

Today's Content:

→ Prefix & Suffix Strings

→ LPS of a given string

→ LPST of a given string

Problems based on LPST:

→ Pattern Matching by LPST

→ Cyclic Rotations

→ Min characters to be added at start to make string palindrome

Q) S_1 & S_2 are given of N length

if $(S_1 == S_2)$ TC: $O(N)$

S_1 : a b a c
 ↑ ↑ ↑ ↓ not same
 S_2 : a b a d

Q): Given Pattern (P) and Text (T) check, if pattern is present as a substring in T, $\text{len}(T) \geq \text{len}(P)$

Try run:

	Pat
$P_k = \begin{matrix} 0 & 1 & 2 & 3 \\ a & c & d & a \end{matrix}$	$[0-3] = \{b c a c\} = a c d a$ not match
$T_N = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \\ b & c & a & c & d & a \end{matrix}$	$[1-4] = \{c a c d\} = a c d a$ not match
	$[2-5] = \{a c d a\} = a c d a$ match

Idea: For every substring of len k in Text compare with Pattern?

→ // no. of substring of len k in Text

TC: $(N - k + 1) * k$ → // time taken to compare a string of len $= k$
→ if $k \approx N/2$

TC: $(N - N/2 + 1) * (N/2) = \lceil N/2 + 1 \rceil \lceil N/2 \rceil \approx O(N^2)$

Note:

Insert a Single String S_N in HashSet & string TC: $O(N)$?

Given a String s of N size:

Prefn Strings: All substrings starting at 0^{th} index

Suffn Strings: All substrings ending at last index $\{N-1\}$

Ex: $s = a \overset{0}{b} \overset{1}{c} \overset{2}{a}$

Ex: $s = d \overset{0}{e} \overset{1}{a}$

Prefn Strings

a
a b
a b c
a b c a

Suffn Strings

a
c a
b c a
a b c a

Suffn

a
e a
d e a

LPS of a String: length of longest Prefix String = Suffix String
 value =

Note: Neglect complete while calculating

Ex1: $S = a \ b \ c \ a \ b$ LPS=2

<u>Prefix</u>		<u>Suffix</u>
a	←→	b
a b	←→	a b
a b c		c a b
a b c a		b c a b
a b c a b		a b c a b

Note: Neglect complete string

Ex2: $S = a$ LPS=0

<u>Prefix</u>	<u>Suffix</u>
a	a

Ex3: $S = a \ a \ a \ a \ a$ LPS=4

<u>Prefix</u>	<u>Suffix</u>
a	a
a a	a a
a a a	a a a
a a a a	a a a a
a a a a a	a a a a a

How to calculate it LPS?

$S = S_0 \ S_1 \ S_2 \ S_3 \ S_4$

<u>Prefix</u>	<u>Suffix</u>	<u>Iterations</u>
S_0	S_4	→ 1 ite
$S_0 S_1$	$S_3 S_4$	→ 2 ite
$S_0 S_1 S_2$	$S_2 S_3 S_4$	→ 3 ite
$S_0 S_1 S_2 S_3$	$S_1 S_2 S_3 S_4$	→ 4 ite

Total ite = 10

$S_N = S_0 \ S_1 \ S_2 \ S_3 \ \dots \ S_{N-3} \ S_{N-2} \ S_{N-1}$

<u>Prefix</u>	<u>Suffix</u>	<u>Iterations</u>
$S[0, 0] = S[N-1, N-1]$		1 ite
$S[0, 1] = S[N-2, N-1]$		2 ite
$S[0, 2] = S[N-3, N-1]$		3 ite
$S[0, 3] = S[N-4, N-1]$		4 ite
\vdots	\vdots	\vdots
$S[0, N-2] = S[1, N-1]$		N-1 ite

