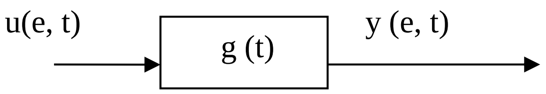
5.1 Power spectral densities and transfer functions
A Gaussian noise signal u(e, t) is the input signal of a linear system which is described by its impulse response g(t).



Some power spectral densities are known:

$$S_{uy} = \frac{S_1}{(1 - j\omega b)(1 + j\omega T_1)} \qquad\qquad S_{yy} = \frac{S_1}{1 + \omega^2 T_1^2}$$

a) Determine the transfer function G(jω) of the linear system.
b) Is the system described by G(jω) a causal system? Explain your statement.
c) Calculate the autocorrelation function $s_{uu}(\tau)$ of the input signal u(e, t).

5.2 Autocorrelation function
A stationary random process x (ζ, t) has the autocorrelation function

$$s_{xx}(\tau) = e^{-\alpha|\tau|} \quad for\ \alpha > 0$$

Another random process is defined by

$$y(\zeta, t) = \begin{cases} \int_0^t x(\zeta,\lambda)d\lambda & for\ t \geq 0 \\ 0 & otherwise \end{cases}$$

a) Calculate the mean $m_y^{(1)}(t)$
b) Determine the autocorrelation function of the random process y (ζ, t)
c) How is variance of the random process y (ζ, t)?
d) Is y (ζ, t) is a stationary random process?

5.3 Cross-correlation function
From a stationary random process x(ζ, t) the mean (=0), the standard deviation (=1) and the autocorrelation function

$$s_{xx}(\tau) = a\, e^{-\alpha|\tau|} + b$$

are known.
A random process y (ζ, t) is given by

$$y(\zeta, t) = \begin{cases} 0 & for\ t \leq t_0 \\ \int_{t_0}^t x(\zeta,\lambda)d\lambda & for\ t > t_0 \end{cases}$$

a) Determine the constants a and b.
b) Determine the cross-correlation function

$$s_{xy}(t_1, t_2) = E\{x(\zeta, t_1)\, y\, (\zeta, t_2)\}$$

c) Is the process y (ζ, t) stationary?