Some formal aspects of two dimensional gravity and its field theory dual

PHN-600B Presentation

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Introduction

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Introduction to ModMax Electrodynamics ([1],[8

- ModMax is a **nonlinear generalization of Maxwell theory** in $D = 4$ that preserves:
  - Conformal invariance
  - Electric-magnetic duality invariance
- ModMax Lagrangian:

$$\mathcal{L} = \cosh\gamma\,\mathcal{S} + \sinh\gamma\sqrt{\mathcal{S}^2 + \mathcal{P}^2}$$

where:

$$\mathcal{S} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2), \quad \mathcal{P} = -\frac{1}{4}F_{\mu\nu}\widetilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$$

- γ is a dimensionless parameter, $\gamma \to 0$ recovers linear Maxwell theory.
- Despite being nonlinear, exact solutions like plane waves and Liénard-Wiechert fields persist.

Conformal and Duality Invariance I

Conformal Invariance:

- The action is invariant under scaling:

$$x^\mu \to b x^\mu, \quad A_\mu \to b^{-1} A_\mu, \quad F_{\mu\nu} \to b^{-2} F_{\mu\nu}$$

Duality Invariance:

- The field equations are invariant under $SO(2)$ electric-magnetic duality rotations:

$$\begin{pmatrix} G'_{\mu\nu} \\ \widetilde{F}'_{\mu\nu} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} G_{\mu\nu} \\ \widetilde{F}_{\mu\nu} \end{pmatrix}$$

- Duality condition:

$$\mathcal{L}_S^2 - 2S\mathcal{L}_S\mathcal{L}_P - \mathcal{L}_P^2 = 1$$

where $\mathcal{L}_S = \partial\mathcal{L}/\partial\mathcal{S}$ and similarly for $\mathcal{L}_P$.

ModMax Coupled to Gravity I

- ModMax theory admits coupling to gravitational backgrounds via standard minimal coupling:

$$\mathcal{L} = \sqrt{-g}\left(\cosh\gamma\,\mathcal{S} + \sinh\gamma\sqrt{\mathcal{S}^2 + \mathcal{P}^2}\right)$$

- Coupling to point charges (electric, magnetic, dyons) preserves exact Liénard-Wiechert field solutions.
- A reformulation using auxiliary axion-dilaton-like scalar fields suggests a possible **effective field theory origin** and link to string theory.
- Thermodynamic and geometric effects of ModMax (e.g., black holes, Taub-NUT metrics) differ from Maxwell, signaling new physics in strong-field regimes.

Mathematical Definition of Central Charge I

Central Charge ([2],[5], [7]) in 2D CFT:

- The energy-momentum tensor generates the Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$$

- The constant c is the **central charge**.

Gravity Perspective (Brown–Henneaux [2]):

- In 3D gravity with AdS_3 boundary conditions:

$$c = \frac{3\ell}{2G_3}$$

where ℓ is the AdS radius and G_3 is Newton's constant in 3D.

2D Gravity on AdS_2 [3] (e.g., Jackiw-Teitelboim gravity):

Mathematical Definition of Central Charge II

- With Maxwell-dilaton gravity and a twisted energy-momentum tensor:

$$\tilde{T}_{\pm\pm} = T_{\pm\pm} \pm A\partial_{\pm} j_{\pm}$$

the central charge becomes:

$$c = 12kG^2 Q^2 \ell^4$$

where Q is electric charge, ℓ the AdS_2 radius, and k the level of the U(1) current.

Physical Significance of Central Charge I

1. Holography and AdS/CFT:

- The central charge governs the density of states in a 2D CFT.
- In AdS/CFT, it encodes the number of degrees of freedom on the boundary.

2. Entropy via Cardy Formula:

$$S = 2\pi\sqrt{\frac{c}{6}\left(\Delta - \frac{c}{24}\right)}$$

- This formula matches black hole entropy when c comes from asymptotic symmetries.
- Example: AdS_2/CFT_1 — entropy from twisted CFT matches gravity result.

3. Indicator of Anomalies:

- Central charge appears in quantum anomalies (e.g., conformal anomaly).
- Shows failure of classical symmetry at the quantum level.

