DAY - 29 : The Lesson of Balance

Given: Total lesson time is $C + P = 120$ minutes.
Effectiveness is given by:

$$E = C \times P$$

(a) Expressing E in terms of C only

$$C + P = 120 \Rightarrow P = 120 - C$$
$$\therefore \quad E = C \times (120 - C)$$
$$\boxed{E(C) = 120C - C^2}$$

(b) Finding when E is maximum (without calculus)

We have:

$$E(C) = 120C - C^2$$

Completing the square:

$$E(C) = -\left(C^2 - 120C\right)$$
$$E(C) = -\Big(C^2 - 120C + 60^2\Big) + 60^2$$
$$\Rightarrow E(C) = -\big(C - 60\big)^2 + 3600$$

Since the term $-\big(C - 60\big)^2 \leq 0$, the maximum value occurs when:

$$(C - 60)^2 = 0 \Rightarrow C = 60$$
$$\text{At } C = 60, \quad P = 120 - 60 = 60$$
$$\boxed{E_{\max} = 60 \times 60 = 3600}$$

(c) If $C = 2P$, finding new effectiveness and compare with maximum

Given $C = 2P$ and $C + P = 120$:

$$2P + P = 120 \Rightarrow 3P = 120 \Rightarrow P = 40$$
$$C = 2P = 80$$

Now,

$$E_{\text{new}} = C \times P = 80 \times 40 = 3200$$

Compare with maximum:

$$\frac{E_{\text{new}}}{E_{\max}} = \frac{3200}{3600} = \frac{32}{36} = \frac{8}{9}$$
$$\boxed{E_{\text{new}} = \tfrac{8}{9} E_{\max}}$$

(d) Proportion of unused learning potential

Unused effectiveness:

$$E_{\max} - E_{\text{new}} = 3600 - 3200 = 400$$

Proportion of unused potential:

$$\frac{E_{\max} - E_{\text{new}}}{E_{\max}} = \frac{400}{3600} = \frac{1}{9}$$

$$\boxed{\text{Unused proportion} = \tfrac{1}{9}}$$

Final Results

(a) $E(C) = 120C - C^2$

(b) $C = 60,\ P = 60,\ E_{\max} = 3600$

(c) $C = 80,\ P = 40,\ E_{\text{new}} = 3200 = \frac{8}{9}E_{\max}$

(d) Unused proportion $= \frac{1}{9}$