

Map Types Module

Module

MapTypes

Uses

N/A

Syntax

Exported Constants

None

Exported Types

CompassT = {N, S, E, W}

LanduseT = {Recreational, Transport, Agricultural, Residential, Commercial}

RotateT = {CW, CCW}

Exported Access Programs

None

Semantics

State Variables

None

State Invariant

None

Point ADT Module

Template Module

PointT

Uses

N/A

Syntax

Exported Types

[PointT = ? —SS]

Exported Access Programs

Routine name	In	Out	Exceptions
PointT	\mathbb{Z}, \mathbb{Z}	PointT	
x		\mathbb{Z}	
y		\mathbb{Z}	
translate	\mathbb{Z}, \mathbb{Z}	PointT	

Semantics

State Variables

xc: [\mathbb{Z} —SS]

yc: [\mathbb{Z} —SS]

State Invariant

None

Assumptions

The constructor PointT is called for each object instance before any other access routine is called for that object. The constructor cannot be called on an existing object.

Access Routine Semantics

PointT(x, y):

- transition: $[\text{xc}, \text{yc} := x, y \text{---SS}]$
- output: $\text{out} := \text{self}$
- exception: None

x():

- output: $\text{out} := xc$
- exception: None

y():

- output: $\text{out} := yc$
- exception: None

translate($\Delta x, \Delta y$):

- $[\text{output: out: PointT}(\text{xc} + \Delta x, \text{yc} + \Delta y) \text{---SS}]$
- exception: None

Line ADT Module

Template Module

LineT

Uses

[\[PointT, MapTypes for CompassT and RotateT —SS\]](#)

Syntax

Exported Types

LineT = ?

Exported Access Programs

Routine name	In	Out	Exceptions
LineT	PointT, CompassT, \mathbb{N}	LineT	invalid_argument
strt		PointT	
end		PointT	
orient		CompassT	
len		\mathbb{Z}	
flip		LineT	
rotate	RotateT	LineT	
translate	\mathbb{Z}, \mathbb{Z}	LineT	

Semantics

State Variables

s : PointT

o : CompassT

L : \mathbb{N}

State Invariant

None

Assumptions

The constructor `LineT` is called for each object instance before any other access routine is called for that object. The constructor cannot be called on an existing object.

Access Routine Semantics

`LineT(st, ornt, l)`:

- transition: $s, o, L := st, ornt, l$
- output: $out := self$
- exception: $[l \leq 0 \implies invalid_argument \text{ ---SS}]$

`strt()`:

- output: $out := \text{PointT}(st.x, st.y)$
- exception: None

`end()`:

- output: $[PointT(x, y) \text{ where, } ((o = N \implies y = strt().y() + l) \parallel (o = S \implies y = strt().y() - l) \parallel (o = E \implies y = strt().x() + l) \parallel (o = W \implies y = strt().x() - l)) \text{ ---SS}]$
- exception: None

`orient()`:

- output: $out := o$
- exception: None

`len()`:

- output: $out := L$
- exception: None

`flip()`:

- output: $[\text{LineT}(strt(), o, l) \text{ where, } (o = rotate(CW), o = rotate(CW)) \text{ ---SS}]$
- exception: None

rotate(r):

• output:			<i>out</i> :=
	<i>r</i> = CW	<i>o</i> = N	[E —SS]
		<i>o</i> = S	[W —SS]
		<i>o</i> = W	[N —SS]
		<i>o</i> = E	[S —SS]
	<i>r</i> = CCW	<i>o</i> = N	[W —SS]
		<i>o</i> = S	[E —SS]
		<i>o</i> = W	[S —SS]
		<i>o</i> = E	[N —SS]

- exception: None

translate(Δx , Δy):

- output: [LineT(newPoint, o, *l*) where, (newPoint = st.translate(Δx , Δy) —SS]
- exception: None

Path ADT Module

Template Module

PathT

Uses

PointT, LineT, MapTypes

Syntax

Exported Types

PathT = ?

Exported Access Programs

Routine name	In	Out	Exceptions
PathT	PointT, CompassT, N	PathT	
append	CompassT, N		invalid_argument
strt		PointT	
end		PointT	
line	N	LineT	outside_bounds
size		N	
len		N	
translate	\mathbb{Z} , \mathbb{Z}	LineT	

Semantics

State Variables

s : sequence of LineT

State Invariant

None

Assumptions

- The constructor `PathT` is called for each object instance before any other access routine is called for that object. The constructor cannot be called on an existing object.

