AN ACADEMIC PROPOSAL
ON THEORETICAL STUDY AND DATA ANALYSIS OF GRAVITATIONAL WAVE SIGNALS FROM BINARY STARS COALESCENCE

Submitted to the Department of Physics, St. Xavier's College,
In Partial fulfillment for the requirement of Bachelor's Degree of Science in Physics

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This proposal of project work entitled 'THEORETICAL STUDY AND DATA ANALYSIS OF GRAVITATIONAL WAVE SIGNALS FROM BINARY STARS COALESCENCE' submitted to the Department of Physics, St. Xavier's College by Shivaji Chaulagain has been certified by the supervisor and accepted by the Head of Department of Physics.

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1 Introduction

1.1 Background

1.1.1 History

Gravitational waves are a fundamental prediction of Albert Einstein's theory of general relativity, proposed in 1916 [7]. However, their direct detection was a scientific challenge that spanned nearly a century. It wasn't until 2015 that the Laser Interferometer Gravitational-Wave Observatory (LIGO) made history by detecting gravitational waves from a binary black hole merger, marking the first direct observation of these elusive waves. Subsequent discoveries, including the detection of binary neutron star mergers, have further solidified the existence of gravitational waves and opened a new era in astrophysics, offering a unique way to explore the universe's most enigmatic phenomena.

1.1.2 Gravitational Waves

Gravitational waves* are 'ripples' in space-time caused by some of the most violent and energetic processes in the Universe. Einstein's mathematics showed that massive accelerating objects (things like neutron stars or black holes orbiting each other) would disrupt space-time in such a way that 'waves' of undulating space-time would propagate in all directions away from the source. These cosmic ripples would travel at the speed of light, carrying with them information about their origins, as well as clues to the nature of gravity itself.

1.1.3 Black Hole

A black hole is a region of spacetime where gravity is so strong that nothing, not even light, can escape. The theory of general relativity predicts that a sufficiently compact mass can deform spacetime to form a black hole. Black holes are thought to be formed when massive stars collapse at the end of their lives. When a star has exhausted the nuclear fuel in its core, it can no longer support its own weight and collapses inward. If the star's core is massive enough, it will collapse to a point of infinite density, forming a black hole.

1.1.4 Binary Black Hole

Binary black holes are pairs of black holes that orbit around a common center of mass due to their gravitational attraction. As they orbit each other, they emit gravitational waves, which cause them to spiral closer and closer together(see fig 2). Eventually, the two black holes will merge, releasing a tremendous amount of energy in the form of gravitational waves. The existence of stellar-mass binary black holes was finally confirmed when LIGO detected GW150914 [1], a distinctive gravitational wave signature of two merging stellar-mass black holes of around 30 solar masses each, occurring about 1.3 billion light-years away.

1.1.5 Neutron Star

Neutron stars are formed when a massive star runs out of fuel and collapses. The very central region of the star – the core – collapses, crushing together every proton and electron into a neutron. If the core of the collapsing star is between about 1 and 3 solar masses, these newly-created neutrons can stop the collapse, leaving behind a neutron star (Stars with higher masses will continue to collapse into stellar-mass black holes). Neutron stars are the smallest and densest known class of stellar objects. Most neutron stars are observed as pulsars. Pulsars

*https://www.ligo.org/science/overview.php are rotating neutron stars observed to have pulses of radiation at very regular intervals that typically range from milliseconds to seconds. Pulsars have very strong magnetic fields which funnel jets of particles out along the two magnetic poles. These accelerated particles produce very powerful beams of light.

1.1.6 Binary Neutron Star

Binary neutron stars are pairs of neutron stars that orbit around a common center of mass due to their gravitational attraction. Binary neutron stars are some of the most energetic objects in the universe. As they orbit each other, they emit gravitational waves, which cause them to spiral closer and closer together. Eventually, the two neutron stars will merge, releasing a tremendous amount of energy in the form of gravitational waves and electromagnetic radiation. The first binary pulsar, PSR B1913+16, was discovered in 1974 by Hulse and Taylor [8], who won the Nobel Prize in Physics in 1993 for their work on this system. On August 17, 2017, the LIGO and Virgo detectors observed the GW170817 gravitational wave signal [2], emanating from the elliptical galaxy NGC 4993, resulting from the final moments of a binary neutron star pair's inspiral and subsequent merger.

1.1.7 Sources of Gravitational Waves

Gravitational wave sources fall into four main categories (see fig 1):

Short-lived and well-modeled Sources: This includes compact object pairs like neutron stars or black holes in tight orbits. They emit waves during inspiral, merger, and damping phases, making them theoretically modelable and detectable by observatories like LIGO.

Short-lived and Poorly Understood Sources: Supernovae and long-duration gamma-ray bursts (GRBs) may emit gravitational waves due to asymmetries in their explosions. These sources are challenging to model due to uncertainties in initial conditions and dynamics.

Long-lived and Well-modeled Sources: Non-axisymmetric, rapidly spinning neutron stars produce periodic, nearly monochromatic waves. These sources, found within our galaxy, emit detectable waves over many cycles.

