2. The Case of S-State Interactions Only

In the overall c.m. system, let the momenta of the proton and 2 neutrons be $\mathbf{P}_3$, $\mathbf{P}_1$, and $\mathbf{P}_2$ respectively, so that

$$\mathbf{P}_1 + \mathbf{P}_2 + \mathbf{P}_3 = 0. \tag{2.1}$$

To simplify matters, we do not consider the isobaric formalism at this stage, so that we have to antisymmetrize only with respect to the 2 neutrons. This antisymmetrization naturally comes about through the spin part of the wave function, which we take as

$$(2)^{-\frac{1}{2}}(\alpha_1\beta_2 - \alpha_2\beta_1)\sigma_3, \tag{2.2}$$

with obvious notation. The spatial part of the wave function should therefore be symmetric. As we know from earlier work[5], the triton ground state is almost entirely space symmetric, so that the above idealization is a rather good approximation.

We find it most convenient to express the kinematics of the problem in terms of the momenta $\mathbf{P}_1$ and $\mathbf{P}_2$ of the 2 neutrons. The kinetic energy is then seen from (2.1) to be

$$\frac{\mathbf{P}_1^2 + \mathbf{P}_2^2 + \mathbf{P}_1 \cdot \mathbf{P}_2}{M}. \tag{2.3}$$

The 3-body wave function is also expressed in terms of $\mathbf{P}_1$ and $\mathbf{P}_2$ via eq. (1). The matrix elements of the respective interactions are then found to be

$$(\mathbf{P}_1\mathbf{P}_3|V_{np}|\mathbf{P}_1'\mathbf{P}_3') = \delta(\mathbf{P}_1 + \mathbf{P}_3 - \mathbf{P}_1' - \mathbf{P}_3')(\mathbf{P}_1 + \frac{1}{2}\mathbf{P}_2|V_{np}|\mathbf{P}_1' + \frac{1}{2}\mathbf{P}_2), \tag{2.4}$$

$$(\mathbf{P}_2\mathbf{P}_3|V_{np}|\mathbf{P}_2'\mathbf{P}_3') = \delta(\mathbf{P}_2 + \mathbf{P}_3 - \mathbf{P}_2' - \mathbf{P}_3')(\mathbf{P}_2 + \frac{1}{2}\mathbf{P}_1|V_{np}|\mathbf{P}_2' + \frac{1}{2}\mathbf{P}_1), \tag{2.5}$$

$$(\mathbf{P}_1\mathbf{P}_2|V_{nn}|\mathbf{P}_1'\mathbf{P}_2') = \delta(\mathbf{P} - \mathbf{P}')(\mathbf{p}|V_{nn}|\mathbf{p}'), \tag{2.6}$$

where

$$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2, \quad \mathbf{P}' = \mathbf{P}_1' + \mathbf{P}_2', \quad 2\mathbf{p} = \mathbf{P}_2 - \mathbf{P}_1, \quad 2\mathbf{p}' = \mathbf{P}_2' - \mathbf{P}_1'. \tag{2.7}$$

For s-state interactions alone, we take, like Yamaguchi[8],

$$\langle\mathbf{p}|V_{np}|\mathbf{p}'\rangle = -\frac{\lambda_0}{M}g(p)g(p'), \tag{2.8}$$

$$\langle\mathbf{p}|V_{nn}|\mathbf{p}'\rangle = -\frac{\lambda_1}{M}f(p)f(p'). \tag{2.9}$$

Using these results, the 3-body Schrödinger equation becomes in the above notation

$$(\mathbf{P}_1^2 + \mathbf{P}_2^2 + \mathbf{P}_1 \cdot \mathbf{P}_2 + \alpha_0^2)\Psi(\mathbf{P}_1, \mathbf{P}_2) = \lambda_0 \int d^3\mathbf{P}_1' g(\mathbf{P}_1 + \frac{1}{2}\mathbf{P}_2) g(\mathbf{P}_1' + \frac{1}{2}\mathbf{P}_2)\Psi(\mathbf{P}_1', \mathbf{P}_2) + (1 \leftrightarrow 2), \tag{2.10}$$

$$+\lambda_1 \int d^3\mathbf{p}' f(\mathbf{p}) f(\mathbf{p}')\Psi(\frac{\mathbf{P}_2 - \mathbf{P}_1}{2}, \frac{\mathbf{P}_2' - \mathbf{P}_1'}{2}),$$

where

$$-E = \alpha_0^2/M$$

is the binding energy of the triton.