Lagrangian and Hamiltonian formalisms of classical mechanics are more appealing mathematically because they deal with scalar quantities like kinetic and potential energies. Newtonian mechanics on the other hand deals with vector quantities such as force; the obvious advantage of course is that it provides an easier physical understanding of the process as an introduction (which is why we are introduced to it first in school!). We will get familiar with these two formalisms before moving on to quantum mechanics (which closely follows Hamiltonian mechanics). The Lagrangian is defined as

$$L = T - V \tag{1}$$

where T is the kinetic energy and V is the potential energy. In what are called conservative systems (where total energy is conserved) V is independent of velocity of the particle, and for such systems, the Lagrangian equation of motion (EOM) is

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0 \tag{2}$$

where the subscript k signifies the k^{th} position coordinate in a system of N possible coordinates. Think of the 3D space. If there is one particle being described here, we will have 3 positions ($q_1, q_2, q_3 3$) and 3 velocities ($\dot{q}_1, \dot{q}_2, \dot{q}_3$, this will be called momenta very soon). For p particles, we will have $N = 3p$ coordinates. The Hamiltonian function (or Hamilton's function) is related to L in the sense that

$$H = \sum_{i=1}^{N} p_i \dot{q}_i - L. \tag{3}$$

The EOMs in the Hamilton's formalism are

$$\frac{\partial H}{\partial p_i} = \dot{q}_i, \frac{\partial H}{\partial q_i} = -\dot{p}_i \tag{4}$$

(1) Write down the Lagrangian for a simple pendulum constrained to move in a vertical plane. Find from it the equation of motion and show that for small displacements from equilibrium the pendulum performs simple harmonic motion.

(2) Write down the Lagrangian for a simple harmonic oscillator and obtain the expression for the time period.

(3) Consider a pendulum consisting of a small mass m attached to one end of an inextensible cord of length l rotating about the other end which is fixed. The pendulum moves on a spherical surface, hence the name spherical pendulum. The inclination angle ϕ in the xy-plane can change independently. (a) Obtain the equations of motion for the spherical pendulum. (b) Discuss the conditions for which the spherical pendulum motion behaves as a (i) simple pendulum and (ii) conical pendulum.

(4) Two blocks of mass m and M connected by a massless spring of spring constant k are placed on a smooth horizontal table. Determine the equations of motion using Lagrangian mechanics.

(5) A one-dimensional harmonic oscillator has Hamiltonian $H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2$ where q is the coordinate of this oscillator and p is the linear momentum. Write down Hamilton's EOMs and find the general solution.