DEFINITION. (Wronskian)

Let $y = f_1(x) = \ y1(x) and\ y\ = f_2\ (x) = y2(x) - - - -(1)$ be solutions of (1). The function W defined by

$w(f_1, f_2) = f_1 f_2^{'} - f_1^{'} f_2$ is called the Wronskian of f_1, f_2

$$w(f_1, f_2) = \begin{vmatrix} f_1 & f_2 \\ f_1^{'} & f_2^{'} \end{vmatrix}$$

Note:

There are three variables means $w(f_1, f_2, f_3)$

$$w(f_1, f_2, f_3) = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1^{'} & f_2^{'} & f_3^{'} \\ f_1^{''} & f_2^{''} & f_3^{''} \end{vmatrix}$$

Expand the determinant.

Important note:

Determinant ≠ 0 Linearly Independent

Determinant=0 Linearly dependent.

Problems:

1. Use wornskian to show that $\{cosx, sinx,\}$ are linearly independent.

   Solutions:

$$\text{Here,} \quad \begin{array}{r|l} f_1 = cosx & f_2 = sinx \\ f_1' = -sinx & f_2' = cosx \end{array}$$

$$w(f_1, f_2) = \begin{vmatrix} f_1 & f_2 \\ f_1^{'} & f_2^{'} \end{vmatrix}$$

$$\text{and} \quad f_1 f_2' - f_1' f_2 = cos^2x + sin^2x = 1$$

Det$erminant\ \neq 0$

Therefore, the given functions linearly independent.

2.Use wornskian to check that { $\sin(x), \cos(x) e^{x}$}are linearly independent or not .

Solution:

There are three variables means $w(f_1, f_2, f_3)$

$$w(f_1, f_2, f_3) = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1^{'} & f_2^{'} & f_3^{'} \\ f_1^{''} & f_2^{''} & f_3^{''} \end{vmatrix}$$

Here, $f_1 = sinx$ | $f_2 = cosx$ | $f_3 = e^x$

$f_1' = cosx$ | $f_2' = -sinx$ | $f_3' = e^x$

$f_1'' = -sinx$ | $f_2'' = -cosx$ | $f_3'' = e^x$

$$w(f_1, f_2, f_3) = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1^{'} & f_2^{'} & f_3^{'} \\ f_1^{''} & f_2^{''} & f_3^{''} \end{vmatrix}$$

$$== \begin{vmatrix} sinx & cosx & e^x \\ cosx & -sinx & e^x \\ -sinx & -cosx & e^x \end{vmatrix} = -2e^x$$

Det$erminant$ $\neq 0$

Therefore, the given functions linearly independent

Do it your self

1. Use wornskian to show that $\{x, e^x, e^{-x}\}$ are linearly independent.
2. Use wornskian to check that $\{x, sinx, cosx\}$ are linearly independent or not .