Long-lived, Diffuse Sources: This category includes the primordial background of gravitational waves predicted by cosmological theories and could result from phenomena like cosmic string snapping or numerous distant binary coalescences. Detecting very-low-frequency signals may involve correlations across multiple detectors

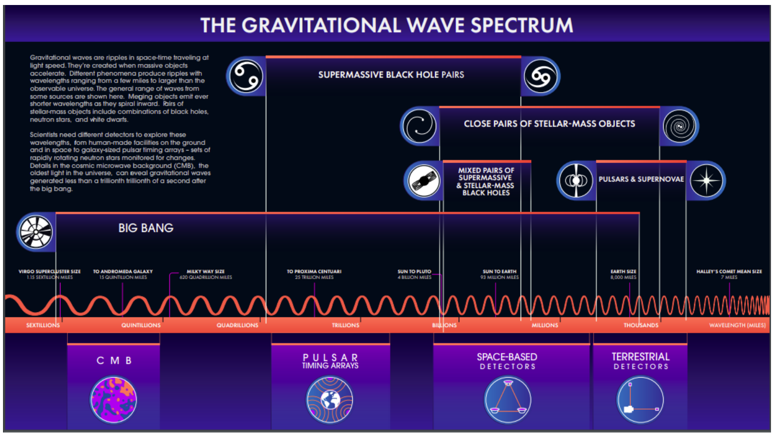


Figure 1: Gravitational-wave spectrum (credit: NASA)

1.1.8 Types of Gravitational Waves

Gravitational waves are categorized into four main groups based on their sources:

Continuous Gravitational Waves: Emitted by rapidly spinning, non-axisymmetric objects, like certain neutron stars, producing periodic, nearly monochromatic waves.

Compact Binary Inspiral Gravitational Waves: Arise from the inspiral and merger of compact object pairs, such as binary neutron stars or black holes, generating a characteristic chirping signal.

Stochastic Gravitational Waves: Result from various unresolved sources, forming a background of waves, requiring statistical analysis and correlations across detectors for detection.

Burst Gravitational Waves: Associated with sudden, transient events like supernovae or gamma-ray bursts, challenging to predict and model due to their unpredictability.

1.1.9 Phase of Gravitational Waves

The phase of a gravitational wave signal is the time evolution of the wave's oscillation. Gravitational wave signals can be divided into three phases(see fig 2):

- **Inspiral:** This is the phase during which the two objects spiral towards each other, emitting gravitational waves as they do so. The inspiral phase is characterized by a gradual increase in frequency and amplitude.
- **Merger:** This is the phase during which the two objects collide and merge. The merger phase is characterized by a rapid increase in frequency and amplitude, followed by a rapid decrease.
- **Ringdown:** This is the phase during which the newly formed object settles down into a stable configuration. The ringdown phase is characterized by a damped oscillation at a characteristic frequency.

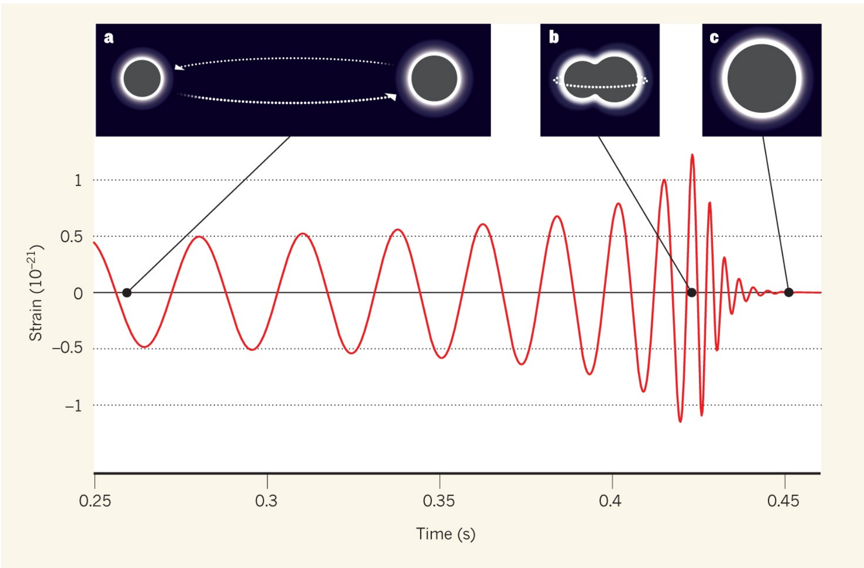


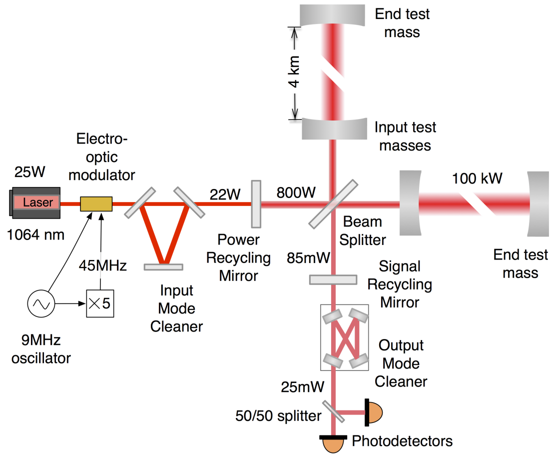
Figure 2: A gravitational wave from merging black holes.(a) Inspiral Phase.(b) Merger Phase.(c) Ringdown Phase

1.1.10 Method of Gravitational Wave Detection

Gravitational waves can be detected using two main methods: laser interferometry and pulsar timing arrays.

