U-II

Hindustan College of Science & Technology, Farah, Mathura

Wave Mechanics –Tutorial-1

Que.1. Find the de-Broglie wavelength for an electron of energy V eV

Que.2. A particle of charge q and mass m is accelerated from rest through a potential difference V. Calculate its de-Broglie wavelength, if particle is an electron and potential difference V=50 volt.

(λ=12.28/√V A)

Que3. Calculate the velocity and kinetic energy of a neutron having de-Broglie wavelength 1 A.

(1.7366 A)

Que.4. A Proton is moving with a speed of $2x10^8$ m/Sec. Find the Wavelength of the matter wave associated with it.

($3.96x10^3$ m/sec; 0.082eV)

Que.5. Calculate the de-Broglie wavelength associated with a proton moving with a velocity equal to (1/20) th velocity of light.

($1.47x10^{-5}$ A)

Que.6. Calculate the de-Broglie wavelength of neutron of energy 28.8eV. (given h=$6.62x10^{-34}$J-Sec, m=$1.67x10^{-27}$Kg)

($2.643x10^{-14}$ m)

Que.7. Calculate the de-Broglie wavelength of an α particle accelerated through a potential difference of 200 Volts.

(0.05336 A)

Que.8. Calculate the de- Broglie weave length of a neutron having Kinetic energy of 1eV.

($7.16x10^{-3}$ A)

Que.9. Find the de-Broglie wavelength of a neutron of energy 12.8 MeV. (m=$1.675x10^{-27}$kg,h=$6.62x10^{-34}$J-sec,c=$3x10^8$m/sec, 1ev=$1.6x10^{-19}$joule)

(0.287 A)

Que.10. Calculate de-Broglie wavelength associated with nitrogen at 3.0 atmospheric pressure and 27 °C mass of N_2 atom=$4.65x10^{-26}$kg

($8.0x10^{-5}$ A)

Que.11. Show that the de- Broglie wavelength for a material particle of rest mass mo and charge q accelerated from rest through a potential difference of V Volt relativistically is given

(0.2754 A)

$\lambda=h/\sqrt{2m_oqv[1+qv/2m_oc^2]}^{1/2}$

Que.12. A particle of rest mass m_o has a kinetic energy K show that its de-Broglie wavelength is given

$\lambda=hc/\sqrt{K(K+2m_oc^2)}$

hence, calculate the wavelength of an electron of K.E> 1 MeV what will be the value of λ if K<<moc²

Or

A particle of rest mass m_o has a K.E. K what will be the value of λ if K<<m_oc^2 ?

Que.13. An electron has a speed of $1.05x10^4$m/sec with the accuracy of 0.01%. Calculate the uncertainty in the position of electron.

($1.115x10^{-4}$m)

Que.14. Hydrogen atom, say, has a radius of 0.5A calculate the K.E. needed by an electron to be confined to the atom.

(15.1eV)

Que.15. The Hydrogen atom is 0.53A in radius. Use Uncertainty principle to estimate the minimum energy, and electron can have in this atom.

(13.5eV)

Que.16. What is the minimum Uncertainty in the frequency of a photon whose life time is about 10^{-8} sec.

($15.92x10^6$ Hz)

Wave Mechanics

Tutorial sheet-1 Solution

Q.1 $\lambda = \frac{h}{\sqrt{2meV}}$ where $h = 6.62 \times 10^{-34}$
$m = 9.1 \times 10^{-31}$ kg.
$e = 1.6 \times 10^{-19}$

$\Downarrow$

$$\boxed{\lambda = \frac{12.28}{\sqrt{V}} \text{ A}^\circ}$$

Q.2 Here $V = 50$ volt, $\lambda = \frac{12.28}{\sqrt{V}} = 1.73 \text{ A}^\circ$

Q.3 given that $\lambda = 1 \text{ A}^\circ = 1 \times 10^{-10}$ m

To find: Velocity (v) = ? & KE = ?

$\Rightarrow$ $\lambda = \frac{h}{m \cdot v}$ so $v = \frac{h}{m \cdot \lambda} = \cancel{\frac{9.62 \times 10^{-34}}{(9.67 \times 10^{-27}) \times (1 \times 10^{-10})}}$

$$= \frac{6.62 \times 10^{-34}}{(1.67 \times 10^{-27})(1 \times 10^{-10})} = \boxed{v = 3.96 \times 10^{3} \text{ m/sec}}$$

Now $K.E = \frac{1}{2} m v^2$

$$= \frac{1}{2} \times (1.67 \times 10^{-27}) \times (3.96 \times 10^{3})^2$$

$$\boxed{KE = 1.309 \times 10^{-20} \text{ Joule}} \Rightarrow KE = \frac{1.309 \times 10^{-20}}{1.6 \times 10^{-19}}$$

$$\boxed{KE = 0.082 \text{ eV}}$$

Q.4 given $V_{(proton)} = 2 \times 10^{8}$ m/s

To find = $\lambda = ?$

Sol^n $\Rightarrow$ Since the velocity is under the range of relativistic velocity, so their must be an effect of mass variation.

