Maximally symmetric nonlinear extension of electrodynamics and charged particles

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Abstract

We consider couplings of electrically and magnetically charged sources to the maximally symmetric non-linear extension of Maxwell's theory called ModMax. The aim is to reveal physical effects which distinguish ModMax from Maxwell's electrodynamics. We find that, in contrast to generic models of non-linear electrodynamics, Lienard-Wiechert fields induced by a moving electric or magnetic particle, or a dyon are exact solutions of the ModMax equations of motion. We then study whether and how ModMax non-linearity affects properties of electromagnetic interactions of charged objects, in particular the Lorentz force, the Coulomb law, the Lienard-Wiechert fields, Dirac's and Schwinger's quantization of electric and magnetic charges, and the Compton Effect. In passing we also present an alternative form of the ModMax Lagrangian in terms of the coupling of Maxwell's theory to axion-dilaton-like auxiliary scalar fields which may be relevant for revealing the effective field theory origin of ModMax.

Contents

1 Introduction

In [1] there was discovered a non-linear extension of free $D = 4$ Maxwell's electrodynamics, called ModMax, which preserves all the symmetries of the latter, namely the four-dimensional conformal symmetry and electric-magnetic duality. This is the unique specimen with such properties among the variety of models of non-linear electrodynamics (see [2] for a review). This theory exhibits interesting features, such as a peculiar form of birefringence [1, 3] and, in spite of its intrinsic non-analyticity, has plane waves [1] and topologically non-trivial knotted null electromagnetic field configurations [4] as exact solutions in its Hamiltonian formulation. Properties of the ModMax Lagrangian formulation were studied in more detail in [5, 6] and its Hamiltonian formulation in [7].

ModMax arises as a weak field limit of a generalized two-parameter Born-Infeld theory [1, 8]. Further generalizations of these theories were discussed in [9, 10]. Quite remarkably, as was shown recently [11–13], both, ModMax and the generalized Born-Infeld theory arise as different $T\bar{T}$-like deformations of Maxwell's and the Born-Infeld theory, and the generalized Born-Infeld theory is a $T\bar{T}$ deformation of ModMax. Their supersymmetric extensions were constructed in [14, 15], and in [16] (super)conformal higher-spin generalizations of ModMax were derived. A possibility of linking the generalized Born-Infeld theory to String Theory by uncovering a string-like nature of the former was discussed in [17].

Effects of ModMax and its generalizations on properties and thermodynamics of charged black holes (e.g. Taub-NUT, Reissner-Nordström ones and others) have been studied in a number of papers [3, 18–27]. The aim of this article is to study how the ModMax non-linearity affects properties of interactions of electrically and magnetically charged point particles, in particular the Lorentz force, the Coulomb law, the Lienard-Wiechert fields, Dirac's and Schwinger's quantization of electric and magnetic charges, and the Compton effect. We will show that the Lienard-Wiechert fields created by a moving electric or magnetic particle, or a dyon are exact solutions of the ModMax equations of motion, while this is not the case for most of non-linear electrodynamics models. In passing we will also present an alternative form of the ModMax Lagrangian in terms of the coupling of Maxwell's theory to axion-dilaton-like auxiliary scalar fields which may be relevant for revealing the effective field theory origin of ModMax.

We will show that for a certain choice of the definition of physical electric and magnetic charges, which is associated with an appropriate rescaling of the source-free ModMax Lagrangian, and the standard minimal electromagnetic coupling of the charges, there is no difference in the Coulomb law and Lorentz forces describing interactions of two electric particles (one of which is a test particle) in ModMax and Maxwell's theory. The difference appears if magnetic monopoles or dyons are present. On the other hand, as is the case of vacuum birefringence of ModMax [1, 3], the Compton scattering differs from that in Maxwell's theory for any scaling of the ModMax Lagrangian.

Notation and conventions. We use the almost minus signature of the Lorentz metric $(+, -, -, -)$ and natural units in which the speed of light c and the Planck constant $\hbar$ are set to one.

2 ModMax electrodynamics

The Lagrangian density of ModMax has the following form [1]

$$\mathcal{L} = \cosh\gamma\, S + \sinh\gamma\, \sqrt{S^2 + P^2} \tag{2.1}$$

$$= \frac{\cosh\gamma}{2}\,(\mathbf{E}^2 - \mathbf{B}^2) + \frac{\sinh\gamma}{2}\,\sqrt{(\mathbf{E}^2 - \mathbf{B}^2)^2 + 4(\mathbf{E}\cdot\mathbf{B})^2}\,,$$

where

$$S = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\,(\mathbf{E}^2 - \mathbf{B}^2)\,, \qquad P = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \mathbf{E}\cdot\mathbf{B} \tag{2.2}$$

