DAY - 27

Solution:

We are given:

- Number of layers = 6.
- Innermost layer thickness t_1 = 1 cm.
- Each successive layer is 20% thicker than the previous, so common ratio r = 1.2.
- The thicknesses form a geometric sequence: $t_k = 1 \cdot r^{k-1}$ for k=1,...,6.

(a) Total radius after all 6 layers

Total radius R is the sum of all six layer thicknesses:

$$R = \sum_{k=1}^{6} t_k \; = \; \sum_{k=0}^{5} 1 \cdot r^{\,k}$$

This is a finite geometric series with first term 1, ratio r=1.2, and n=6 terms. Use the geometric sum formula:

$$R = \frac{r^6 - 1}{r - 1}.$$

Compute r^6 step by step:

$$\begin{aligned}
r^1 &= 1.2, \\
r^2 &= 1.2^2 = 1.44, \\
r^3 &= 1.44 \times 1.2 = 1.728, \\
r^4 &= 1.728 \times 1.2 = 2.0736, \\
r^5 &= 2.0736 \times 1.2 = 2.48832, \\
r^6 &= 2.48832 \times 1.2 = 2.985984.
\end{aligned}$$

Now plug into the sum formula:

$$R = \frac{2.985984 - 1}{1.2 - 1} = \frac{1.985984}{0.2} = 1.985984 \times 5 = 9.92992 \text{ cm}.$$

So,

$$\boxed{R = 9.92992 \text{ cm}}$$

(You can also verify by summing individual thicknesses: 1 + 1.2 + 1.44 + 1.728 + 2.0736 + 2.48832 = 9.92992.)

(b) Fraction of volume left after peeling off outer two layers

Peeling off the outer two layers removes the last two thicknesses t_6 and t_5. The remaining radius R_inner equals the sum of the first 4 layers:

$$R_{\text{inner}} = \sum_{k=1}^{4} t_k = \frac{r^4 - 1}{r - 1}.$$

We already computed r^4 = 2.0736, so

$$R_{\text{inner}} = \frac{2.0736 - 1}{0.2} = \frac{1.0736}{0.2} = 1.0736 \times 5 = 5.368 \text{ cm}.$$

Volume of a sphere scales with the cube of the radius. If V is original volume and V_inner the remaining volume, then

$$\frac{V_{\text{inner}}}{V} = \left(\frac{R_{\text{inner}}}{R}\right)^3.$$

Substitute the exact geometric-sum expressions first (exact algebraic form):

$$\frac{V_{\text{inner}}}{V} = \left(\frac{\frac{r^4 - 1}{r - 1}}{\frac{r^6 - 1}{r - 1}}\right)^3 = \left(\frac{r^4 - 1}{r^6 - 1}\right)^3.$$

Now numeric evaluation with r=1.2:

$$\begin{aligned}
r^4 - 1 &= 2.0736 - 1 = 1.0736, \\
r^6 - 1 &= 2.985984 - 1 = 1.985984, \\
\frac{r^4 - 1}{r^6 - 1} &= \frac{1.0736}{1.985984} \approx 0.5405884438142503, \\
\left(\frac{r^4 - 1}{r^6 - 1}\right)^3 &\approx 0.5405884438142503^{\,3} \approx 0.1579793318035828.
\end{aligned}$$

So the fraction of the onion's total volume remaining after removing the outer two layers is approximately

$$\boxed{\frac{V_{\text{inner}}}{V} \approx 0.1579793318 \approx 15.79793318\%}.$$

We can also express the fraction exactly as (1.0736/1.985984)³.

(c) Estimating number of layers if a similar onion has total radius ≈ 25 cm

Using the same ratio r=1.2, the total radius after n layers is

$$R_n = \frac{r^n - 1}{r - 1}.$$

If R_n ≈ 25 cm, then

$$r^n - 1 \approx (r - 1) \cdot 25 = 0.2 \times 25 = 5,$$

so

$$r^n \approx 1 + 5 = 6.$$

Take logarithms to estimate n:

$$n \approx \frac{\ln(6)}{\ln(1.2)}.$$

A quick calculation gives

$$n \approx \frac{\ln 6}{\ln 1.2} \approx 9.83.$$

So Lila could estimate the onion has **about 10 layers** (since n must be an integer and 9.83 rounds up to 10).

Final answers (concise)

(a) Total radius R ≈ 9.93 cm.

(b) Fraction of volume left after removing outer two layers ≈ 0.158.

(c) For R ≈ 25 cm with the same 20% pattern, the number of layers is **about 10** (estimate).