i.e $\lambda = \frac{h}{p} = \frac{h}{m \cdot v}$ —(1)

where $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ so eq^n (1) will be

$$\lambda = \frac{h\sqrt{1-v^2/c^2}}{m_o v}$$

$$= \frac{6.62\times10^{-34}\sqrt{1-\left(\frac{2\times10^8}{3\times10^8}\right)^2}}{(1.67\times10^{-27})(2\times10^8)}$$

$$= \frac{(6.62\times10^{-34})(0.74)}{(1.67\times10^{-27})(2\times10^8)}$$

$$= \frac{4.898\times10^{-34}}{3.34\times10^{-19}}$$

$$\boxed{\lambda = 1.46\times10^{-15}\,m} \quad \text{or} \quad \boxed{\lambda = 1.46\times10^{-5}\,A^\circ}$$

Q.5 Given $v = \frac{1}{20}c = 1.5\times10^7\ m/s$

To find $\lambda = ?$

Sol^n Since velocity is must smaller than the relativistic limit, so concept of mass variation is not applied.

$$\lambda = \frac{h}{m\cdot v} = \frac{6.62\times10^{-34}}{(1.67\times10^{-27})(1.5\times10^7)}$$

$$\boxed{\lambda = 2.64\times10^{-14}\,m} \quad \text{OR} \quad \cancel{\lambda = 2.64\times10^{-4}}\,A$$

OR

$$\boxed{\lambda = 2.64\times10^{-4}\,A^\circ}$$

Q.6. Given $K\cdot E = 28.8\ eV = \cancel{288}\ 28.8\times1.6\times10^{-19}$

$$\boxed{KE = 46.08\times10^{-19}\ \text{Joule}}$$

Sol^n first of all we will check the status of relativistic case i.e. Rest mass energy of neutron $= m_n c^2 = (1.67\times10^{-27})\times(3\times10^8)^2$

$$\boxed{m_n c^2 = 1.503\times10^{-10}\ \text{Joul}}$$

in ev it is $\Rightarrow$ $\boxed{m_n c^2 = 939.4\ MeV}$

Here we can see that the K.E given in numerical is much less than the standard value of neutrons K.E. so will ~~not~~ ignore the case of relativity.

So

$$\lambda = \frac{h}{\sqrt{2m(KE)}} = \frac{6.62\times10^{-34}}{\sqrt{2\times(1.67\times10^{-34})\times(46.08\times10^{-19})}}$$

$$\boxed{\lambda = 5.336\times10^{-12}}$$

$$\boxed{\lambda = 0.053\ A^{\circ}}$$

Q.7 given

$V = 200$ volts

α particle $\Rightarrow$ $m_\alpha = 4\,m_p = 4\times1.67\times10^{-27}$

$$\boxed{m_\alpha = 6.68\times10^{-27}}$$

$q_\alpha = 2e = 2\times1.6\times10^{-19}$

$$\boxed{q = 3.2\times10^{-19}}$$

Sol^n

$$\lambda = \frac{h}{\sqrt{2m_\alpha qV}} = \frac{6.62\times10^{-34}}{\sqrt{2\times(6.68\times10^{-27})\times(3.2\times10^{-19})\times200}}$$

$$= 7.16\times10^{-13}\ m$$

$$\boxed{\lambda = 7.16\times10^{-3}\ A^{\circ}}$$

Q.8 sol^n of this question is as same as ~~of~~ question-6

Q.9 Sol^n of this question is also as that of question-6

Q.10 given:

$m = 4.65\times10^{-26}$ kg

$T = 300$ K

$K = 1.38\times10^{-23}$

$$\lambda = \frac{h}{\sqrt{3mKT}} = \frac{6.62\times10^{-34}}{\sqrt{3\times(4.65\times10^{-26})\times(300)\times(1.38\times10^{-23})}}$$

$$\boxed{\lambda = 2.75\times10^{-11}\,m} \text{ OR } \boxed{\lambda = 0.27\,A^{\circ}}$$

Q.11 Prove that

$$\lambda = \frac{h}{\sqrt{2m_0qV\left(1+\frac{qV}{2m_0c^2}\right)}}$$

$\Rightarrow$ 1 — $\boxed{\lambda = \frac{h}{P}}$, Here first of all we will find the value of P.

i.e
$$E^2 = P^2c^2 + m_0^2c^4$$

$$(KE + m_0c^2)^2 = P^2c^2 + m_0^2c^4 \quad (\because E = KE + m_0c^2)$$

$$(qV + m_0c^2)^2 = P^2c^2 + m_0^2c^4 \quad (\because KE = qV)$$

$$q^2V^2 + \cancel{m_0^2c^4} + 2m_0c^2qV = P^2c^2 + \cancel{m_0^2c^4}$$

$$q^2V^2 + 2m_0qVc^2 = P^2c^2$$

$$\frac{q^2V^2}{c^2} + 2m_0qV = P^2 \quad \text{OR} \quad P^2 = 2m_0qV + \frac{q^2V^2}{c^2}$$

