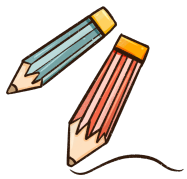
Bit Manipulation 3



Agenda

- MSB & LSB
- How to store negative numbers
- 2's Complement
- Ranges
- Importance of constraints

Changes in your scheduled lectures.

Contest

Friday – Sunday

10 AM 8 PM

3 hr contest

Contest Discussion

Sunday – 9 PM

MSB and LSB

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
10 :	0	0	0	0	1	0	1	0

Leftmost Bit / Most Significant Bit (MSB)

0^{th} bit / Rightmost bit / Least Significant Bit (LSB)

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8 bit

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
A =	1	0	0	0	0	0	0	0

MSB

$\Rightarrow 2^7$

$= 128$

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
B =	0	1	1	1	1	1	1	1

$= 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$

$= 127$

Storing Negative Numbers

→ Take MSB as sign bit

MSB = 0 → +ve

MSB = 1 → -ve

Sign bit

10 : 0 0 0 0 1 0 1 0

-10 : 1 0 0 0 1 0 1 0

4 : 0 0 0 0 0 1 0 0

-4 : 1 0 0 0 0 1 0 0

$10 + (-4) = 6$

Note:
This might or might not be 100% correct. Stay sharp.

10 : 0 0 0 0 1 0 1 0

\+ -4 : 1 0 0 0 0 1 0 0

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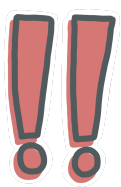
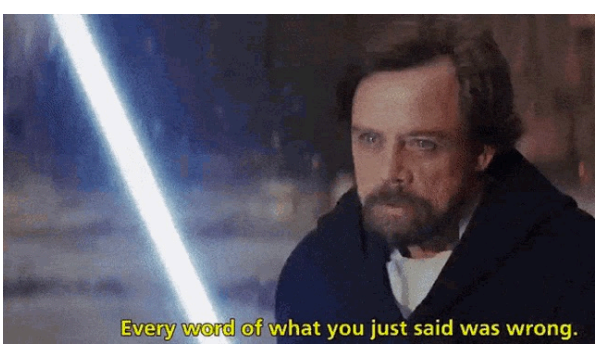
1 0 0 0 1 1 1 0

↳ -14

Not 6

Concept of storing negative numbers with sign bit is wrong.





2's complement

$a \longrightarrow -a$

$= 2$'s complement of a

$= \sim a + 1$

10 : 0 0 0 0 1 0 1 0

-10 = 2's complement of 10

$= \sim 10 + 1$

~ 10 : 1 1 1 1 0 1 0 1

$+1$: 0 0 0 0 0 0 0 1

-10 : 1 1 1 1 0 1 1 0

2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

$= 2^7 + 2^6 + 2^5 + 2^4 + 2^2 + 2^1$

$= 246$

Is $246 == -10$?

Absolute Not.

MSB

$\overline{-2^7}$ $\overline{2^6}$ $\overline{2^5}$ $\overline{2^4}$ $\overline{2^3}$ $\overline{2^2}$ $\overline{2^1}$ $\overline{2^0}$

MSB base value is -ve

-10 : 1 1 1 1 0 1 1 0

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

$= -2^7 + 2^6 + 2^5 + 2^4 + 2^2 + 2^1$

$= -128 + 118$

$= -10$

$10 + (-10) = 0$

```
  10 : 0 0 0 0 1 0 1 0
+ -10 : 1 1 1 1 0 1 1 0
-------------------------
       0
-------------------------
```

```
  4 : 0 0 0 0 0 1 0 0
 -4 : ~4 +1

 ~4 : 1 1 1 1 1 0 1 1
 +1 : 0 0 0 0 0 0 0 1
-------------------------
 -4 : 1 1 1 1 1 1 0 0
-------------------------
```

$10 + (-4) = 6$

(1) ← Discard the carry

8 bit

10 :	0	0	0	0	1	0	1	0
−4 :	1	1	1	1	1	1	0	0
	0	0	0	0	0	1	1	0
	-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$\Rightarrow$ 6