1. **Laser Interferometry:** This method involves measuring tiny changes in the distance between two mirrors separated by a long distance. When a gravitational wave passes through the detector, it stretches and squeezes space, causing slight changes in the mirror distance. Lasers and highly sensitive detectors are used to measure these changes with great precision. Laser interferometry detectors include LIGO, VIRGO, GEO600 and KAGRA.

(a) LIGO



(b) LIGO's Interferometers

Figure 3: Example of Laser Interferometry

2. **Pulsar Timing Arrays:** Pulsar timing arrays (PTAs) are networks of radio telescopes that are used to monitor the arrival times of pulses from millisecond pulsars. PTAs are less sensitive than laser interferometer gravitational wave detectors. However, PTAs can detect lower-frequency gravitational waves, which makes them well-suited for detecting gravitational waves from sources such as supermassive black hole mergers and cosmic strings. Examples are NANOGrav, EPTA and IPTA.

1.1.11 Noises

"Noise" is unwanted and random fluctuations or signals that can obscure or mask the true gravitational wave signals. Some common sources of noise in gravitational wave detectors include:

- **Seismic Noise:** Ground vibrations caused by seismic activity, as well as other environmental factors like wind and temperature fluctuations, can introduce noise into the detector. To filter out these disturbances, the optical components are suspended to a series of several pendulums, each hanging from the above.

- **Thermal Noise:** This source of noise in gravitational wave detectors is associated with the thermal vibrations of the mirrors and their suspensions. The steel wire that suspends the mirror is typically at room temperature, in thermal equilibrium with the surrounding environment. These thermal fluctuations can induce motion in the mirror, which, in turn, changes the length of the interferometer's arm.

- **Quantum Noise:** Quantum mechanics imposes fundamental limits on the precision of measurements. Quantum noise arises due to the inherent uncertainty in position and momentum of particles. It can affect the accuracy of interferometric measurements used in many gravitational wave detectors.
- **Shot Noise:** Photon shot noise is the major limiting source of disturbance at frequencies above 200 Hz and results from the finite number of photons arriving at the photo-detector. Shot noise can be reduced by increasing the laser power. In order to be able to detect gravitational waves with frequency 100 Hz, the intensity of the laser would need to be of the order of 100 W, a value beyond the capability of any existing continuous laser.
- **Mirror Coating Noise:** The reflective coatings on mirrors used in interferometers can introduce noise when they fluctuate or deform, affecting the quality of the laser beam's reflection.
- **Electromagnetic Interference (EMI):** External electromagnetic signals, including radio frequency interference (RFI) and electronic noise, can interfere with detector operations.

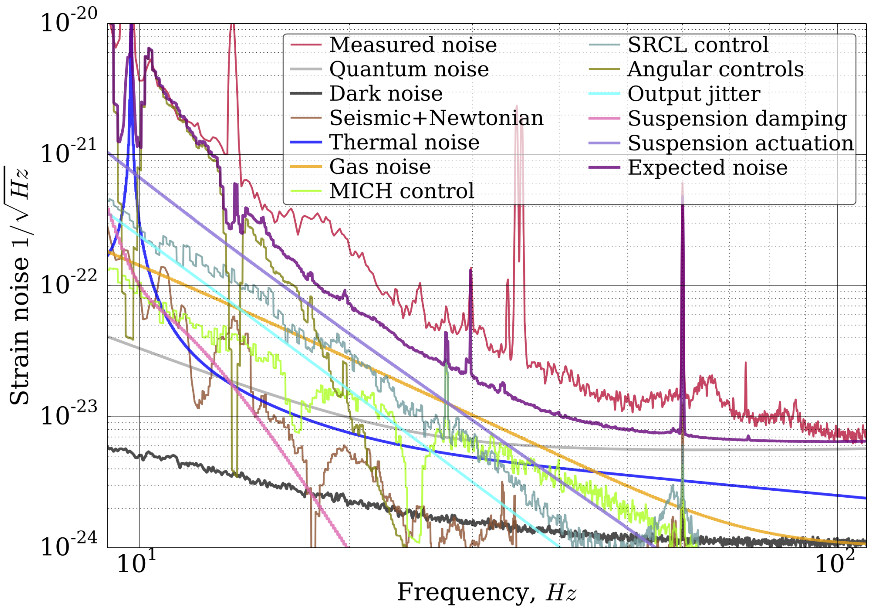


Figure 4: Accounting of sources of noise that combine to form the limiting sensitivity of the detectors. The units on the y-axis are those of amplitude spectral density.

