6.1 Calculate the pdf $f_y(y)$. Remember input Signal is Gaussian noise. Pg 71

For pdf $f_y(y)$:-

$\rightarrow F^{-1}(S_{yy}) = S_{yy}(\tau)$

$\rightarrow$ pdf of Gaussian Noise :

$$\boxed{f_y(y) = \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{-\frac{(y-m_y^{(1)})^2}{2\sigma^2}}}$$

For σ, $m_y^{(1)} = ?$

$S_{yy} = \dfrac{S_1}{1+\omega^2 T_1^2}$: we know inverse transform

$$e^{-a|\tau|} = \frac{2a}{a^2+\omega^2} \quad \text{—— a}$$

$$\Rightarrow \frac{S_1}{1+\omega^2 T_1^2} \Rightarrow \frac{S_1}{\frac{1}{T_1^2}\left(\frac{1}{T_1^2}+\omega^2\right)}$$

$$S_{yy} \Rightarrow \frac{S_1}{2T_1}\left(\frac{\frac{2}{T_1}}{\frac{1}{T_1^2}+\omega^2}\right)$$

$$S_{yy}(\tau) = \frac{S_1}{2T_1} F^{-1}\left(\frac{2/T_1}{\frac{1}{T_1^2}+\omega^2}\right)$$

using a : $S_{yy}(\tau) = \dfrac{S_1}{2T_1} e^{\frac{-|\tau|}{T_1}}$

$$\Rightarrow m_y^{(1)} = \sqrt{\lim_{\tau \to \infty} \frac{S_1}{T_1} e^{-\frac{|\tau|}{T_1}}}$$

$$\Rightarrow \boxed{m_y^{(1)} = 0}$$

As: $\sigma^2 = m_y^{(2)} - \left(m_y^{(1)}\right)^2$

$$\Rightarrow m_y^{(2)} = \lim_{\tau \to 0} \left\{ \frac{S_1}{2T_1} e^{-\frac{|\tau|}{T_1}} \right\}$$

$$\Rightarrow m_y^{(2)} = \frac{S_1}{2T_1}$$

$$\sigma^2 = \frac{S_1}{2T_1}$$

$$\Rightarrow \boxed{\sigma = \sqrt{\frac{S_1}{2T_1}}}$$

$$\rightarrow \boxed{f_y(y) = \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{-\frac{(y - m_y^{(1)})^2}{2\sigma^2}}}$$

$$f_y(y) = \frac{1}{\sqrt{\cancel{2}\pi}\sqrt{\frac{S_1}{\cancel{2}T_1}}} e^{-\frac{\{(y-0)\}^2}{2\left(\sqrt{\frac{S_1}{2T_1}}\right)^2}}$$

$$\boxed{f_y(y) = \sqrt{\frac{T_1}{\pi S_1}}\, e^{\left(\frac{-y^2 T_1}{S_1}\right)}}$$