Convert binary to decimal for the following 4 bit numbers

$-2^3 \quad 2^2 \quad 2^1 \quad 2^0$

1 0 1 1

$= (1 \times -2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

$= -8 + 0 + 2 + 1 = -5$

Quiz 1

$-2^3 \quad 2^2 \quad 2^1 \quad 2^0$

1 0 1 0

$= (1 \times -2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$

$= -8 + 0 + 2 + 0 = -6$

$-2^3 \quad 2^2 \quad 2^1 \quad 2^0$

0 0 1 1

$= (0 \times -2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

$= 0 + 0 + 1 + 1 = 3$

Quiz 2

$-2^3 \quad 2^2 \quad 2^1 \quad 2^0$

1 0 0 0

$= (1 \times -2^3) + 0 + 0 + 0$

$= -8$

$-2^3 \quad 2^2 \quad 2^1 \quad 2^0$

0 1 1 1

$= (0 \times -2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

$= 0 + 4 + 2 + 1 = 7$

Convert binary to decimal for the following 8 bit numbers

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

0 0 0 1 0 1 0 1

Quiz 3

$= 2^4 + 2^2 + 2^0$

$= 16 + 4 + 1 = 21$

1 0 0 1 0 1 0 1

Quiz 4

$= -2^7 + 2^4 + 2^2 + 2^0$

$= -128 + 21 = -107$

1 0 0 1 0 0 1 1

$= -2^7 + 2^4 + 2^0$

$= -128 + 16 + 1$

$= -111$

N Bit Number

MSB

— — — — — — . . . . — —

↓

-2^{N-1}

Will MSB base value be always negative ?

Unsigned

→ MSB as +ve

→ Only store +ve nos

Declare a 4 bit no.

2^3 2^2 2^1 2^0

— — — —

Declare a 8 bit no.

2^7 2^6 2^5 ..... 2^0

— — — ..... —

Signed

→ MSB as -ve

→ Both -ve & +ve

Declare a 4 bit no.

-2^3 2^2 2^1 2^0

— — — —

Declare 8 bit no.

-2^7 2^6 2^5 ..... 2^0

— — — ..... —

How will the system know signed/unsigned ?

Java, Python, JS ⟶ Only signed int

C, C++, C# ⟶ Signed & Unsigned

Default

```
int x = 5;          ← signed
unsigned int y = 10; ← unsigned
```

5 : 0 0 0 0 0 1 0 1

-5 = us +1

us : 1 1 1 1 1 0 1 0

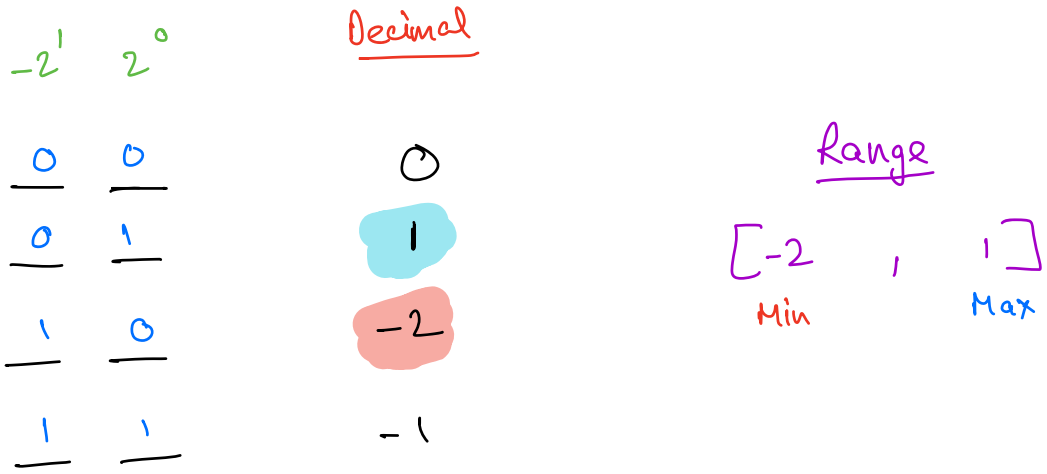
+1 : 1

-5 : 1 1 1 1 1 0 1 1

Break till 10:15 PM

Ranges in Signed Datatypes

2 Bit Signed Number



3 Bit Signed Number

-2^2	2^1	2^0	Decimal
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	-4
1	0	1	-3
1	1	0	-2
1	1	1	-1

