(1) 1D oscillator: A block whose mass m is 1000g is tied to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11cm$ from its equilibrium position at $x = 0$. Assume that the surface is frictionless and the block is released from rest at $t = 0$.
  (a) What are the angular frequency, the frequency and the period of the resulting motion?
  (b) What is the amplitude of the oscillation?
  (c) What is the maximum speed ν_m of the oscillating block, and where is the block when it occurs?
  (d) What is the magnitude a_m of the maximum acceleration of the block?
  (e) What is the phase constant ϕ of the motion?
  (f) What is the displacement function $x(t)$ for the spring-block system?
  (g) What is the mechanical energy E of the linear oscillator when the block's position is at $x = 11cm$ and its speed is $\nu = 0cm/s$?
  (h) What is the potential energy U and kinetic energy K of the oscillator when the block is at $x = \frac{1}{2}x_m$? What are they when the block is at $x = -\frac{1}{2}x_m$?
  (i) Write a simple program in MATLAB or Python to graph the position, velocity and acceleration of a particle as a function of time executing a simple harmonic motion given the input parameters of initial position, force constant, mass and position of maximum displacement are given. Make a plot for the above parameters.
  (j) Also plot, using your program $x(t)$ versus $v(t)$. What is the shape of this graph? Why? What you have drawn is a 'phase diagram'! This is the set of all possible paths that the particle can take given a certain set of initial conditions. If you change the initial conditions, you will simply trace out another curve (that does not cross the previous one) and so on.

(2) 2D oscillator The restoring force for a particle moving in 2D is $\mathbf{F} - k_x\hat{x} - k_y\hat{y}$ giving us the components $F_x = -k_x$ and $F_y = -k_y$. This gives us the equations of motion

$$\ddot{x} + \omega_x^2 x = 0 \tag{1}$$

$$\ddot{y} + \omega_y^2 y = 0 \tag{2}$$

where $\omega_x = \sqrt{k_x/m}$ and $\omega_y = \sqrt{k_y/m}$ with solutions

$$x(t) = A\cos(\omega_x t - \alpha) \tag{3}$$

$$y(t) = B\cos(\omega_y t - \beta) \tag{4}$$

where the frequency, amplitude and phase can be different along $\hat{x}$ and $\hat{y}$ directions.
  (a) We can obtain the equation for the path of the particle by eliminating the time t between the two quantities $x(t)$ and $y(t)$. Add and subtract α inside the cos term in $y(t)$, and use the sum of angles in cosines trigonometric identity to get a term containing $(\alpha - \beta)$.
  (b) Let's define $\delta \equiv \alpha - \beta$, and assume $\omega_x = \omega_y = \omega_0$. Use the expression in Eq. 3 to get the expression

$$Ay - Bx\cos(\delta) = -B\sqrt{A^2 - x^2}\sin(\delta). \tag{5}$$

  (c) Square Eq. 5 on both sides and group the terms so that all the x and y terms are to one side of the equation.
  (d) Set $\delta = \pm\pi/2$. What is the form of the equation you have obtained? Write a program to plot x versus y.
  (e) Set $\delta = \pm\pi/2$ and $A = B$. What is the form of the equation you have now? Write a program to plot x versus y.

(f) Set $\delta = 0$. What is the form of the equation now? How about if you set $\delta = \pm\pi$? Plot these two cases as well.

(g) In your program, now make a plot of x versus y for $\delta = 90°, 120°, 150°, 180°, 210°, 240°, 270°, 300°, 330°$ and $360°$. Make sure to use a subplot feature of the software so that we can see the 10 plots as 10 panels in the same figure with a label under each subplot panel of that particular δ value.

(h) Can we comment on the direction in which the particle is moving in 2D in each of the above cases?

(i) Now assume that $\omega_x \neq \omega_y$, $A = B$ and $\delta = 0$. In a program, generate two arrays x and y using the parameters $\omega_x = [1, 3, 5, 7]$ and $\omega_y = [2, 4, 6, 8]$ and $A = B = 1$ as a function of time according to Eqs 3 and 4. Now plot x versus y for each of these cases (you should get 16 plots in all) in one figure with 16 panels and their corresponding labels of the (ω_x, ω_y) set of parameters. What sort of behaviors do you see? Are all figures showing closed loops? Such figures have a name – Lissajous figures (after the French physicist Jules Lissajous in the 19th century).