1.1.12 Discrete Fourier Transform (DFT)

Discrete Fourier Transform (DFT)* is a mathematical technique used to convert a discrete time-domain signal into its frequency-domain representation. It involves computing the complex exponential sum of the signal and its frequency components. DFT converts a sequence of N complex number* $x_0, x_1, ..., x_{N-1}$ to a new sequence of N complex number,

$$X_k = \sum_{n=0}^{N-1} x_n . e^{-2\pi ikn/N} \tag{1}$$

for $0 \leq k \leq N-1$.
The x_n are thought of as the values of a signal, at equally spaced time interval. The output X_k is

*For more understanding of signal processing, refer to [10, 13]
*in practise, time domain signal/sample signal x_n are real but outputs are complex

a complex number which encodes the amplitude and phase of a sinusoidal wave with frequency $\frac{k}{N}$ cycles per time unit. The frequency corresponding to k is given by $f_k = k.F_s/N$, where F_s is the sampling frequency.

The inverse DFT "x_n" is:

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k.e^{2\pi ikn/N} \tag{2}$$

The traditional implementation of the DFT has a time complexity of $O(N^2)$, where N is the number of data points. This means that the computation time increases quadratically with the size of the input.

However, thanks to the Fast Fourier Transform (FFT) algorithm [6], the computational cost of computing the DFT can be significantly reduced. The FFT is an efficient algorithm for calculating the DFT, and its time complexity depends on the specific implementation but is typically $O(N.logN)$.

1.1.13 Windowing

Windowing † in signal processing involves multiplying a signal by a mathematical function known as a window function. This function is typically zero-valued outside of a specific interval and smoothly tapers at its edges. The primary purpose of windowing is to select a portion of a signal for analysis while reducing the disruptive effects of discontinuities at the signal's edges. Windowing is especially important in spectral analysis, such as when performing a fast Fourier transform (FFT), to mitigate spectral leakage, which occurs when energy from one frequency bin spreads to others due to non-periodic signals or incompatible window lengths.
Some of the most common window functions include:

- **Rectangular window:** The simplest window function, it has a constant value of one within the selected interval and zero outside. However, it exhibits the highest spectral leakage, making it less ideal for applications where precise spectral analysis is crucial.
- **Hanning Window:** Offering a better trade-off between spectral leakage and main lobe width compared to the rectangular window, the Hanning window tapers smoothly from one at the center to zero at the edges. This results in reduced spectral leakage but a slightly wider main lobe in the frequency domain.
- **Hamming Window:** Similar to the Hanning window, the Hamming window tapers smoothly, but it has a narrower main lobe in the frequency domain. It provides better spectral leakage reduction at the expense of a slightly wider main lobe compared to the Hanning window.
- **Blackman Window:** This window function minimizes spectral leakage effectively, making it suitable for applications requiring accurate spectral analysis. However, it has a wide main lobe in the frequency domain, which can affect frequency resolution.

1.1.14 Significance of Gravitational Waves

Gravitational waves are of immense significance in astrophysics and science at large. They affirm Einstein's theory of general relativity, offer a novel tool for astronomy, and enable the study of cataclysmic cosmic events. Detection of gravitational waves from binary black hole and neutron star mergers has opened up a new era of multi-messenger astronomy, enhancing our understanding of the universe's extreme conditions. These waves also probe extreme physics, aiding the investigation of dark matter, dark energy, and the properties of massive celestial

† `https://dspillustrations.com/pages/posts/misc/spectral-leakage-zero-padding-and-frequency-resolution.html`

objects. Gravitational wave observations inspire scientific curiosity, technological innovation, and hold the promise of unveiling new cosmic mysteries in the future.

1.2 Motivation

The research project is deeply rooted in the transformative impact of gravitational wave astronomy on our understanding of the universe. Gravitational wave detectors, such as LIGO and Virgo, have ushered in a new era of exploration, enabling us to directly observe and interpret the gravitational whispers of the cosmos' most cataclysmic events. Among these celestial phenomena, binary systems composed of neutron stars and black holes stand as captivating and mysterious cosmic laboratories where the boundaries of our understanding are continually pushed to the extreme. I intend to study these events: GW170817, GW190521,and GW190814.

GW170817: Studying GW170817 is crucial as it revolutionized astronomy. It was the first event observed in both gravitational waves and light, showcasing the power of using different methods to study space. This event also provided insights into neutron stars and confirmed how heavy elements like gold are formed in space, advancing our understanding of the cosmos.

GW190521: GW190521, observed as a binary black hole merger, captured attention due to its unique characteristics. It featured an unusual mass ratio and is believed to have involved the formation of an intermediate-mass black hole. This event challenges conventional models of black hole evolution and dynamics, adding to the intrigue of our study.

