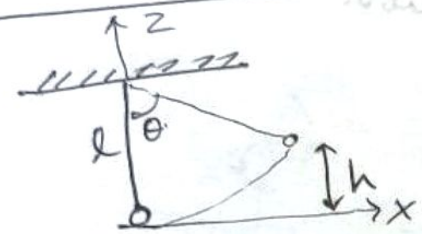
1)



tangential displacement $= l\theta$

velocity $= l\dot{\theta}$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}ml^2(\dot{\theta})^2$$

$$h = l - l\cos\theta$$

$$\Rightarrow h = l(1-\cos\theta) \qquad PE = mgh = mgl(1-\cos\theta)$$

$$L = \frac{1}{2}ml^2(\dot{\theta})^2 - mgl(1-\cos\theta)$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} \text{———}$$

for small displacements, $\cos\theta = 1 - \frac{\theta^2}{2!}$

$$\Rightarrow L = \frac{1}{2}ml^2(\dot{\theta})^2 - mgl\left(1 - \left(1 - \frac{\theta^2}{2!}\right)\right)$$

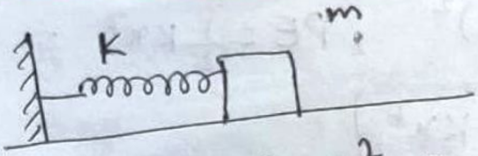
$$\Rightarrow \boxed{L = \frac{1}{2}ml^2(\dot{\theta})^2 - mgl\left(\frac{\theta^2}{2!}\right)}$$

Lagrangian equation of motion,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{\partial L}{\partial \theta}$$

$$\Rightarrow \frac{d}{dt}(ml^2\dot{\theta}) = -mgl\theta$$

$$\Rightarrow \cancel{m}l^2\ddot{\theta} = -\cancel{m}g\cancel{l}\theta \Rightarrow \boxed{\ddot{\theta} = -\left(\frac{g}{l}\right)\theta}$$

2) 

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{x})^2$

$PE = \frac{1}{2}Kx^2$.

$\boxed{L = \frac{1}{2}m(\dot{x})^2 - \frac{1}{2}Kx^2}$

Lagrangian equation of motion

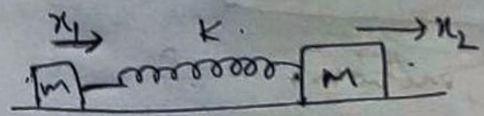
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$$

$\Rightarrow \frac{d}{dt}(m\dot{x}) = -Kx$

$\Rightarrow m\ddot{x} = -Kx \quad \Rightarrow \boxed{\ddot{x} = -\left(\frac{K}{m}\right)x}$

$\omega = \sqrt{\frac{K}{m}} \Rightarrow$ time period $= \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{K}}}$

3) 

4) 

Potential energy of the system, $PE = \frac{1}{2}K(x_2 - x_1)^2$

$KE = \frac{1}{2}m(\dot{x}_1)^2 + \frac{1}{2}M(\dot{x}_2)^2$

$\Rightarrow L = \frac{1}{2}m(\dot{x}_1)^2 + \frac{1}{2}M(\dot{x}_2)^2 - \frac{1}{2}K(x_2 - x_1)^2$

Lagrangian equation of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = \frac{\partial L}{\partial x_1}$$

$$\Rightarrow \frac{d}{dt}(m\dot{x}_1) = -K(x_2 - x_1)(-1)$$

$$\Rightarrow m\ddot{x}_1 = +K(x_2 - x_1) \longrightarrow (1)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = \frac{\partial L}{\partial x_2}$$

$$\Rightarrow \frac{d}{dt}(M\dot{x}_2) = -K(x_2 - x_1)$$

$$\Rightarrow M\ddot{x}_2 = -K(x_2 - x_1) \longrightarrow (2)$$

from (1) and (2),

$$(1)+(2) \Rightarrow \ddot{x}_1 = -\frac{M}{m}\ddot{x}_2$$

$$(1)-(2) \Rightarrow m\ddot{x}_1 - M\ddot{x}_2 = 2K(x_2 - x_1)$$

$$-\cancel{2}M\ddot{x}_2 = \cancel{2}K(x_2 - x_1)$$

$$\Rightarrow \ddot{x}_2 = -\frac{K}{M}(x_2 - x_1)$$

5) $H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2$

(p = linear momentum , q = coordinate)

$$\frac{\partial H}{\partial p} = \dot{q}$$

$$\frac{\partial H}{\partial q} = -\dot{p}$$

$\Rightarrow$ $p = \dot{q}$

$\omega^2 q = -\dot{p}$

$\Rightarrow$ $\omega^2 q = -(\ddot{q})$

$\Rightarrow$ $\boxed{\ddot{q} = -\omega^2 q}$ $\longrightarrow$ this is the equation of motion of a 1-D harmonic oscillator

$\Rightarrow$ general solution is of the form

$$q = A \sin(\omega t + \phi)$$