Time Complexity 1

→ Big O

→ TLE (Time limit Exceeded)

→ Why TLE?

} Next Session

Today's session

→ Count no of iterations in our codes

Approx 20 questions

Pizza

Pen + Paper

No answers in the chat

Q1. Sum of first N Natural Numbers

$1 + 2 + 3 + 4 \quad \ldots\ldots - N$

$a = 1$

$d = 1$

$$\frac{N(N+1)}{2}$$

AP

$$\frac{N}{2}\left(2a + (N-1)d\right)$$

Q2. [3,10]

Standard Mathematical Notation

$[3,10] \rightarrow 3, 4, 5, 6, 7, 8, 9, 10$

8

Q3. [a, b]

$[5,9] \rightarrow 5, 6, 7, 8, 9$ - (5)

$[a,b] \rightarrow b - a + 1$

Q4.

```
function func (int N) {
    int s=0
    for (i=0; i<=100; i++) {
        s = s+i
    }
}
```

i = 0, 1, 2, 3, 4 100

101 iterations

Q5.

No quiz

```
function func(int N) {
    int s=0
    for (i=35; i<=87; i++) {
        s=s+i
    }
}
```

i = 35, 36, 37, 38 87

i → [35, 87]

87 - 35 + 1

= 53

Q6.

```
func ( int N) {
    int s=0
    for (i=1 ; i<=N; i++) {
        s = s+i
    }
}
```

i = 1, 2, 3, 4, 5 . . . . . N

i → [1, N]

N - 1 + 1

= N

Q7.

No quiz

```
func (int N, int M) {
    int s=0
    for (i=1 ; i<=N; i++) {
        if (i%2==0) {
            s = s+i
        }
    }
    for (i=1 ; i<=M; i++) {
        if (i%2==1) {
            s = s+i
        }
    }
}
```

N

M

Total = N + M

Q8.

```
func ( int N) {
    int s=0
    for (i=1 ; i<=N; i=i+2) {
        s = s+i
    }
}
```

Take N=9

i = 1, 3, 5, 7, 9 → 5

Take N=12

i = 1, 3, 5, 7, 9, 11 → 6

Taking odd no.s from 1 to N

$\frac{N}{2}$

N=9

$\frac{9}{2} = 4.5 = 4$

N=12

$\frac{12}{2} = 6$

Correct answer → $\frac{N+1}{2}$

N=9 ⇒ $\frac{9+1}{2} = 5$

N=12 ⇒ $\frac{12+1}{2} = \frac{13}{2}$

$= 6$

Q9.

```
func (int N) {
    int s=0
    for (i=1; i*i <= N; i++) {
        s = s+i
    }
}
```

$i * i <= N$

$\Rightarrow i^2 <= N$

$\Rightarrow i <= \sqrt{N}$

$i \rightarrow [1, \sqrt{N}]$

a ↑ b ↑

b-a+1

$\Rightarrow \sqrt{N} - 1 + 1 = \sqrt{N}$

$\Rightarrow \sqrt{N}$ iterations

Q10.

```
func (int N) {
    int i = N
    while (i > 1) {
        i = i / 2
    }
}
```

← integer division

10 → 5 → 2 → 1

Iteration	value of i
1	N/2
2	N/4
3	N/8
4	N/16

Divide by 2 till you reach 1

$i = N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \frac{N}{8} \rightarrow \frac{N}{16} \dots\dots 1$

$N = 128$

$\log_2 128 = \log_2 2^7 = 7$

$\log_2 N$

$i = 128 \rightarrow 64 \rightarrow 32 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$

1 2 3 4 5 6 7

$\log_a a^x = x$ ← Log Property

Q11.

```
func (int N) {
    int s = 0
    for (i = 0; i <= N; i = i * 2) {
        s = s + i
    }
}
```