GW190814: GW190814 remains an intriguing enigma in the realm of gravitational wave astronomy. This event, which may have involved a neutron star-black hole binary, presents a unique challenge. Our aim is to uncover the true nature of the merger, further our understanding of compact object binaries, and potentially unveil new astrophysical phenomena.

Through these three events, the correlation among the gravitational wave events GW170817, GW190521, and GW190814 can be studied by comparing their waveforms, source properties, and astrophysical implications. Analyzing these events together allows us to identify commonalities and differences in the characteristics of binary systems, such as mass distributions, spins, and orbital dynamics. This comparative analysis can deepen our understanding of compact object mergers and their broader astrophysical context, enhancing our knowledge of extreme cosmic phenomena.

2 Literature Review

2.1 Einstein Field Equations

General relativity asserts that the curvature of space-time causes gravity [12, 14] ‡. The presence of matter curves space-time and the curvature in turn determines the behaviour of matter. The Einstein field equations for the gravitational field is:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}, \tag{3}$$

where $g_{\mu\nu}$ is the metric tensor of the four dimensional space-time and $T_{\mu\nu}$ is the stress energy tensor that encodes the matter content and R is Ricci scalar also known as scalar curvature.
Furthermore, we are interested in the study of gravitational radiation, treated as a small perturbation that propagates through a flat space-time. In this weak-field situation there exist coordinate systems where the components of the metric can be decomposed as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| << 1 \quad throughout\ space-time, \tag{4}$$

where $\eta_{\mu\nu}$ is the Minkowski metric that describes a space-time with no curvature. Such coordinates are particularly suitable to solve equation 3, which predicts gravitational waves. Applying ansatz 4 to equation 3 and solving for the perturbative radiation field to first order in $h_{\mu\nu}$ yields the linearized Einstein equations

$$-\partial^{\alpha}\partial_{\alpha}h'_{\mu\nu} - \partial^{\alpha}\partial^{\beta}h'_{\alpha\beta} + \partial^{\alpha}\partial_{\mu}h'_{\nu\alpha} + \partial^{\alpha}\partial_{\mu}h'_{\mu\alpha} = 16\pi T_{\mu\nu} \tag{5}$$

on the field $h'_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h_{\mu\nu}$. In the linear approximation this implies that two perturbations $h_{\mu\nu}$ and $h'_{\mu\nu}$ represent the same physical phenomenon if they are related by a transformation of the form

$$h'_{\mu\nu} \to h_{\mu\nu} + \partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}, \tag{6}$$

where ξ^{α} is a vector field. Without loss of generality, a field ξ^{α} can be found such that the following guage condition is verified

$$\partial^{\alpha}h'_{\mu\alpha} = 0 \qquad (Lorentz\ guage), \tag{7}$$

in clear analogy with lorentz guage condition for the electromagnetic tensor $\partial_{\alpha}A^{\alpha}$. In the case of the linearized Einstein equations we need to impose the additional condition of being far away from the sources, so that the weak field condition 4 is satisfied. This taken into account, equation 5 in the lorentz guage simplifies to

$$\partial^{\alpha}\partial_{\alpha}h'_{\mu\nu} = 0 \quad (in\ vacuum) \tag{8}$$

Thus, we can always arrive at the guage

$$h' = 0 \tag{9}$$

$$h'_{0i} = 0 \qquad (i = 1,2,3) \quad (in\ a\ sourcefree\ region) \tag{10}$$

$$h'_{00} = 0 \qquad (if\ no\ sources\ are\ present\ anywhere) \tag{11}$$

which is referred to as radiation guage. In this transverse-traceless guage $h'_{\mu\nu} = h_{\mu\nu}$. The Einstein field equations in vacuum far away from the source of the field take the form

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) h_{\mu\nu} = 0, \tag{12}$$

‡For Einstein paper, refer to `https://einsteinpapers.press.princeton.edu/`

a wave equation for the gravitational radiation that admits the plain wave solution

$$h_{\mu\nu} = a_{\mu\nu}.e^{ik_\alpha x^\alpha}, \tag{13}$$

where $a_{\mu\nu}$ is a four-dimensional symmetric tensor containing the amplitude of the different components of the wave and k^α is the wave vector. If we further orient the direction of propagation of the wave along the z-axis so that $k^\alpha = (\omega, 0, 0, \omega)$ then $a_{\alpha z} = 0$ for all α. These condition reduce the number of independent components of $a_{\mu\nu}$ from ten to only two

$$a_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a_{xx} & a_{xy} & 0 \\ 0 & a_{xy} & -a_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{14}$$

hence the final form of the solution to the source-free, linearized Einstein equations for the perturbative field is

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} .e^{i\omega(z-t)}, \tag{15}$$

with h_+ and $h_\times$ representing the two polarization states of the wave. A general gravitational wave can be written as a linear combination of the *plus* and *cross* components $h = h_+\hat{e}_+ + h_\times\hat{e}_\times$ in the orthonormal basis of vectors

$$\hat{e}_+ = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \hat{e}_\times = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{16}$$

Rotation of the x- and y-axes in the transverse plane by an angle ψ change the polarization components in the follow way

$$\begin{aligned} h'_+ &= h_+ \cos 2\psi + h_\times \sin 2\psi \\ h'_\times &= -h_+ \sin 2\psi + h_\times \cos 2\psi, \end{aligned} \tag{17}$$

which indicates that general relativistic gravitational waves have spin two.

