Reg. No.: 23BAI10882

Name : Aarya Solanki

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TERM END EXAMINATIONS (TEE) – December 2024 - January 2025

Programme	:	**B.Tech.**	**Semester**	:	**Interim Semester -2024-25**
Course Title	:	**Applied Linear Algebra**	**Course Code**	:	**MAT3002**
Date/Session	:	**26 Dec 2024/Session I**	**Slot**	:	**A12+A13**
Time	:	**3 Hrs.**	**Max. Marks**	:	**100**

Answer ALL the Questions

Q. No.	Question Description	Marks

PART A – (60 Marks)

1 (a) By the Gauss elimination method Find the values of A, B, C in the following partial fraction. $\frac{5x^2+23x-58}{(x-1)(x-3)(x+4)} = \frac{A}{x-1} + \frac{B}{x-3} + \frac{C}{x+4}$ — 12

OR

(b) To infer the surface shape of an object from images taken of a surface from three different directions is given below — 12

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

The unknowns are the incident intensities that will determine the shape of the object. Find the values of x_1, x_2 and x_3 using LU decomposition.

2 (a) Let a, b and c be fixed real numbers. Let V be the set of points in three-dimensional Euclidean space that lie on the plane P given by — 12

$$ax + by + cz = 0$$

Define addition and scalar multiplication on V coordinate-wise. Verify that V is a vector space

OR

(b) Finding a basis for a row space and column subspace of the following matrices — 12

$$\begin{bmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ -3 & 0 & 6 & -1 \\ 3 & 4 & -2 & 1 \\ 2 & 0 & -4 & 2 \end{bmatrix}$$

3 (a) Let $T: R^3 \to R^3$ be a linear operator and $B = \{v_1, v_2, v_3\}$ is a basis for R^3 suppose that, $T(v_1) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $, T(v_2) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $T(v_3) = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, 12

(a) Is $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ in $R(T)$

(b) Find the basis for $R(T)$

(c) Find the null space for $N(T)$.

OR

(b) Suppose that $T: P_2 \to P_2$ is a linear operator such that $T(x^2) = 2x - 1$, $T(-3x) = x^2 - 1$, $T(-x^2 + 3x) = 2x^2 - 2x + 1$. 6
Is it possible to determine $T(2x^2 - 3x + 2)$? "If yes, find it; and if no, explain why."

(c) Let $V = R^2$ with bases $B = \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$ *and* $B' = \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$ 6

I. Find the transition matrix from B and B'.

II. Let $[v]_B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and find $[v]_{B}'$.

4 (a) Apply the Gram–Schmidt orthogonalization process to find an orthogonal basis and then an orthonormal basis for the subspace U of R^4 spanned by 12
$v_1 = (1,1,1,1), v_2 = (1,2,4,5), v_2 = (1,-3,-4,-2)$

OR

(b) Given the orthonormal basis $S = \left\{\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right), (0, 1, 0), \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)\right\}$ for R^3, 6
write the vector $(1, 2, 3)$ as a linear combination of the vector in S.

(c) Show that u and v be vectors in R^n. Then $\| u + v \| = \| u \| + \| v \|$ if and if the vectors have the same direction. 6

5 (a) Let $A = \begin{bmatrix} 2 & 2 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$ $, b = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ 12

I. Find the least square solution of $Ax = b$

II. Find the orthogonal projection of b *onto* $W = col(A)$, and decomposition of vector $b = W_1 + W_2$

OR

(b) Obtain the singular value decomposition of the matrix $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$. 12

PART B – (40 Marks)

6 Calculate all the Eigen values & Corresponding Eigen vectors of the matrix 8

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

7 Which of the following two subsets is a subspace of R^2? 8

(a) The set of points on the line given by $x+2y=0$

(b) The set of points on the line given by $x+2y=1$

8 Define a linear transformation, $T: P_2 \rightarrow P_3$ by 8

$T\{f(x)\} = x^2 f''(x) - 2f'(x) + xf(x)$, Find the matrix representation of T relative to the standard bases for P_2 & P_3.

9 Let R^4 have the Euclidean inner product, If $u = (1,3,-4,2)$, 8

$v = (4,-2,2,1), w = (5,-1,-2,6)$ in R^4 , show that

a) $\langle 3u - 2v, w\rangle = 3\langle u, w\rangle - 2\langle v, w\rangle$

b) Normalize u and v

10 Find the QR – factorization of $A = \begin{bmatrix} -1 & 3 \\ 1 & 5 \end{bmatrix}$. 8

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