Total iter = $1 + 2 + 3 + \dots + N-1$

$= \frac{(N)(N-1)}{2} \approx O(N^2)$

Given S_N return $LPS[N]$

$LPS[i] = [lps \text{ value of substring } S[0, i]]$

Ex: $S = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ a & a & b & a & a & b & a \end{matrix}$

$LPS[7] = \underline{0} \quad \underline{1} \quad \underline{0} \quad \underline{1} \quad \underline{2} \quad \underline{3} \quad \underline{4}$

$LPS[i] = lps \text{ of substring } S[0 - i]$

String

Lps Value

$LPS[0] = lps \text{ of substring } S[0 - 0] = a \quad 0$

$LPS[1] = lps \text{ of substring } S[0 - 1] = aa \quad 1$

$LPS[2] = lps \text{ of substring } S[0 - 2] = aab \quad 0$

$LPS[3] = lps \text{ of substring } S[0 - 3] = aaba \quad 1$

$LPS[4] = lps \text{ of substring } S[0 - 4] = aabaa \quad 2$

$LPS[5] = lps \text{ of substring } S[0 - 5] = aabaaab \quad 3$

$LPS[6] = lps \text{ of substring } S[0 - 6] = aabaaaba \quad 4$

Obs: $S_N, lps[N]$

↳ We are calculating lps for N strings

↳ TC: $N \times O(N^2) \approx O(N^3)$

↳ Time taken To calculate $LPS[i]$ of $S_N \approx TC: O(N)$

Q1) Search for a given Pattern P in Text T

Note: Both contains only lower case alphabets
a b ... z

Ex1:

$T_N = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \\ a & a & b & a & c & d \end{matrix}$ } Idea: Since lps will match prefix & suffix, append Pattern at start of Text & create a new string

$P_k = a \ b \ a \ c$

$C_{N+k} = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ a & b & a & c & a & a & b & a & c & d \end{matrix}$
Pattern == Text[5-8]

$lps[10] = \begin{matrix} 0 & 0 & 1 & 0 & 1 & 1 & 2 & 3 & 4 & 0 \end{matrix}$
= k length of Pattern

Steps: Given P_k, T_N : TC: $O(N+k)$ SC: $O(N+k)$

Step1: Created a new Concatenated String

$C = P_k + @ + T_N \xrightarrow{\quad \quad \quad} \begin{matrix} \underline{TC} & \underline{SC} \\ O(N+k) & O(N+k) \end{matrix}$
↳ Added to differentiate pattern & Text

Step2: Create lps[N+k] for String C $\rightarrow O(N+k) \ O(N+k)$

Step3: Search for k in lps[], $\rightarrow O(N+k) \ O(1)$

: If $lps[i] == k$ Pattern present

If k is not lps[] Pattern not found

Edge Case:

$P_y = a b a b$

$T_s = a b a c a$

$C_q = P_y + T_s$ issue: We are not able to separate Pattern & Text

	0	1	2	3	4	5	6	7	8
$C_q =$	a	b	a	b	a	b	a	c	a

$lps[q] = \underline{0} \quad \underline{0} \quad \underline{1} \quad \underline{2} \quad \underline{3} \quad \underline{4} \quad - \quad - \quad -$

↳ $4 = k$ present present * wrong

Idea: To separate Pattern & Text add a separator = @

↳ Any character not present in both strings

$P_y = a b a b$

$T_s = a b a c a$

	0	1	2	3	4	5	6	7	8	9
$C_{10} =$	a	b	a	b	@	a	b	a	c	a

$lps[l] = \underline{0} \quad \underline{0} \quad \underline{1} \quad \underline{2} \quad \underline{0} \quad \underline{1} \quad \underline{2} \quad \underline{3} \quad \underline{0} \quad \underline{1}$

↳ In $lps[l]$, if $k=4$ not present: pattern not present

Q2) # Count no: of Substrings of T are same as Pattern (P)