FAANG

N = 100

i = 0	0 * 2 = 0
0	0 * 2 = 0
0	0 * 2 = 0

i will never update

Infinite

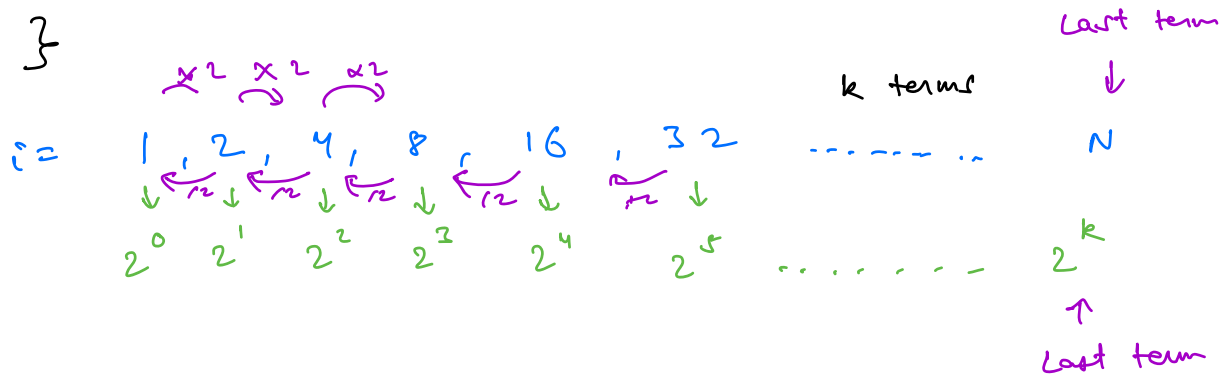
Q12.

```
func (int N) {
    int s=0
    for(i=1; i<N; i=i*2) {
        s = s+i
    }
}
```

N = 100

i → 1 .... ≤99

i=100



$$2^k = N$$

Taking log with base 2 both sides

$$\log_2(2^k) = \log_2(N)$$

$$\Rightarrow \quad k = \log_2 N$$

Log property

$$\log_a a^x = x$$

Q13.

No quiz

```
func ( ) {
    for( i=1 ; i<=4 ; i++) {          ← 4
        for( j=1 ; j<=3 ; j++) {      ← 3
            print( "Hello World")
        }
    }
}
```

12

Q14.

```
func ( ) {
    for( i=1 ; i<=10 ; i++) {
        for( j=1 ; j<=N ; j++) {
            print( "Hello World")
        }
    }
}
```

i	j	Iterations
1	1 → N	N ✓
2	1 → N	N + N ✓
3	1 → N	N + N + N ✓
4	1 → N	N + N + N + N
.....		
10	1 → N	N + N + N + N + ... + N

10 times

Total = $\underbrace{N + N + N + \ldots + N}_{10 \text{ times}}$

= $10N$

Q15.

```
func ( ) {
    for( i=1 ; i<=N ; i++) {
        for( j=1 ; j<=N ; j++) {
            print( "Hello World")
        }
    }
}
```

N

i	j	Iterations
1	1 → N	N
2	1 → N	N + N
3	1 → N	N + N + N
...		...
N	1 → N	N + N + ... N

Total = $\underbrace{N + N + N + N + \ldots N}_{N \text{ terms}}$

$= N * N$

$= N^2$ iterations

Q16.

```
func ( ) {
    for( i=1 ; i<=N ; i++) {
        for( j=1 ; j<=i ; j++) {
            print( "Hello World")
        }
    }
}
```

i	j	Iterations
1	1 → 1	1
2	1 → 2	2
3	1 → 3	3
4	1 → 4	4
5	1 → 5	5
⋮		⋮
N	1 → N	N

Total = $1 + 2 + 3 + 4 \ldots\ldots + N$

$= \frac{N(N+1)}{2}$ iterations

Q17.

```
func ( ) {
    for(i=1 ; i<=N ; i++) {
        for(j=1 ; j<=N ; j=j*2) {
            print("Hello World")
        }
    }
}
```

N terms

i	j		iterations
1	1 → N	x2	$\log_2 N$ +
2	1 → N	x2	$\log_2 N$ +
3	1 → N	x2	$\log_2 N$ +
4	1 → N	x2	$\log_2 N$ +
⋮			⋮ +
N	1 → N	x2	$\log_2 N$

j → 1, 2, 4, 8, 16, 32, 64, 128 ....