Range

$[-4, 3]$

Min Max

4 Bit Signed Number

Quiz 5

-2^3 2^2 2^1 2^0

Max - 0 1 1 1 = 7

Min - 1 0 0 0 = -8

Range

[-8 , 7]

5 Bit Signed Number

-2^4 2^3 2^2 2^1 2^0

Max - 0 1 1 1 1 = 15

Min - 1 0 0 0 0 = -16

Range

[-16 , 15]

Generalisation

	Min	Max
2 bits	-2	1
3 bits	-4	3
4 bits	-8	7
5 bits	-16	15
N bits	-2^{N-1}	$2^{N-1}-1$

N bits

Max

Max Value $= 2^{N-2} + 2^{N-3} + 2^{N-4} + \ldots + 2^0$

$= 2^0 + 2^1 + 2^2 + \ldots + 2^{N-2}$

GP

$a = 2^0 = 1$

$r = 2$

No of terms, $t = N-1$

$$\text{Sum} = a\left(\frac{r^t - 1}{r-1}\right)$$

$$= 1\left(\frac{2^{N-1} - 1}{2-1}\right)$$

$$= \frac{2^{N-1} - 1}{1} = 2^{N-1} - 1$$

Min Value

$= -2^{N-1}$

Data type ranges

N bits — Min -2^{N-1} , Max $2^{N-1}-1$

1 byte = 8 bits

1 B

Range = $[-2^7, 2^7 - 1]$

= $[-128, 127]$

Java byte

C++ → char

2 bytes = 16 bits

Range = $[-2^{15}, 2^{15} - 1]$

= $[-32768, 32767]$

Java short

old C++ → int

4 bytes = 32 bits

$$\text{Range} = \left[-2^{31} , 2^{31} - 1 \right]$$

Java, Present C++ int

$$\approx \left[-2 \times 10^{9} , 2 \times 10^{9} \right]$$

8 bytes = 64 bits Quiz 6

$$\text{Range} = \left[-2^{63} , 2^{63} - 1 \right]$$

Java, C++, C# long

$$\approx \left[-8 \times 10^{18} , 8 \times 10^{18} \right]$$

Approxiamations

$2^{10} = 1024 \approx 1000$

$\Rightarrow 2^{10} \approx 10^3$

Cube both sides

$\Rightarrow 2^{30} \approx 10^9$

Multiply both sides by 2

$2^{31} \approx 2 \times 10^9$

$2^{30} \approx 10^9$

Square both sides

$2^{60} \approx 10^{18}$

Multiply both sides with 2^3

$$2^{63} \simeq 8 \times 10^{18}$$

Importance of Constraints

Q Sum of array

$1 \leq N \leq 10^5$

$0 \leq arr[i] \leq 10^5$

```
long sum = 0
for (i=0; i<N; i++) {
    sum += (long) arr[i];
}
```

Highest value of sum?

Arr = $10^5, 10^5, 10^5, 10^5 \ldots\ldots 10^5$

Terms, N = 10^5

$$Sum = 10^5 \times 10^5$$

$$= 10^{10}$$

Highest value in 4 bytes Int $= 2 \times 10^9$

Overflow

---

2. $c = a * b$

$$1 \le a, b \le 10^6$$

max 2×10^9

$10^6 \quad 10^6 \rightarrow 10^{12}$

← `int c = a * b;` Overflow

`long c = (long)a * b;`

C++: int, long, long long
Java: byte, short, int, long, BigInteger
Python:

Numerical values $\leq 10^{18}$

Computation of large numbers ($n \geq 10^{18}$) is very very slow.

Doubts

Thank You

No assn/hw

Reverse Bits

4 bit

2^3 2^2 2^1 2^0

A = 1 0 1 0 → 0 1 0 1

y

res

res = 0

```
while ( A > 0 ) {
    y = A & 1
    mul = ____
    res += (y * mul)
    A = A >> 1
}
```

← figure out

mul

In first iteration — 2^3 4 bits

Second — 2^2

Third 2^1

In question, you have 32 bits

$$2^x = 1 << x$$

Good Night

Thank You

Friday