Probability for Computer Science (BCS 3022): Tutorial-1
Fall Semester

1. Show that if $P(A \mid B) = 1$, then $P(B^c \mid A^c) = 1$.

2. Let A, B, and C be events relating to the experiment of rolling a pair of dice.

   (a) If
   $$P(A \mid C) > P(B \mid C) \quad \text{and} \quad P(A \mid C^c) > P(B \mid C^c),$$
   either prove that $P(A) > P(B)$ or give a counterexample by defining events A, B, and C for which that relationship is not true.

   (b) If
   $$P(A \mid C) > P(A \mid C^c) \quad \text{and} \quad P(B \mid C) > P(B \mid C^c),$$
   either prove that
   $$P(AB \mid C) > P(AB \mid C^c)$$
   or give a counterexample by defining events A, B, and C for which that relationship is not true.

   Hint: Let C be the event that the sum of a pair of dice is 10; let A be the event that the first die lands on 6; let B be the event that the second die lands on 6.

3. Let X be a random variable with distribution function $F()$. In each of the following cases determine whether X is a discrete random variable or a continuous random variable. Also find the p.d.f./p.m.f. of X:

   (i)
   $$F(x) = \begin{cases} 0, & x < -2, \\ \frac{1}{3}, & -2 \le x < 0, \\ \frac{1}{2}, & 0 \le x < 5, \\ \frac{3}{4}, & 5 \le x < 6, \\ 1, & x \ge 6. \end{cases}$$

   (ii)
   $$F(x) = \begin{cases} 0, & x < 0, \\ 1 - e^{-x}, & x \ge 0. \end{cases}$$

4. Let X be the number of accidents per week in a factory. Let the pmf of X be
   $$f(x) = \frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}, \quad x = 0, 1, 2, \ldots$$
   Find the conditional probability
   $$P(X \ge 4 \mid X \ge 1).$$

5. Given
   $$E(X+4) = 10 \quad \text{and} \quad E\big[(X+4)^2\big] = 116,$$
   determine (a) $\mathrm{Var}(X+4)$, (b) $\mu = E(X)$, and (c) $\sigma^2 = \mathrm{Var}(X)$.

6. Let X equal the number of people selected at random that you must ask in order to find someone with the same birthday as yours. Assume that each day of the year is equally likely, and ignore February 29.

   (a) What is the pmf of X?

(b) Give the values of the mean, variance, and standard deviation of X.

(c) Find $P(X > 400)$ and $P(X < 300)$.

7. In the casino game called **high–low**, there are three possible bets. Assume that 1 Rs. is the size of the bet. A pair of fair six-sided dice is rolled and their sum is calculated. If you bet **low**, you win 1 Rs. if the sum of the dice is $\{2, 3, 4, 5, 6\}$. If you bet **high**, you win 1 Rs. if the sum of the dice is $\{8, 9, 10, 11, 12\}$. If you bet on $\{7\}$, you win 4 Rs. if a sum of 7 is rolled. Otherwise, you lose on each of the three bets. In all three cases, your original rupee is returned if you win. Find the expected value of the game to the bettor for each of these three bets.

8. When coin 1 is flipped, it lands on heads with probability 0.4; when coin 2 is flipped, it lands on heads with probability 0.7. One of these coins is randomly chosen and flipped 10 times.

   (a) What is the probability that the coin lands on heads on exactly 7 of the 10 flips?

   (b) Given that the first of these ten flips lands heads, what is the conditional probability that exactly 7 of the 10 flips land on heads?

9. Let X be a binomial random variable with parameters n and p. Show that

$$\mathbb{E}\left[\frac{1}{X+1}\right] = \frac{1-(1-p)^{n+1}}{(n+1)p}.$$

10. Each game you play is a win with probability p. You plan to play 5 games, but if you win the fifth game, then you will keep on playing until you lose.

    (a) Find the expected number of games that you play.

    (b) Find the expected number of games that you lose.

11. If X is a binomial random variable with expected value 6 and variance 2.4, find $P\{X = 5\}$.