4. Near-Horizon Symmetries:

Physical Significance of Central Charge II

- Emergence of a CFT near black hole horizons with well-defined central charge.
- Supports holographic descriptions even in 2D (AdS_2 cases).

Central Charge for AdS$_2$ Gravity [Hartman et al.][5] I

Setup: 2D Maxwell-dilaton gravity on AdS$_2$ with constant electric field E, AdS radius ℓ, and gauge field A_μ.

1. Classical Action in Conformal Gauge:

$$S = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left(\eta\left(R + \frac{8}{\ell^2}\right) - \frac{\ell^2}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

2. Twisted Stress Tensor: Conformal diffeomorphisms do not preserve boundary conditions $\rightarrow$ must be accompanied by a $U(1)$ gauge transformation. This leads to a *twisted* energy-momentum tensor:

$$\widetilde{T}_{\pm\pm} = T_{\pm\pm} \pm \frac{E\ell^2}{4}\partial_\pm j_\pm$$

3. Commutator of Twisted Stress Tensor:

$$\widetilde{T}_{--}(x), \widetilde{T}_{--}(y)] = \cdots + \frac{\pi k E^2\ell^4}{8}\partial_y^3\delta(x-y)$$

Central Charge for AdS_2 Gravity [Hartman et al.][5] II

4. Central Charge: From the above commutator, extract the central term:

$$c = \frac{3kE^2\ell^4}{4}$$

Conclusion: Twisting by a conserved $U(1)$ current yields a Virasoro algebra with nonzero central charge, despite pure AdS_2 gravity having $c = 0$.

Central Charge for JT Gravity ([6],[9]) coupled to ModMax [HR,DRC][8] I

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4D to 3D reduction and central charge [4] I

We start with the 4D action for *EH* gravity coupled to *ModMax* electrodynamics. We then perform suitable dimensional reduction to obtain the action in 3D and solve for the system thereafter.
The ansatz for the metric is taken to be

$$ds^2_{(4)} = e^{2\alpha\phi} ds^2_{(3)} + e^{2\beta\phi}(dx^3)^2 \tag{1}$$

$$dx^2_{(3)} = g_{\mu\nu}(x^\rho) dx^\mu dx^\nu \tag{2}$$

$$R_{(4)} = e^{-2\alpha\phi}\left(R_{(3)} - \frac{1}{2}(\partial\phi)^2 - d\alpha\Box\phi\right) \tag{3}$$

μ, ν are indices representing reduced dimensions and x^3 is the compact dimension. Note that we can drop the dalembertian term in the Ricci scalar as it is a total derivative.
Further for this form of choice for dissection of the metric to one lower dimension (from $d+1$ to d), we observe that $\mathcal{L}$ becomes $e^{(\beta+(d-2)\alpha)\phi}\sqrt{-g}\mathcal{R} + \dots$ where $\mathcal{R}$ is the Ricci scalar in d dimensions. So it is

4D to 3D reduction and central charge [4] II

required to set $\beta + (d-2)\alpha = 0$, which gives us $\beta = -\alpha$ in $4D$ to $3D$ reduction. Further to ensure that we obtain a term of the form $\frac{1}{2}\sqrt{-g}\,(\partial\phi)^2$ in the action we require $\alpha^2 = \dfrac{1}{2(d-1)(d-2)}$ which gives $\alpha = \frac{1}{2}$ for our case.
The reduced metric determinant can be determined using

$$\sqrt{-g_{(4)}} = \sqrt{-g_{(3)}}\,e^{(\beta+d\alpha)\phi} = e^{2\alpha\phi}\sqrt{-g_{(3)}}$$

The 3D projected *ModMax* lagrangian can now be calculated as

$$\begin{aligned}
s = \frac{1}{2}F_{AB}F^{AB} &= \frac{1}{2}F_{AB}F_{CD}g^{AC}g^{BD} \\
&= \frac{1}{2}F_{\mu\nu}F^{\alpha\beta}g^{\mu\alpha}g^{\nu\beta} + e^{-2\beta\phi}\left(F_{M3}F_{P3}g^{PM}\right) \\
&= \frac{1}{2}F_{\mu\nu}F^{\mu\nu} + e^{-2\beta\phi}(\partial_\mu\chi)^2
\end{aligned}$$

