Stochastic Signals and Systems

Exercises

1 Task 2.1

Given is the probability density function

$$f_x(x) = \begin{cases} 2x & \text{for } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate cummulative distribution function $F_x(x)$.

2 Task 2.2

Given is the joint density function

$$f_{xy}(x,y) = \begin{cases} 24xy & \text{for } x \geq 0,\ y \geq 0 \text{ and } x + y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate $f_x(x)$, $f_y(y)$, $F_{xy}(x,y)$, $F_x(x)$ and $F_y(y)$ using MATLAB.

3 Task 2.3

Given is the following signal

$$x(t) = \sin(2\pi f t)$$

where, frequency f is 1 Hz.

Find the Auto-Correlation Function of the signal $x(t)$ using both Rectangular Window and Hamming Window. Use MATLAB for calculation.

Hint: To generate Hamming Window in MATLAB, use "w = hamming(L)" where L is the length of the sequence over which you wish to apply the filtering window & for a

rectangular window, use "w = rectpuls(x, a)" for a rectangular pulse of unit amplitude centered around x = 0 and width = a. To center the rectangular pulse at x = g and width = a, use "w = rectpuls(x - g, a)".

4 Task 2.4

Given is the following random process with random noise added to it

$$x(\zeta, t) = \sin(2\pi f t) + \alpha \cdot \eta(\zeta, t)$$

where, frequency f is 1 Hz, α is 0.05 and $\eta(\zeta, t)$ is random noise.

Find the Auto-Correlation Function of the random process $x(\zeta, t)$ using both Rectangular Window and Hamming Window. Also, find the Probability Density Function of the random noise. Use MATLAB for calculation.

Hint: To generate a random noise signal in MATLAB, use "n = randn(a, b)" where randn is a normal distribution random number generator and result n is a noise matrix of size a x b. Take care to note that the dimensions of your random signal and your ideal signal should match for consistent result.