P: a a $T_N: a^0 a^1 a^2 a^3$
 h

$C_{N+k+1} = a^0 a^1 a^2 a^3 a^4 a^5 a^6$

$lps[] = 0 \ 1 \ 0 \ 1 \ 2 \ 2 \ 2 \Rightarrow \text{Count of } 2 = 3$

Idea: find no: of k in $lps[]$, no: of substrings of $T = P$

TC: $O(N+k)$ SC: $O(N+k)$

Q3) Given a Binary String S_N

Find no: of start \rightarrow end rotations of S will give same string S

Note: At max N rotations are possible \rightarrow

Ex: $S = 1 \ 0 \ 1 \ 0 \ \text{ans} = 2$

$S_1 = 0 \ 1 \ 0 \ 1 \ == S$

$S_2 = 1 \ 0 \ 1 \ 0 \ == S + 1$

$S_3 = 0 \ 1 \ 0 \ 1 \ == S$

$S_4 = 1 \ 0 \ 1 \ 0 \ == S + 1$

obs: N^{th} rotation of given string will

be original string, Atleast ans will increase by +1

// Rotation explanation

$S = a \ b \ c \ d$

After₁ = $b \ c \ d \ a == S$

After₂ = $c \ d \ a \ b == S$

After₃ = $d \ a \ b \ c == S$

After₄ = $a \ b \ c \ d == S$

obs: After every rotation we are comparing it with S ,

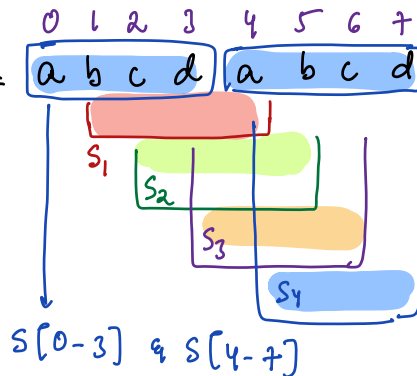
$P = S \quad T = S + S$ // will contain all rotations

$C = P @ T$ $T = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ a & b & c & d & a & b & c & d \end{matrix}$

$C = S @ SS$

↓ calculate freq of S
in C

TC: $3N \rightarrow O(N)$
SC: $3N \rightarrow O(N)$



obs: In general we only need to have one abcd in T but we have 2, because of this in our ans, we are getting extra 1, in your final ans remove extra, $ans-1$

Ex:

$S = 1010$

$P = 1010 \quad T = 10101010$

$C = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 1 & 0 & 1 & 0 & @ & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{matrix}$

$Lps[] = \begin{matrix} 0 & 0 & 1 & 2 & 0 & 1 & 2 & 3 & 4 & 3 & 4 & 3 & 4 \end{matrix}$

↳ cnt of $k=3$, return $cnt-1=2$

-1 because T contains an extra S

4Q) Given a string S_n , min character to be added at start of string to make entire string palindrome

{ Sunday doubts session }

Ex1: $S =$ a d a c d ans =

$S =$ a b b a a c d ans =

$S =$ a b c b a d e f ans =

$S =$ a a a e a a a g h ans =

$S =$ a a d a b a ans =

$S =$ a b c ans =

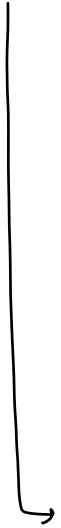
→ ans =

=

Idea: $S =$ ^{0 1 2 3 4}
a b a c d

Edge Case: $S = a\ a\ a$

Steps:



//Lps[] : How to optize this

for a given S_N , to calculate $Lps[N]$

$Lps[0] = 0$ ✓

$i = 1; i \leq N; i++ \{$ $TC \rightarrow O(N)$ ✓

 // Say we found $Lps[i]$ ✓

$n = Lps[i-1]$

 while ($s[i] \neq s[n]$)

 if ($n == 0$) { $n = -1$; break }

$n = Lps[n-1]$

 }

$Lps[i] = n + 1$

 }

Sunday Problem Solving Set : 10am : \rightarrow 12:30pm