

DAY - 26

Solution:

Given:

$$\begin{aligned}\text{Each small box (per box)} &= S = 2, & n_s &= 5, \\ \text{Each medium box (per box)} &= M = 3S = 3 \cdot 2 = 6, & n_m &= 4, \\ \text{Each large box (per box)} &= L = 2M = 2 \cdot 6 = 12, & n_l &= 2.\end{aligned}$$

Initial totals (at t=0):

$$\begin{aligned}\text{Small total} &= n_s \cdot S = 5 \cdot 2 = 10, \\ \text{Medium total} &= n_m \cdot M = 4 \cdot 6 = 24, \\ \text{Large total} &= n_l \cdot L = 2 \cdot 12 = 24, \\ T_0 &= 10 + 24 + 24 = 58.\end{aligned}$$

(a) Start (per box):

$$\boxed{S = 2, \quad M = 6, \quad L = 12}$$

(b) Medium boxes after 8 minutes (doubling every 4 minutes):

$$\text{Number of doublings in 8 minutes} = \frac{8}{4} = 2.$$

$$M(8) = M \cdot 2^2 = 6 \cdot 4 = 24.$$

$$\text{Total medium after 8 min} = n_m \cdot M(8) = 4 \cdot 24 = 96.$$

$$\boxed{\text{Each medium box at } t = 8 : 24 \text{ coins}}$$

(c) Large-box gold production :

Define for one large box:

$$L_0 = 12, \quad \text{for } k = 1, 2, \dots, 8 : \quad a_k = \left\lfloor \frac{L_{k-1}}{6} \right\rfloor, \quad L_k = L_{k-1} + a_k.$$

Computing iteratively:

$$\begin{aligned}\text{Minute 1 : } L_0 = 12 &\Rightarrow a_1 = \left\lfloor \frac{12}{6} \right\rfloor = 2, & L_1 &= 12 + 2 = 14, \\ \text{Minute 2 : } L_1 = 14 &\Rightarrow a_2 = \left\lfloor \frac{14}{6} \right\rfloor = 2, & L_2 &= 14 + 2 = 16, \\ \text{Minute 3 : } L_2 = 16 &\Rightarrow a_3 = \left\lfloor \frac{16}{6} \right\rfloor = 2, & L_3 &= 16 + 2 = 18, \\ \text{Minute 4 : } L_3 = 18 &\Rightarrow a_4 = \left\lfloor \frac{18}{6} \right\rfloor = 3, & L_4 &= 18 + 3 = 21, \\ \text{Minute 5 : } L_4 = 21 &\Rightarrow a_5 = \left\lfloor \frac{21}{6} \right\rfloor = 3, & L_5 &= 21 + 3 = 24, \\ \text{Minute 6 : } L_5 = 24 &\Rightarrow a_6 = \left\lfloor \frac{24}{6} \right\rfloor = 4, & L_6 &= 24 + 4 = 28, \\ \text{Minute 7 : } L_6 = 28 &\Rightarrow a_7 = \left\lfloor \frac{28}{6} \right\rfloor = 4, & L_7 &= 28 + 4 = 32, \\ \text{Minute 8 : } L_7 = 32 &\Rightarrow a_8 = \left\lfloor \frac{32}{6} \right\rfloor = 5, & L_8 &= 32 + 5 = 37.\end{aligned}$$

Total extra gold produced by one large box in 8 minutes:

$$A_{\text{large}} = \sum_{k=1}^8 a_k = 2 + 2 + 2 + 3 + 3 + 4 + 4 + 5 = 25.$$

Extra gold per large box in 8 min = 25, $L(8) = L_8 = 37$.

(d) Grand total of coins after 8 minutes:

$$\text{Small total} = 10 \quad (\text{unchanged}),$$

$$\text{Medium total} = 96 \quad (\text{from (b)}),$$

$$\text{Large total} = n_l \cdot L(8) = 2 \cdot 37 = 74,$$

$$T(8) = 10 + 96 + 74 = 180.$$

$T(8) = 180 \text{ coins}$

Gold vs silver breakdown:

$$\text{Gold coins} = n_l \cdot A_{\text{large}} = 2 \cdot 25 = 50,$$

$$\text{Silver coins} = T(8) - \text{Gold} = 180 - 50 = 130.$$

(e) Total coin value in silver-coin equivalents (gold = 5 × silver):

$$V_{\text{silver}} = 130 \cdot 1 = 130, \quad V_{\text{gold}} = 50 \cdot 5 = 250,$$

$$V_{\text{total}} = 130 + 250 = 380.$$

$V_{\text{total}} = 380 \text{ (silver-coin equivalents)}$
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Final boxed answers:

- (a) $S = 10$, $M = 24$, $L = 24$ (per box at start),
- (b) Each medium at $t = 8$: 24,
- (c) Extra gold per large box in 8 min: 25,
- (d) Total coins at $t = 8$: 180,
- (e) Total value (silver-equivalents): 380.