4D to 3D reduction and central charge [4] III

$$
\begin{aligned}
p &= \frac{1}{2}F_{AB}\tilde{F}^{AB} = \frac{1}{2}F_{AB}F_{CD}\epsilon^{ABCD} \\
&= 2e^{-2\beta\phi}\sqrt{-g_{(3)}}\,(F_{01}F_{23} + F_{02}F_{31} + F_{21}F_{03}) \\
&= e^{-2\beta\phi}\epsilon^{abc}F_{ab}F_{c3} \\
&= 2e^{-2\beta\phi}\epsilon^{abc}F_{ab}\partial_c\chi
\end{aligned}
$$

$$
\mathcal{L}^{(3)}_{\mathrm{MM}} = \frac{1}{2}\left(s\cosh\zeta - \sqrt{s^2+p^2}\sinh\zeta\right) \tag{4}
$$

here ζ is the *ModMax* parameter.
The 4D lagrangian is integrated along the compact dimension and we get the 3D reduced action as

$$
I = \frac{1}{16\pi G_3}\int d^3x\sqrt{-g_{(3)}}\left(R - 2\Lambda - 4\kappa\mathcal{L}^{(3)}_{\mathrm{MM}}\right) \tag{5}
$$

The variation of the action yields (and setting them all to zero to find the stationary action)

$$\delta I = \frac{1}{16\pi G_3}\int d^3x\sqrt{-g}\left(\Psi_{\mu\nu}\delta g^{\mu\nu} + \Psi_\mu \delta A^\mu + \Psi_\phi \delta\phi + \Psi_\chi \delta\chi\right) \tag{6}$$

where

$$\Psi_\phi = -2\kappa\left[-2\beta e^{-2\beta\phi}(\partial\chi)^2\cosh\zeta + \frac{\sinh\zeta}{\sqrt{s^2+p^2}}\left(se^{-2\beta\phi}(\partial\chi)^2 + p^2\right)\right] = 0 \tag{7}$$

$$\Psi_\chi = -4\kappa\nabla_\mu\left[-e^{-2\beta\phi}\partial^\mu\chi\cosh\zeta + e^{-2\beta\phi}\frac{\sinh\zeta}{\sqrt{s^2+p^2}}\left(s(\partial\chi)^2 + \frac{p}{2}\epsilon^{ab\mu}F_{ab}\right)\right] = 0 \tag{8}$$

$$\Psi_\nu = 4\kappa\nabla_\mu\left[F^\mu_\nu\cosh\zeta - \frac{\sinh\zeta}{\sqrt{s^2+p^2}}\left(sF^\mu_\nu + pe^{-2\beta\phi}\epsilon^{\mu bc}g_{b\nu}\partial_c\chi\right)\right] = 0 \tag{9}$$

4D to 3D reduction and central charge [4] V

$$\Psi_{\mu\nu} = \left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + \Lambda g_{\mu\nu} + 2\kappa\mathcal{L}_{MM}g_{\mu\nu} -$$
$$\kappa\left[2\left(\cosh\zeta - \frac{s\sinh\zeta}{\sqrt{s^2+p^2}}\right)\left(F_{\mu}^{\beta}F_{\nu\beta} + e^{-2\beta\phi}\partial_\mu\chi\partial_\nu\chi\right) - \frac{p^2\sinh\zeta}{\sqrt{p^2+s^2}}g_{\mu\nu}\right] = 0 \tag{10}$$

We can extract some information from the Eq 10 upon contracting it with the contravariant metric tensor. We thus obtain

$$-\frac{R}{2} + 3\Lambda - 2\kappa\left[-3\mathcal{L}_{MM} + 2\left(\cosh\zeta - \frac{s\sinh\zeta}{\sqrt{s^2+p^2}}\right)\left(F^2 + e^{-2\beta\phi}(\partial\chi)^2\right) - \frac{3p^2}{\sqrt{p^2+s^2}}\sinh\zeta\right] = 0 \tag{11}$$

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