Q12 $\log_2 N$

Total = $\underbrace{\log_2 N + \log_2 N + \log_2 N \text{ ---- } \log_2 N}_{N \text{ terms}}$

= $N \log_2 N$

Q18.

```
func (int N) {
    for(i=1 ; i<=2^n ; i=i+1) {
        print ( )
    }
}
```

No quiz

$i \to [1, 2^n]$

2^n iterations

Q19.

```
func (int N) {
    for(i=1; i<=N; i=i+1) {
        for(j=1; j<=2^i; j++) {
            print()
        }
    }
}
```

i	j	Iterations
1	$1 \to 2^1$	2
2	$1 \to 2^2$	2^2
3	$1 \to 2^3$	2^3
4	$1 \to 2^4$	2^4
5	$1 \to 2^5$	2^5
⋮	⋮	⋮
N	$1 \to 2^N$	2^N

Total iterations

$$= 2 + 2^2 + 2^3 + 2^4 + 2^5 \ldots\ldots 2^N$$

$a = 2$ ← first term

$r = 2$ ← common ratio

Geometric Progression (GP)

$$\text{Sum} = a\,\frac{(r^n - 1)}{r - 1}$$

$$= 2\,\frac{(2^N - 1)}{2 - 1}$$

$$= 2(2^N - 1)$$

Comparison Functions

$y = f(x)$

10th class

$y = \log x$

$y = \sqrt{x}$

$y = x$

$y = x^2$

$\log(x) < \sqrt{x} < x < x\log(x) < x\sqrt{x} < x^2 < x^3 < 2^x$

N^2 ↗ Lower order function

N^3 ↖ Higher order function

$N\sqrt{N}$ → higher order

$\log N$ → lower order

Big O Notation

→ what ?
→ why / where ?
→ How ?

} Next session

→ Calculate the no of iterations
→ Neglect all lower order terms
→ Neglect constant coefficient