Access Routine Semantics

`PathT(st, ornt, l):`

- transition: $[s_0 := \text{LineT}(st, ornt, l) \text{---SS}]$
- output: $out := self$
- exception: None

`append(ornt, l):`

- transition: $[s := s + \text{LineT}(z, ornt, l) \text{ where, } (z = adjPt(ornt)). \text{---SS}]$
- exception: $[s_{|s|-1}.rotate(CW).rotate(CW).orient = ornt \implies invalid_argument \text{---SS}]$

`strt():`

- output: $[out := s_0.st \text{---SS}]$
- exception: None

`end():`

- output: $[out := s_{|s|-1}.end \text{---SS}]$
- exception: None

`line(i):`

- output: $[out := s_i \text{---SS}]$
- exception: $[i < 0 || i \geq s.len \implies outside_bounds \text{---SS}]$

`size:`

- output: $[out := s.len \text{---SS}]$
- exception: None

len:

- output: $[out := +\forall(i : \mathbb{N} | i \in [0 \dots s.len - 1] : pointsInLine(i)) - SS]$
- exception: None

translate($\Delta x, \Delta y$):

- output: Create a new PathT object with state variable s' such that:

$$\forall(i : \mathbb{N} | i \in [0 \dots |s| - 1] : s'[i] = s[i].translate(\Delta x, \Delta y))$$

- exception: None

Local Functions

pointsInLine: LineT \rightarrow (set of PointT)

pointsInLine (l)

$$\equiv \{i : \mathbb{N} | i \in [0 \dots (l.len - 1)] : l.strt.translate.($$

$$(\begin{array}{|l|l|} \hline l.orient = N & (0, 1) \\ \hline l.orient = S & (0, -1) \\ \hline l.orient = W & (-1, 0) \\ \hline l.orient = E & (1, 0) \\ \hline \end{array}) - SS]$$

adjPt: CompassT \rightarrow PointT

adjPt($ornt$) \equiv

$ornt = N$	$s[s - 1].end.translate[(0, 1) - - - SS]$
$ornt = S$	$s[s - 1].end.translate[(0, -1) - - - SS]$
$ornt = W$	$s[s - 1].end.translate[(-1, 0) - - - SS]$
$ornt = E$	$s[s - 1].end.translate[(1, 0) - - - SS]$

Generic Seq2D Module

Generic Template Module

Seq2D(T)

Uses

N/A

Syntax

Exported Types

Seq2D(T) = ?

Exported Constants

None

Exported Access Programs

Routine name	In	Out	Exceptions
Seq2D	seq of (seq of T), \mathbb{R}	Seq2D	invalid_argument
set	PointT, T		outside_bounds
get	PointT	T	outside_bounds
getNumRow		\mathbb{N}	
getNumCol		\mathbb{N}	
getScale		\mathbb{R}	
count	T	\mathbb{N}	
count	LineT, T	\mathbb{N}	invalid_argument
count	PathT, T	\mathbb{N}	invalid_argument
length	PathT	\mathbb{R}	invalid_argument
connected	PointT, PointT	\mathbb{B}	invalid_argument

Semantics

State Variables

s : seq of (seq of T)

scale: \mathbb{R}

nRow: \mathbb{N}
nCol: \mathbb{N}

State Invariant

None

Assumptions

- The Seq2D(T) constructor is called for each object instance before any other access routine is called for that object. The constructor can only be called once.
- Assume that the input to the constructor is a sequence of rows, where each row is a sequence of elements of type T. The number of columns (number of elements) in each row is assumed to be equal. That is each row of the grid has the same number of entries. $s[i][j]$ means the i th row and the j th column. The 0th row is at the bottom of the map and the 0th column is at the leftmost side of the map.

Access Routine Semantics

Seq2D(S , scl):

- transition: $[s, scale, nRow, nCol := S, scl, S.len, S[0].len \text{ ---SS}]$
- output: $out := self$
- exception: $[scl < 0 \parallel \neg(validRow) \parallel \neg(validCol) \parallel S.len \neq S[0].len \implies invalid_argument \text{ ---SS}]$

set(p, v):

- transition: $[s[p.y][p.x] := v \text{ ---SS}]$
- exception: $[p.y < 0 \parallel p.x < 0 \parallel p.y \geq nRow \parallel p.x \geq nCol \implies outside_bounds \text{ ---SS}]$

get(p):

- output: $[out := s[p.y][p.x] \text{ ---SS}]$
- exception: $[p.y < 0 \parallel p.x < 0 \parallel p.y \geq nRow \parallel p.x \geq nCol \implies outside_bounds \text{ ---SS}]$

getNumRow():

- output: $out := nRow$

- exception: None

getNumCol():

- output: $out := nCol$
- exception: None

getScale():

- output: $out := scale$
- exception: None

count(t : T):