2.2 Effects of Gravitational Waves on Test Particles

Consider two free falling particles A and B in a background lorentz frame and choose TT guage introduced in above section associated to this frame. [12, 14, 17]
Let's denote ξ^α the connecting vector between A and B. Free particles obey the geodesic equation for their 4-velocity u^α

$$u^\alpha \nabla_\alpha u^\beta = 0 \tag{18}$$

In a curved space-time, the second derivative of the vector ξ^α is non zero, which means that there is an acceleration between particles A and B. We will get the result

$$\frac{d^2\xi^i}{dt^2} = \frac{1}{2}\frac{d^2 h_{ij}}{dt^2}\xi^j \tag{19}$$

where i,j = 1,2 representing the x- and y-directions. A ring of particles placed at rest on the xy-plane in an initially wave-free region of space-time encounters a gravitational wave travelling along the z-direction. The arrival of the wave modifies the proper distance between the particles. The *plus* polarization of the wave stretches and squeezes the ring along the x- and y-axes, oscillating between the shapes displayed on panel (b) of figure 5.

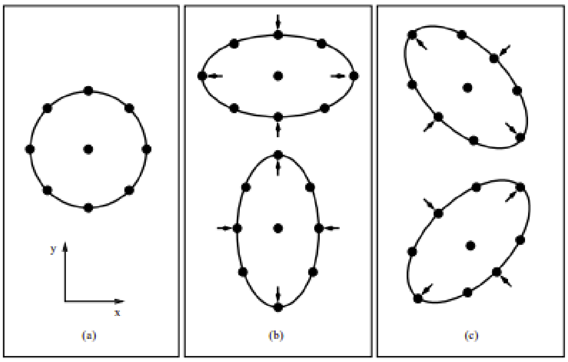


Figure 5: Effect of the two polarizations of a gravitational wave propagating through a ring of test particles. (a) A ring of free particles before a wave travelling in the z-direction reaches them. (b) Distortion produced in the ring by the plus polarization h_+, which modifies the proper distance between the particles in the ring, altering its geometry as the phase of the gravitational wave changes through a complete oscillation cycle. (c) Same as (b) but for the cross polarization h_+

Once the effect of a passing gravitational wave on a pair of test particles is known, we can proceed to devise mechanisms for detecting that effect, for example, by means of a laser and an interferometer of arm's length L. Assuming the wavelength of the gravitational wave is much larger than the size the interferometer, this simplifies the integration of equation 19

$$\delta x = \frac{h_{xx}}{2}x, \quad \delta y = \frac{h_{yy}}{2}y. \tag{20}$$

When a gravitational wave passes along the z-direction with polarization $h_{xx} = -h_{yy} = h_+(t)$, the change in proper distance and the phase difference between the two beams at the origin are

$$\frac{\delta L(t)}{L} = h_+(t), \qquad \Delta\phi = 2\pi\frac{L}{\lambda}h_+(t) \tag{21}$$

The interferometer response, i.e. the difference in phase between the beams recombining at the beam splitter is proportional to

$$h(t) \propto \delta(\Delta\phi) \equiv F_+ h_+(t) + F_\times h_\times(t), \tag{22}$$

where F_+ and $F_\times$ are the antenna patterns of the detector. The quantity h is the *gravitational wave strain*.
In this derivation it is implicitly assumed that the spatial variation of the gravitational wave in the interferometer's arms is negligible. The temporal variation of $h(t)$ in during the short time $\approx 2L$ that it takes for the light to travel back and forth to the mirror is not taken into account. We will study details in coming days.

2.3 Gravitational Waves Of Binary Stars

Gravitationally-bound binaries, which consist of compact objects orbiting each other, such as neutron stars or black holes, undergo a coalescence process during which they release a portion of their energy as gravitational radiation. These coalescing binaries hold special significance in gravitational-wave astronomy, as they are highly promising candidates for initial detection. The gravitational radiation emanating from pairs of neutron stars and/or black holes can be precisely described using various analytical and numerical theoretical approaches. We will study theoretical and numerical model thoroughly later. For further study and references, refer to [14, 9].

2.4 Gravitational Wave Events

A century ago, Einstein proposed gravitational waves as spacetime ripples resulting from mass acceleration, although doubts lingered. Detecting these waves posed a formidable challenge spanning decades. Additionally, Einstein's theory predicted the existence of black holes—cosmic entities formed through intense spacetime curvature. The understanding of black holes became closely linked with the pursuit of gravitational wave detection. It wasn't until the late 1950s that calculations began to solidify the belief in the existence of gravitational waves. Finally, in the 1970s, indirect evidence emerged when astronomers observed a double pulsar system PSR 1913+16 [8] for which measurements of the decay of the orbital period with time are consistent with the energy losses expected for gravitational-wave emission, confirming the energy loss predicted by gravitational wave theory.

