**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: mean = 10 + 45

std = 8

probability = 1 - stats.norm.cdf(60,55,8) = 0.26598552904870054

Answer is B. 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: The probability that employees are older than 44 is 1-stats.norm.cdf(44,38,6) = 0.15865525393145707 and The probability that employees are between 38 and 44 is = 0.3413447460685429 . Since The probability that employees are older than 44 is less than The probability that employees are between 38 and 44 the statement is false.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: Probability that employee is less than 30 = stats.norm.cdf(30,38,6) = 0.09121121972586788

Total outcome = 400

P(x<30) = num\_employee / Total outcome

0.0912 = num\_employee/400

num\_employee = 0.0912 \* 400 = 36.48 (approx. 36) hence the statement is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: Following central limit theorem :

X1 + X2 = N(μ+ μ, σ2+σ2) = N(2 μ,2 σ2)

2X1 = N(2 μ,2^2 σ2) = N(2 μ,4 σ2)

the difference between 2 *X*1 and *X*1 + *X*2 = N(2 μ - 2 μ, 4 σ2 + 2 σ2 ) = N(0, 6 σ2 )

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: By using brute force technique substituting the values given in the option into stats.norm.cdf and matching the difference between the 2 cdf output we get answer as D.

stats.norm.cdf(151.5 ,100,20) - stats.norm.cdf(48.5,100,20) = 0.99

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Total profit = N(5,3\*\*2) + N(7,4\*\*2) = N(12,5\*\*2)

Using stats.norm.interval(0.95,12\*45,5\*45) = (99.00810347848784, 980.9918965215122)

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: Z Score = stats.norm.ppf(0.05) = -1.6448536269514729

Using 5th percentile = mean + zscore \* std = 540 + (-1.6448) \* 225 = 169.91

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: The probability for N(5, 32) loss = 0.0477903522728147 and The probability for N(7, 42) loss = 0.040059156863817086 since the probability of loss for 1st division is greater than 2nd division N(5, 32) is more likely to give loss.