2 — $\boxed{P = \sqrt{2m_0qV\left(1+\frac{qV}{2m_0c^2}\right)}}$

So by eqn 1 & 2

$$\boxed{\lambda = \frac{h}{\sqrt{2m_0qV\left(1+\frac{qV}{2m_0c^2}\right)}}} \text{ Proved.}$$

Q.12 P.T. $\lambda = \frac{hc}{\sqrt{K(K+2m_0c^2)}}$

$\Rightarrow$ 1 — $\boxed{\lambda = \frac{h}{m \cdot V}}$ = $\cancel{\frac{h}{\left(\frac{m_0}{\sqrt{1-V^2/c^2}}\right) \cdot V}}$

as we know ~~that~~ which $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$

$$\sqrt{1-v^2/c^2} = \frac{m_0}{m}$$

$$\left(1-\frac{v^2}{c^2}\right) = \frac{m_0^2}{m^2}$$ (By squaring both sides)

$$1-\frac{m_0^2}{m^2} = \frac{v^2}{c^2}$$

$$(m^2-m_0^2)c^2 = m^2v^2$$

$$\cancel{m^2c^2 - m_0^2c^2 = m^2v^2}$$

$$\boxed{c\sqrt{m^2-m_0^2} = m\cdot v}$$ ——— (2)

By eqn (1) & (2)

$$\lambda = \frac{h}{c\sqrt{m^2-m_0^2}}$$

$$= \frac{hc}{c^2\sqrt{(m^2-m_0^2)}}$$ (By multiply g c).

$$= \frac{hc}{\sqrt{c^4(m^2-m_0^2)}}$$

$$= \frac{hc}{\sqrt{c^2(m-m_0)\{c^2(m+m_0)\}}}$$

$$= \frac{hc}{\sqrt{K\{(m-m_0)c^2+2m_0c^2\}}}$$ ($\because (m-m_0)c^2 = K$)

$$\boxed{\lambda = \frac{hc}{\sqrt{K(K+2m_0c^2)}}}$$ proved.

for $K = 1\ MeV$; $\lambda = \dfrac{hc}{\sqrt{K(K+2m_oc^2)}} = \underline{8.78 \times 10^{-3}\ A^o}$.

if $K << m_oc^2$; $K + 2m_oc^2 = \underline{2m_oc^2}$.

so $\lambda = \dfrac{hc}{\sqrt{2m_oKc^2}} = \boxed{\dfrac{h}{\sqrt{2m_oK}} = \lambda}$

Q.13. Given $V = 1.05 \times 10^4\ m/sec$

Accuracy $= 0.01\%$

$\Rightarrow$

$(\Delta x)\cdot(\Delta p) \approx \dfrac{h}{2\pi}$

$(\Delta x)\cdot m\cdot(\Delta V) \approx \dfrac{h}{2\pi}$ $(\because p = m\cdot v)$

$(\Delta x) \approx \dfrac{h}{2\pi\cdot m\cdot(\Delta V)}$ where $\Delta V = (1.05 \times 10^4) \times \dfrac{0.01}{100}$

$\boxed{\Delta V = 1.05}$

$= \dfrac{6.62 \times 10^{-34}}{2 \times (3.14) \times (9.1 \times 10^{-31}) \times (1.05)}$

$\boxed{\Delta x = 1.10 \times 10^{-4}\ m}$

Q.14 given $\Delta x = 0.5\ A^o = 0.5 \times 10^{-10}\ m$

So $(\Delta x)\cdot(\Delta p) = \dfrac{h}{2\pi} \Rightarrow (\Delta p) = \dfrac{h}{2\pi\cdot(\Delta x)}$

$\boxed{\Delta p = 2.1 \times 10^{-24}\ kg\ m/sec}$

So $K.E = \dfrac{(\Delta p)^2}{2m} = \cancel{4.41 \times 10^{-17}}$

$\boxed{KE = 15.1 eV}$

Q.15 Same as ~~eq~~ question-14

Q.16. $(\Delta E)\cdot(\Delta t) \approx \dfrac{h}{2\pi}$

$h(\Delta \nu)\cdot(\Delta t) \approx \dfrac{h}{2\pi}$ $(\because E = h\nu)$

$(\Delta \nu) = \dfrac{\cancel{h}}{\cancel{h}\,2\pi\cdot(\Delta t)}$

$(\Delta \nu) = \dfrac{1}{2\pi\cdot(\Delta t)} =$

$\boxed{\Delta \nu = 15.92 \times 10^6\ sec^{-1}}$