DAY - 30

Question:

The total perimeter of a rectangular plot is 60 cm. The length of the plot is 4 cm more than twice its width. Inside this rectangle, a square garden is left vacant. The side of the square is equal to the difference between the rectangle's length and width. Find:

(a) The length and width of the rectangular plot.

(b) The ratio of the area of the vacant square to the area of the entire plot.

Solution:

Let the length of the rectangle be L cm and the width be W cm.

$$\text{Perimeter of rectangle} = 2(L + W)$$

Given,

$$2(L + W) = 60$$

$$\Rightarrow L + W = 30 \quad (1)$$

Also given, the length is 4 cm more than twice its width.

$$L = 2W + 4 \quad (2)$$

Substituting the value of L from (2) in (1),

$$(2W + 4) + W = 30$$

$$3W + 4 = 30$$

$$3W = 26$$

$$W = \frac{26}{3} \text{ cm} = 8\tfrac{2}{3} \text{ cm}$$

Now, substituting the value of W in equation (2):

$$L = 2W + 4$$

$$L = 2\left(\frac{26}{3}\right) + 4$$

$$L = \frac{52}{3} + \frac{12}{3} = \frac{64}{3} \text{ cm} = 21\tfrac{1}{3} \text{ cm}$$

$$\therefore \boxed{L = \frac{64}{3} \text{ cm}, \quad W = \frac{26}{3} \text{ cm}}$$

(b) Finding the required ratio:

The side of the square garden is equal to the difference between the length and width.

$$\text{Side of square } s = L - W$$

$$s = \frac{64}{3} - \frac{26}{3} = \frac{38}{3} \text{ cm}$$

$$\text{Area of square} = s^2 = \left(\frac{38}{3}\right)^2 = \frac{1444}{9} \text{ cm}^2$$

$$\text{Area of rectangle} = L \times W = \frac{64}{3} \times \frac{26}{3} = \frac{1664}{9} \text{ cm}^2$$

$$\text{Required ratio} = \frac{\text{Area of square}}{\text{Area of rectangle}} = \frac{\frac{1444}{9}}{\frac{1664}{9}} = \frac{1444}{1664}$$

Simplifying the ratio:

$$\frac{1444}{1664} = \frac{361}{416}$$

$$\boxed{\text{Ratio} = \frac{361}{416} \approx 0.8678 \text{ or } 86.78\%}$$

Final Answers:

$$\boxed{\begin{array}{l} \text{(a) } L = \dfrac{64}{3} \text{ cm}, \quad W = \dfrac{26}{3} \text{ cm} \\ \text{(b) Ratio of areas} = \dfrac{361}{416} \text{ (or } 86.78\%) \end{array}}$$