On September 14, 2015, the LIGO collaboration made history by detecting GW150914, the first gravitational wave event. This monumental discovery, results from the merger of two massive black holes.

Further, On August 17, 2017, the Advanced LIGO and Advanced Virgo detectors observed GW170817, marking the first binary neutron star inspiral detection. The signal had a high signal-to-noise ratio (32.4) and a rare false-alarm rate. It suggested component masses between 0.86 and 2.26 solar masses or 1.17 to 1.60 solar masses when considering neutron star spins. The total system mass was estimated at 2.74 solar masses. The event was localized within 28 square degrees, the closest such localization. It was associated with gamma-ray burst GRB 170817A, linking neutron star mergers to short gamma-ray bursts [2].

In May 2019, the Advanced LIGO and Advanced Virgo detectors observed the gravitational wave event GW190521 [3, 11], characterized by a high signal-to-noise ratio and a low estimated false-alarm rate. This event is believed to result from the merger of two black holes, with estimated masses of approximately 85-66 solar masses. The primary black hole's mass is likely within a range produced by pair-instability supernova processes. The remnant black hole has an estimated mass of about 142 solar masses, classifying it as an intermediate mass black hole (IMBH). The remaining 9 solar masses were radiated away as energy in the form of gravitational waves [4, 3]. The source of GW190521 is located at a luminosity distance of $5.3Gpc$, corresponding to a redshift of 0.82. The inferred rate of similar black hole mergers in the universe is approximately 0.13 $Gpc^{-3}yr^{-1}$. These findings provide valuable insights into black hole mergers and their astrophysical implications.

GW190814, observed on August 14, 2019, by LIGO and Virgo, represents a significant gravitational wave event. It involves a compact binary coalescence featuring an unequal mass ratio, with a black hole ranging from 22.2 to 24.3 solar masses and a compact object between 2.50 and 2.67 solar masses. The signal boasted a high signal-to-noise ratio and was localized to a distance of 241 Mpc. This event challenges current astrophysical models due to its unique mass ratio, component masses, and estimated merger rate density, which ranges from 1 to 23 $Gpc^{-3}yr^{-1}$. Furthermore, tests of general relativity revealed no deviations, confirming its predictions for higher-multipole emission. The origin and characteristics of GW190814 raise intriguing questions in the field of compact-object binary formation [5, 16, 15].

3 Objectives

This project embarks on a mission to explore, both theoretically and through data analysis, the intricate gravitational wave signals emanating from these extraordinary celestial collisions.

- To develop a comprehensive understanding of the theoretical foundations of gravitational waves, including the equations of general relativity that govern their propagation, with a specific focus on binary neutron star, binary black hole and neutron star-black hole coalescence events.
- To confirm the consistency of gravitational waveforms observed in events like GW170817, GW190521, and GW190814 with the waveform predictions of General Relativity (GR).
- To gather and prepare actual gravitational wave data from observatories like LIGO and Virgo for detailed analysis.
- To apply advanced data analysis techniques, including matched filtering, parameter estimation, and Bayesian analysis, to extract relevant information from the gravitational wave signals.
- To identify and select specific binary neutron star-binary black hole coalescence events from the dataset for in-depth analysis.
- To gain insights into the extreme physics associated with neutron stars and black holes, particularly their behavior under the influence of strong gravitational fields.
- To propose possible paths for future research in understanding gravitational waves, especially those from binary neutron star-binary black hole systems.

4 Data Sets and Methodologies

In this section, I outline the methodologies and techniques I intend to employ for the gravitational wave data analysis.

- **Data Acquisition:** I will access open datasets[§] provided by LIGO and Virgo collaborations, which contain information about detected gravitational wave events. These datasets will serve as the basis for my analysis.
- **Data Preprocessing:** I will implement data preprocessing techniques to clean and prepare the raw gravitational wave data for analysis. This will involve noise removal, calibration, time alignment, and data quality checks using Python and relevant libraries.
- **Signal Detection:** To detect gravitational wave signals in the data, I will employ matched filtering techniques and waveform template matching. I will use Python packages such as PyCBC and gwpy for this purpose.
- **Parameter Estimation:** For parameter estimation of detected events, I will utilize Bayesian techniques, estimating parameters like mass, spin, and event time, and incorporating prior information.
- **Data Visualization:** I will create visualizations of the data, including time-frequency plots and spectrograms, to aid in the interpretation and understanding of the gravitational wave signals.

[§] `https://gwosc.org/eventapi`

- **Statistical Analysis:** To assess the significance of detected events and estimate uncertainties, I will employ statistical methods and calculations, making use of Python's scientific computing libraries.
- **Quality Control:** I will implement quality control checks to ensure the reliability and accuracy of the analysis results using gwosc python library.
- **Interpretation:** Finally, I will interpret the results in the context of astrophysical implications and contributions to the field of gravitational wave astronomy.

