SYK/JT Duality

An introduction to the SYK/JT duality

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November 29, 2024

AdS/CFT correspondance I

Definition 1: AdS$_d$ is a maximally symmetric spacetime with negative curvature introduced as solution to Einstein's equations with a negative cosmological constant.

$$S = \frac{1}{16\pi G_d} \int dx^d \sqrt{|g|}(\mathcal{R} - \Lambda)$$

It may be written as an embedding in $d+1$ dim Euclidean space:

$$X_0^2 + X_d^2 - \sum_{i=1}^{d-1} X_i^2 = R^2, \text{ where } R^2 = -\frac{(d-1)(d-2)}{\Lambda}$$

which can be visualised as a d dimensional *hyperboloid* surface in one higher dimensional space.

AdS/CFT correspondance II

Definition 2: A Conformal (Quantum) Field Theory refers to the class of (quantum) field theories with conformal symmetry. They are invariant under the conformal group, which includes dilations, translations, and special conformal transformations.

$$\text{Conformal Transformation: } (dx')^2 = \Omega^2(x)(dx)^2$$

Useful Properties of CFTs:

- Fields scale under dilations: $x^\mu \to \lambda x^\mu, \quad \phi(x) \to \lambda^\Delta \phi(\lambda x)$
- Operator-Product Expansion (OPE) organizes products of operators: $O_1(x)O_2(y) \sim \sum_{O_i} C_{12i}(x-y)O_i(y)$
- The stress tensor $T_{\mu\nu}$ satisfies: $T^\mu_\mu = 0$

AdS/CFT correspondance III

The Correspondance:[5] The AdS/CFT posits a duality between:

- A gravitational theory in $d + 1$ dimensions (AdS)
- A conformal field theory in d dimensions

$$H_{\text{CFT}} = H_{\text{AdS}}$$

- Notable case: String theory on $\text{AdS}_5 \times S^5$ corresponds to $N = 4$ Super Yang-Mills in 4 dimensions

Connection to SYK/JT Duality:

- The SYK model relates to a nearly conformal quantum mechanics (in $0 + 1$ dim), while JT gravity emerges as the $2D$ AdS counterpart
- The correspondence generalizes AdS/CFT to lower dimensions and explores holographic principles

2D Dilaton Gravity

The problem with EH action in 2D:

- The EH action

$$\frac{1}{16\pi G}\int d^2x\sqrt{-\det g}\,\mathcal{R}$$

  does not yield any field equations in two spacetime dimensions.
- EH action is topological and does not suppress fluctuations.

Modified action: (G:=1) Introducing an auxilliary field ϕ [4]

$$I = \underbrace{-\frac{S_0}{4\pi}\int_M \sqrt{g}\mathcal{R}}_{\text{topological}} \underbrace{-\frac{1}{2}\int_M \sqrt{g}(\Phi\mathcal{R} + U(\Phi)) + (\text{bdry terms})}_{\text{dynamical}}$$

where U(ϕ) is the dilaton potential

Jackiw-Teitelboim Gravity I

JT gravity corresponds to a linear dilaton (JT) potential $U(\phi) = \Lambda\phi$ with action

$$S_{JT} = \frac{1}{16\pi G}\int d^2x\sqrt{-g}\,\Phi(\mathcal{R} - \Lambda)$$

- Varying the action with respect to ϕ:[1]

$$\frac{\delta S_{JT}}{\delta\phi} = 0 \implies \mathcal{R} = \Lambda$$

on shell, which means the metric is either Anti-de Sitter space or De Sitter space depending upon the sign of Λ.