Ex1

$3N^2 + \sqrt{N} + 10^4$ ← Neglected

↓

$3N^2$ ↳ $O(N^2)$

Ex 2

$5N^2 + 10N^3 + 6N \log N + 10^6$

$5N^2$ ↑ Neglect

$6N \log N$ ↑ Neglect

10^6 ↑ Neglected

↓

$10N^3$

↳ $O(N^3)$

Ex 3

$4N^2 + 3N + 10^6$

↳ $O(N^2)$

Ex 4

Quiz

$N^2 + 10N$

↳ $O(N^2)$

Ex 5

$N \log N + 7N^2 + 8$

↳ $O(N^2)$

Ex 6

$50N^2 + 14N \log N + 31N^2 \log N + 18$

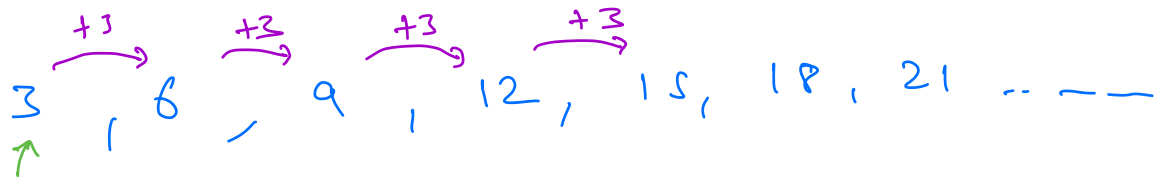
↓

$O(N^2 \log N)$

Doubts

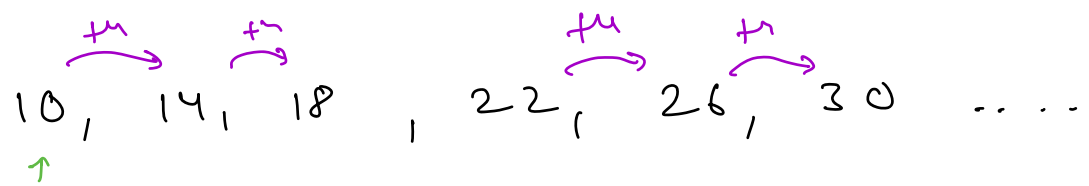
Thank
You

AP
Arithmetic Progression



Common difference, $d = 3$

First term, $a = 3$



$a = 10$

$d = 4$

N^{th} term $= a + (n-1)d$

5^{th} term $= 10 + (5-1)\,4$

$= 10 + 16$

$= 26$

Sum of AP till n terms $= \frac{n}{2}\left[2a + (n-1)d\right]$

$a = 10$

$d = 4$

Sum of first 4 terms

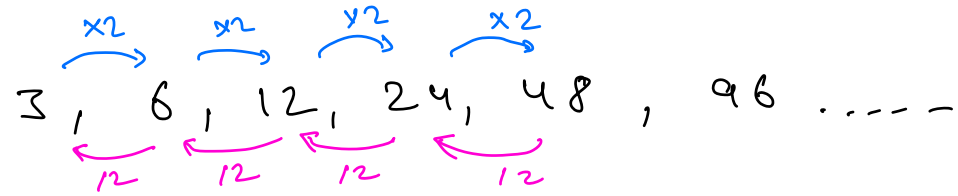
$S_4 = \frac{4}{2}\left[2(10) + (4-1)\,4\right]$

$= 2\left[20 + 12\right]$

$= 2 \times 32$

$= 64$

GP



First term, $a = 3$

Common ratio, $r = 2$

N^{th} term $= ar^{n-1}$

5^{th} term $= 3 \times 2^{5-1}$

$N = 5$

$= 3 \times 2^4$

$= 3 \times 16$

$= 48$

Sum till N terms of GP

$$S_n = \begin{cases} a\left(\frac{r^n - 1}{r - 1}\right), & \text{if } r \neq 1 \\ a\,n, & \text{if } r = 1 \end{cases}$$

First term, $a = 3$

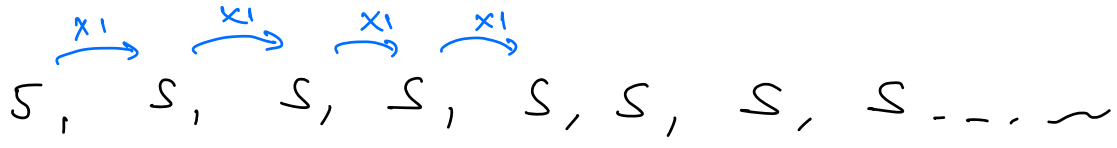
Common ratio, $r = 2$

Sum of first 3 terms $= 3\left(\frac{2^3 - 1}{2 - 1}\right)$

$N = 3$

$= 3\left(\frac{8 - 1}{1}\right)$

$= 3 \times 7 = 21$



a = 5

r = 1

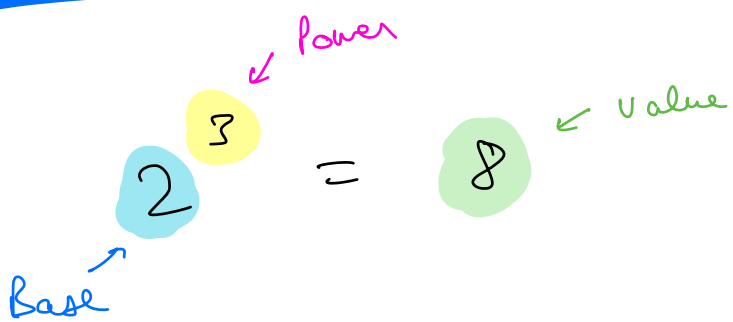
Sum of first 4 = 5 x 4

N = 4

a n

= a * n

Log



$$\log_{2} 8 = 3$$

value ↙ (8)

Power ← (3)

Base ↑ (2)

$\log_2 8 = 3$ ← what power should I raise 2 to get 8?

$2^{x} = 8$

$\Rightarrow 2^{x} = 2^{3}$

$\Rightarrow x = 3$

$$\log_{10} 1000 = 3$$

$$10^{x} = 1000$$

$$\Rightarrow 10^{x} = 10^{3}$$

$$\Rightarrow x = 3$$

$$\log_{2}\left(2^{5}\right) = x = 5$$

what power should I raise 2 to get 2^5

$$2^{x} = 2^{5}$$

$$\Rightarrow x = 5$$

$$\log_a\left(a^{10}\right) = x = 10$$

$$a^{x} = a^{10}$$

$$\Rightarrow \quad x = 10$$

$$\log_a\left(a^{b}\right) = b$$

Important log property

Good Night

Thank You

Monday