I will be using a number of software packages and Python packages to clean and prepare raw gravitational wave data for analysis. Some common examples include:

- **Gwosc:** The GWOSC Python package[¶], known as gwosc, serves as a powerful tool for accessing and working with gravitational wave data and event-related information from the Gravitational-Wave Open Science Center (GWOSC). It simplifies data retrieval, allows for data filtering and visualization, provides event-specific details, and offers access to critical metadata.
- **GWpy:** GWpy[‖] is a Python package that provides a number of tools for processing and analyzing gravitational wave data, including filtering, windowing, and decimation. I use GWpy to remove noise from the data and to make it more periodic.
- **LALSuite:** LALSuite is a software package that provides a number of tools for processing and analyzing gravitational wave data, including whitening and glitch removal. I use LALSuite to remove the effects of the detector noise from the data and to identify and remove glitches.
- **PyCBC:** PyCBC[**] is a Python package that provides a number of tools for analyzing gravitational wave data from merging black holes and neutron stars, including filtering, windowing, and downsampling. I use PyCBC to prepare the data for analysis using specialized techniques that are designed for detecting and analyzing gravitational wave signals from merging black holes and neutron stars.

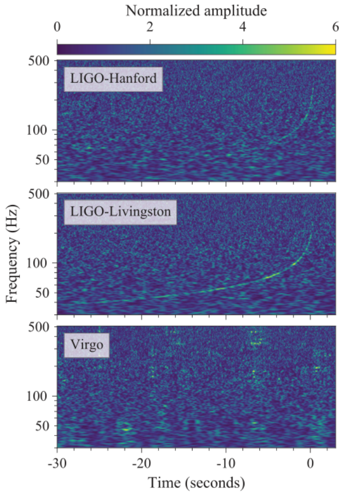
For waveform modeling, we aim to accurately model gravitational waveforms based on General Relativity (GR) for events like GW170817, GW190521, and GW190814. This involves selecting relevant data, understanding GR principles, and estimating critical parameters such as masses, spins, and distances. We generate waveforms, calibrate them to match observed data, and validate the models statistically.

5 Expected Result

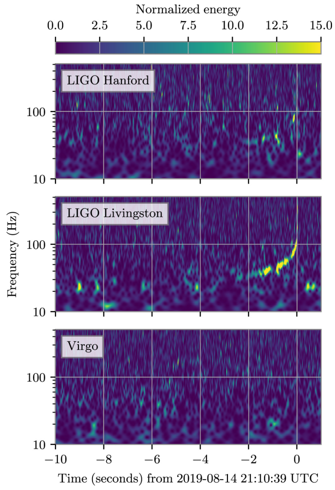
In our analysis of GW170817, GW190521 and GW190814, we expect to confirm the detection of gravitational waves, validating General Theory of Relativity, theoretical and numerical model, and observatory capabilities. Measured parameters of *GW170817* include neutron star masses of approximately 1.36 and 1.17 solar masses, spin angular velocities of about 0.03 and 0.05 radians per second, a distance of around 40 megaparsecs from Earth, and neutron star radii of approximately 12 kilometers whereas of *GW190521*, black hole masses of approximately 85 and 66 times the mass of our Sun and distance over 17 billion light-years away from Earth, corresponding to a redshift of approximately 0.82. And of *GW190814*, black hole mass of approximately 23 times the mass of our Sun and a neutron star mass of approximately 2.6 times

[¶] `https://gwosc.readthedocs.io/en/stable/`
[‖] `https://gwpy.github.io/`
[**] `http://pycbc.org/pycbc/latest/html/`

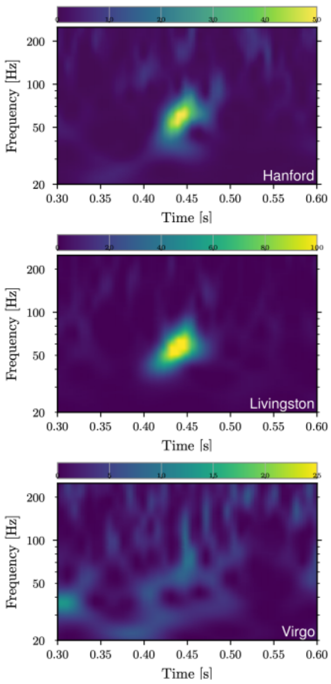
the mass of our Sun, a distance of around 800 million light-years from Earth. However, it is uncertain that what kind of merger caused the GW190814. It is also expected to get similar Time-frequency representations of data containing the gravitational-wave event GW170817, GW190521 and GW190814, observed by the LIGO-Hanford, LIGO-Livingston, and Virgo detectors.



(a) Chirp of GW170817



(b) Chirp of GW190814



(c) Chirp of GW190521

Figure 6: Time-frequency representations of data containing the gravitational-wave event GW170817, observed by the LIGO-Hanford (top), LIGO-Livingston (middle), and Virgo (bottom) detectors

6 Work Plan

Work	Time duration (in months)				
	1-2	3-4	5-6	7-8	9-10
Literature Review					
Problem Identification					
Finding solution					
Project writing					
Documentation					

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