq 0$.
Example: Drawing two cards without replacement involves conditional probability.

7. Independent Events
Two events A and B are independent if:
$P(A \cap B) = P(A) \times P(B)$
Example: Tossing a coin and rolling a die.

8. Addition Theorem
• For any events A and B:
$P(A \cup B) = P(A) + P(B) – P(A \cap B)$

9. Multiplication Theorem
• For independent events:
$P(A \cap B) = P(A) \times P(B)$
• For dependent events:
$P(A \cap B) = P(A) \times P(B|A)$

10. Bayes' Theorem
Used to reverse conditional probabilities.
$P(A|B) = [P(B|A) \times P(A)] / P(B)$
Useful in diagnostics, spam filtering, predictions.

11. Random Variables
A variable whose values depend on outcomes of a random experiment.
• **Discrete RV:** Finite/Countable outcomes (e.g., number of heads).
• **Continuous RV:** Infinite uncountable outcomes (e.g., height).

12. Probability Distribution
Describes how probabilities are distributed over values of a random variable.
• **PMF (Probability Mass Function):** For discrete RVs.
• **PDF (Probability Density Function):** For continuous RVs.

13. Expectation (Mean) and Variance
• Expectation: $E(X) = \Sigma [x \cdot P(x)]$
• Variance: $Var(X) = E(X^2) – (E(X))^2$
• Standard Deviation = $\sqrt{Var(X)}$

14. Common Distributions
• **Bernoulli:** Outcomes 0/1 with probability p.
• **Binomial:** Repeated Bernoulli trials (n trials).
Mean = np, Variance = npq
• **Normal:** Bell-shaped continuous distribution.
• **Poisson:** Rare events over time/space.

15. Law of Large Numbers
As number of trials increases, the sample mean approaches theoretical probability.

16. Central Limit Theorem (CLT)
Regardless of distribution, the sum/average of a large number of independent random variables tends toward a normal distribution.

These concise notes cover all major definitions, formulas, and concepts from probability, suitable for quick revision for exams.