- output: $[out := +(\forall i : \mathbb{N} | i \in [0 \dots nRow] \bullet \forall j : \mathbb{N} | j \in [0 \dots nCol] \bullet s[i][j] = t : 1) \text{---SS}]$
- exception: None

count(l : LineT, t : T):

- output: $[out := +(\forall i : \mathbb{N} | i \in [l.strt.y \dots l.end.y] \bullet \forall j : \mathbb{N} | j \in [l.strt.x \dots l.end.x] \bullet s[i][j] = t : 1 \text{---SS})]$
- exception: $[\neg(validLine(l)) \implies invalid_argument \text{---SS}]$

count(pth : PathT, t : T):

- output: $[out := +(\forall k : \mathbb{N} | k \in [0 \dots pth.size-1] \bullet \forall i : \mathbb{N} | i \in [pth[k].strt.y \dots pth[k].end.y] \bullet \forall j : \mathbb{N} | j \in [pth[k].strt.x \dots pth[k].end.x] \bullet s[i][j] = t : 1 \text{---SS})]$
- exception: $[\neg(validPath(pth)) \implies invalid_argument \text{---SS}]$

length(pth : PathT):

- output: $[out := scl \times pth.len \text{---SS}]$
- exception: $[\neg(validPath(pth)) \implies invalid_argument \text{---SS}]$

connected(p_1 : PointT, p_2 : PointT):

- output: $[out := ((\forall k : \mathbb{N} | k \in [0 \dots pth.size-1] \bullet \exists p_1 : PointT | \bullet pointsInPath(pth)[k] = p_1) \wedge (\forall k : \mathbb{N} | k \in [0 \dots pth.size-1] \bullet \exists p_2 : PointT | \bullet pointsInPath(pth)[k] = p_2)) \implies true \text{---SS})]$
- exception: $[\neg(validPoints(p_1) \wedge validPoints(p_2)) \implies invalid_argument \text{---SS}]$

Local Functions

validRow: $\mathbb{N} \rightarrow \mathbb{B}$

$[\text{validRow}(i) \equiv 0 < i < nRow = \text{true}, \text{ where } i \rightarrow \mathbb{N} \text{ —SS}]$

validCol: $\mathbb{N} \rightarrow \mathbb{B}$

$[\text{validCol}(i) \equiv 0 < i < nCol = \text{true}, \text{ where } i \rightarrow \mathbb{N} \text{ —SS}]$

validPoint: $\text{PointT} \rightarrow \mathbb{B}$

$[\text{validPoint}(\text{point}) \equiv (\text{point}.x \wedge \text{point}.y) \geq 0 \wedge (\text{point}.x \wedge \text{point}.y) < (nRow \wedge nCol) \implies \text{true}, \text{ where } \text{point} \rightarrow \text{PointT} \text{ —SS}]$

validLine: $\text{LineT} \rightarrow \mathbb{B}$

$[\text{validLine}(l) \equiv (\text{validPoint}(l.start) \wedge \text{validPoint}(l.end)) \implies \text{true}, \text{ where } l \rightarrow \text{LineT} \text{ —SS}]$

validPath: $\text{PathT} \rightarrow \mathbb{B}$

$[\text{validPath}(i) \equiv \forall : \mathbb{N} | k \in [0..pth.size - 1] \bullet (\text{validLine}(pth[k])) \implies \text{true}, \text{ where } pth \rightarrow \text{PathT} \text{ —SS}]$

pointsInLine: $\text{LineT} \rightarrow (\text{set of PointT})$

$\text{pointsInLine}(l) [\text{pointsInLine}(l) \equiv i : \mathbb{N} | i \in [0..(l.len-1)] : l.start.translate($

$l.orient = N$	$(0, 1)$
$l.orient = S$	$(0, -1)$
$l.orient = W$	$(-1, 0)$
$l.orient = E$	$(1, 0)$

$\text{—SS}]$

pointsInPath: $\text{PathT} \rightarrow (\text{set of PointT})$

$[\text{pointsInPath}(p) \equiv \cup(k : \mathbb{N} | k \in [0..(p.size - 1)] : (\text{pointsInLine}(p[k]))), \text{ where } p \rightarrow \text{PathT} \text{ —SS}]$

LanduseMap Module

Template Module

LanduseMapT is Seq2D(LanduseT)

DEM Module

Template Module

DEMT is Seq2D(\mathbb{Z})

Critique of Design

In general, the specifications were easy to follow. However, one thing that could be added are hints to use the local functions for certain exceptions earlier on in each module because some time was wasted to create exceptions beforehand, only to realize that one could use the local functions as exceptions.