are the two independent $D = 4$ Lorentz invariants constructed from the electromagnetic field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and its Hodge dual $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\lambda}F_{\rho\lambda}$. $E_i = F_{0i}$ is the electric three-vector field ($i = 1, 2, 3$), $B^i = \tilde{F}^{0i}$ is the magnetic vector field and γ is a dimensionless coupling constant. The conditions of causality and unitarity require this constant to be non-negative $\gamma \geq 0$ [1]. These values of γ also ensure that the Lagrangian density is a convex function of the electric field E_i [1] and that its energy-momentum tensor satisfies the weak, strong and dominant energy conditions [28]. Note that Maxwell's electrodynamics is not a weak field limit of ModMax because of conformal invariance, but is recovered when the ModMax coupling constant tends to zero $\gamma \to 0$.

The Lagrangian field equations of the theory, accompanied by the Bianchi identities, are

$$\partial_\mu G^{\mu\nu} = \cosh\gamma\, \partial_\mu F^{\mu\nu} + \sinh\gamma\, \partial_\mu \Big(\frac{SF^{\mu\nu} + P\tilde{F}^{\mu\nu}}{\sqrt{S^2 + P^2}}\Big) = 0, \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0, \tag{2.3}$$

where

$$G^{\mu\nu} := -2\frac{\partial \mathcal{L}}{\partial F_{\mu\nu}} = \cosh\gamma\, F^{\mu\nu} + \sinh\gamma\, \Big(\frac{SF^{\mu\nu} + P\tilde{F}^{\mu\nu}}{\sqrt{S^2 + P^2}}\Big). \tag{2.4}$$

The equations are non-linear, but they linearize for field configurations for which $P = c\,S$ with c being a constant. So all the solutions of Maxwell's equations with $P = c\,S$ are solutions of ModMax theory.

One can notice that the equations of motion (2.3) are non-analytic and are not well defined when the electromagnetic fields are null, i.e. the fields for which the Lorentz scalar and pseudo-scalar (2.2) are zero

$$S = 0, \qquad P = 0. \tag{2.5}$$

In the null-field limit the ambiguity of the values of the scalar factors $\frac{S}{\sqrt{S^2+P^2}}$ and $\frac{P}{\sqrt{S^2+P^2}}$ in (2.3) range from -1 to $+1$. This might be an issue, since the class of solutions for which the electromagnetic fields are null, such as the plane waves, are not well defined in the ModMax Lagrangian formulation.[1] However, somewhat surprisingly, the Hamiltonian formulation of ModMax comes to the rescue [1]. In the ModMax Hamiltonian formulation the null electromagnatic fields are well defined. Among these configurations, the plane waves [1] and topologically non-trivial knotted electromagnetic fields [4] (generalizing those of Maxwell's theory [29]) are exact solutions of the ModMax Hamiltonian equations (see [1, 2, 4] for more details).

2.1 Conformal and duality invariance

The ModMax action $I = \int d^4x\, \mathcal{L}(S, P)$ is invariant under the $D = 4$ conformal transformations [1, 8], which can be easily checked for the rescaling of the coordinates and the fields with a constant parameter b

$$x^\mu \to b\, x^\mu, \quad A_\mu \to b^{-1}A_\mu, \quad F_{\mu\nu} \to b^{-2}F_{\mu\nu}. \tag{2.6}$$

The ModMax field equations and the Bianchi identities (2.3) are invariant under electric-magnetic duality $SO(2)$ rotations of $G^{\mu\nu}$ and $\tilde{F}^{\mu\nu}$ [1, 5]

$$\begin{pmatrix} G^{\mu\nu}(F') \\ \tilde{F}'^{\mu\nu} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} G^{\mu\nu}(F) \\ \tilde{F}^{\mu\nu} \end{pmatrix}. \tag{2.7}$$

The duality invariance is ensured by the fact that the ModMax Lagrangian density (2.1) satisfies a condition which must hold for any duality-invariant non-linear electrodynamics [30], namely

$$F_{\mu\nu}\tilde{F}^{\mu\nu} - G_{\mu\nu}\tilde{G}^{\mu\nu} = 0 \qquad \Rightarrow \qquad \mathcal{L}_S^2 - \frac{2S}{P}\mathcal{L}_S\mathcal{L}_P - \mathcal{L}_P^2 = 1, \tag{2.8}$$

where $\mathcal{L}_S = \frac{\partial\mathcal{L}}{\partial S}$ and $\mathcal{L}_P = \frac{\partial\mathcal{L}}{\partial P}$.

[1] Note, though, that the vacuum (in which $F_{\mu\nu} = 0$) is a well defined solution of the ModMax Lagrangian field equations (2.3) with zero energy-momentum tensor.