# Exercise 1

## Submission Deadline 11 May 2020, 11:00 AM (GMT +02:00)

### 1.1 Probability

**Task 1.1.1**

In a random experiment three LED lamps are turned on simultaneously.

It must be assumed that every single LED lamp might be faulty.

Define the simplest possible sample space for this random experiment contains the events

$$A_1 = \{exactly\ one\ LED\ lamp\ is\ on\}$$

and

$$A_2 = \{maximum\ of\ two\ LED\ lamps\ are\ on\}$$

Suppose that the probabilities of the events $A_1\ and\ A_2$ are given as

$$P(A_1) = \frac{1}{4} \quad und \quad P(A_2) = \frac{1}{2}$$

**Task 1.1.2**

The union of two disjoint events A and B is the certain event. The conditional probabilitiy of an event X and the events A and B are:

$$P(X|A) = \frac{1}{4}, \qquad P(X|B) = \frac{1}{3}, \qquad P(A|X) = \frac{1}{2}.$$

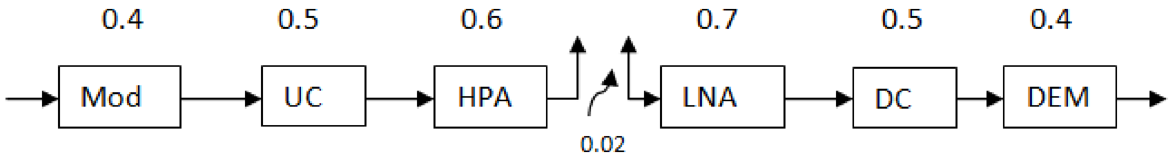
Determine P (A) and P (B).

**Task 1.1.3**

For the modules of an electronic device, resistors of a certain resistance (Ω-value) are necessary. 10000 such resistors have been bought from three different manufacturers.
5000 pieces from manufacturer $A_1$ (1% of the resistors do not meet the specification),
3000 pieces from manufacturer $A_2$ (2% of the resistors do not meet the specification),
2000 pieces from manufacturer $A_3$ (5% of the resistors do not meet the specification).
How is the probability of the event B that an arbitrarily picked resistor is out of specification?

**Task 1.1.4**

Let a radio transmission chain:




The following probabilities for the components breakdown within a time interval T are given:

Modulator (Mod) breakdown, event $A_1$: P ($A_1$) = 0.4
Up-Converter (UC) break down, event $A_2$: P ($A_2$) = 0.5
Power amplifier (HPA) break down, event $A_3$: P ($A_3$) = 0.6
Transmission media break down, event $A_4$: P ($A_4$) = 0.02
Low noise amplifier (LNA) break down, event $A_5$: P ($A_5$) = 0.7
Down converter (DC) break down, event $A_6$: P ($A_6$) = 0.5
Demodulator (DEM) break down, event $A_7$: P ($A_7$) = 0.4

How is the probability that no interrupt occurs within a time interval T?

## 1.2 Probability density function (pdf) and cumulative distribution function (cdf)

**Task 1.2.1**

Let the probability density function (pdf) of a random variable x (ζ):

$$f_x(x) = \begin{cases} \frac{k}{8} e^{-\frac{x}{k}+2} & x \geq 0 \\ 0 & otherwise \end{cases}$$

a) Calculate the constant k.
b) Calculate the mean and the variance $(m_x, \sigma_x^2)$ of the random variable x (ζ).
c) Calculate the probability $P(\{-1 \leq x(\zeta) < 2\})$.