Jackiw-Teitelboim Gravity II

- Varying the action with respect to $g_{\mu\nu}$:

$$\left(\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla^2\right) \phi + g_{\mu\nu} \Lambda \phi = 0$$

Upon contracting with $g_{\mu\nu}$ we obtain equivalently

$$\left(\nabla^2 - 2\Lambda\right) \phi = 0$$

$$\left(\frac{1}{2} g_{\mu\nu} \nabla^2 - \nabla_\mu \nabla_\nu\right) \phi = 0$$

which in the conformal gauge becomes

$$\left(\Box - 2\Lambda e^{\Phi}\right) \phi = 0$$

Sachdev-Ye-Kitaev Model I

Introduction

- Hamiltonian:

$$\frac{1}{2}\sum_{i=1}^{N}\chi_i\partial_t\chi_i - \frac{1}{4!}\sum_{i,j,k,l=1}^{N}\mathcal{J}_{ijkl}\chi_i\chi_j\chi_k\chi_l$$

$$H = \frac{1}{4!}\sum_{i,j,k,l} J_{ijkl}\chi_i\chi_j\chi_k\chi_l$$

  where χ_i are N Majorana fermions satisfying:

$$\{\chi_i, \chi_j\} = \delta_{ij}, \quad \chi_i^\dagger = \chi_i$$

  and J_{ijkl} are random couplings with variance:

$$\langle J_{ijkl}^2 \rangle = \frac{3!J^2}{N^3}$$

- Proposed by Kitaev as a simplification of the Sachdev-Ye model, it combines strong interactions, solvability, and chaotic dynamics.

Sachdev-Ye-Kitaev Model II

Key Features:

- Large N Solvability: In the $N \to \infty$ limit, the model is dominated by "melonic" Feynman diagrams, enabling exact computation of correlation functions.
- Emergent Conformal Symmetry: In the infrared (IR) limit:

$$G(\tau) \sim \frac{\mathrm{sgn}(\tau)}{|J\tau|^{2\Delta}}, \quad \Delta = \frac{1}{4}$$

Disorder Averaging:

- Random couplings lead to a quenched average:

$$\langle J_{ijkl} \rangle = 0, \quad \langle J_{ijkl}^2 \rangle = \frac{3! J^2}{N^3}$$

- Self-averaging ensures that physical properties are universal.

The Duality I

- **Overview of the duality:** [2]
  - Duality between JT gravity and the SYK model - Unique bridge between gravity in lower-dimensional spacetimes and strongly-coupled quantum systems.
  - At a high level, the relationship emerges in the context of holography, where the SYK model, a quantum mechanical system with random interactions, is conjectured to be dual to a lower-dimensional gravity theory, specifically JT gravity.
  - In this context, both theories exhibit similar behavior in certain limits, revealing deep connections between quantum gravity and many-body physics.

The Duality II

- **SYK Model at Large-N:** [3]
  - The SYK model, a quantum system of fermions with random all-to-all interactions, captures features of quantum chaos and black hole dynamics.
  - In large-N limit, (N = $\#$ of fermionic fields), the SYK model simplifies significantly.
  - This large-N limit reduces the degrees of freedom, making the model amenable to a description via classical gravity.

The Duality III

- **JT Gravity in the Semi-Classical Regime:**
  - JT gravity describes the dynamics of a dilaton field in the presence of a negative cosmological constant.
  - In the semi-classical limit, where quantum gravitational effects are considered small, JT gravity captures the essential features of low-dim black hole physics.
  - The theory is defined on a 2D manifold and is governed by a simple action that includes terms for the dilaton and the metric.

The Duality IV

- **Connecting SYK to JT Gravity:**
  - **Key to duality:** In large-N limit SYK model's behavior is controlled by classical gravity
  - Partition function of the SYK model   the partition function of JT gravity.
  - Common features:
    - non-trivial density of states at low energies
    - thermodynamic behavior that mirrors black hole entropy.

The Duality V

- Implications of the Duality
  - This duality offers a new perspective on quantum gravity
  - holographic models that describe gravity may have a direct connection with non-gravitational quantum systems
  - Additionally, the duality allows for the exploration of gravitational phenomena using non-gravitational models, thereby providing a deeper understanding of the nature of spacetime.

References

[1] Roman Jackiw. “Lower dimensional gravity”. In: *Nuclear Physics B* 252.1 (1985), pp. 343–356.

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[3] Douglas Stanford and Gustavo Joaquin Turiaci. “Lessons from JT gravity”. In: (June 2024). Stanford University and University of Washington.

[4] Thomas Strobl. “Gravity in Two Spacetime Dimensions”. In: (Nov. 2000). arXiv:hep-th/0011240v1.

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