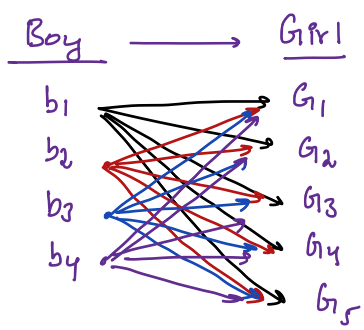
Q) 3 T/F, how many ways we can all answer them?

$\underline{2} * \underline{2} * \underline{2} = 8$ ways

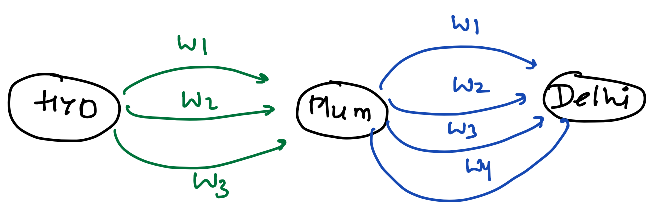
$$\begin{bmatrix} F & F & F \\ F & F & T \\ F & T & F \\ F & T & T \\ T & F & F \\ T & F & T \\ T & T & F \\ T & T & T \end{bmatrix}$$

Q) $\underset{B_1}{\underline{3}} * \underset{B_2}{\underline{3}} * \underset{B_3}{\underline{3}} * \underset{B_4}{\underline{3}} = 81$

Q) 5 girls & 4 boys, how many pair: $\underset{\text{Boy}}{\underline{4}} \overset{\text{and}}{*} \underset{\text{Girl}}{\underline{5}} = 20$



Q):



ways HYD → Delhi via Mum :

↳ HYD → Mum && Mum → Delhi

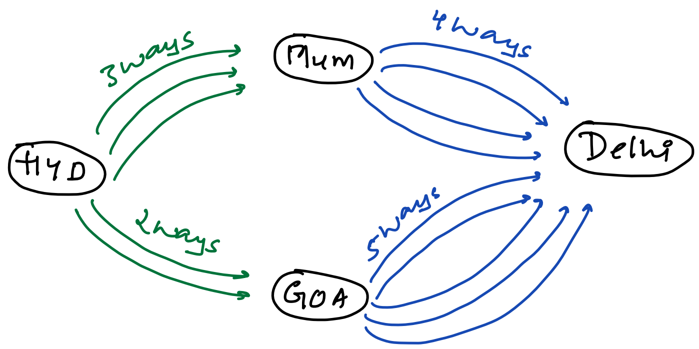
#3 ways #4 ways → 12 ways

4Q) In Class 9 Girls & 8 Boys, number a we can select either a boy or a girl

$\underbrace{\text{9 Girls}}_{\text{9 ways}}$ or $\underbrace{\text{8 Boys}}_{\text{8 ways}}$

9 ways + 8 ways = 17 ways

5Q)



#ways Hyd → Delhi via GOA or Mum

↳ $\underbrace{\text{Hyd} \rightarrow \text{Delhi via Mum}}_{\text{12 ways}}$ or $\underbrace{\text{Hyd} \rightarrow \text{Delhi via GOA}}_{\text{10 ways}}$ = $\boxed{\text{22 ways}}$

Permutations: # ways to arrange {order matters}

Pair (i, j) (j, i) : Both are different

ways to arrange P_1, P_2, P_3 = 6 ways

P_1	P_2	P_3
P_1	P_3	P_2
P_2	P_1	P_3
P_2	P_3	P_1
P_3	P_1	P_2
P_3	P_2	P_1

$\underline{3} * \underline{2} * \underline{1} = 3*2*1 = 3! = 6$

P_1: $P_2 \to P_3$, $P_3 \to P_2$

P_2: $P_1 \to P_3$, $P_3 \to P_1$

P_3: $P_1 \to P_2$, $P_2 \to P_1$

ways to arrange 4, P_1 P_2 P_3 P_4

$\underline{4} * \underline{3} * \underline{2} * \underline{1} = 24$

P_1 {P_2 P_3 P_4} = 6
+
P_2 {P_1 P_3 P_4} = 6
+
P_3 {P_1 P_2 P_4} = 6
+
P_4 {P_1 P_2 P_3} = 6

24

Obs: # ways to arrange N = (N)!

$\hookrightarrow$ $\underline{N} * \underline{N-1} * \underline{N-2} * \underline{N-3} * \dots \underline{1} =$

ways to arrange 2, from 4 people $\{P_1\ P_2\ P_3\ P_4\}$

$\underline{4} * \underline{3} = 12\text{ ways}$

$P_1\ P_2$	$P_2\ P_1$	$P_3\ P_1$	$P_4\ P_1$
$P_1\ P_3$	$P_2\ P_3$	$P_3\ P_2$	$P_4\ P_2$
$P_1\ P_4$	$P_2\ P_4$	$P_3\ P_4$	$P_4\ P_3$

ways to arrange 3, from 5 people = $\{P_1\ P_2\ P_3\ P_4\ P_5\}$

$\underline{5} * \underline{4} \cdot \underline{3} = 60\text{ ways}$

ways to arrange r, from N people ==

$$\underset{1}{\underline{N}}\quad \underset{2}{\underline{N-1}}\quad \underset{3}{\underline{N-2}}\quad \underset{4}{\underline{N-3}}\quad \cdots\cdots\quad \underset{r}{\underline{N-r+1}}$$

ways =

$$\frac{N * N-1 * N-2 * N-3 * \cdots * (N-r+1) * (N-r) * (N-r-1) * (N-r-2) \cdots 1}{(N-r) * (N-r-1) * (N-r-2) \cdots 1}$$

ways = $\dfrac{N!}{(N-r)!}$

$$\boxed{\#\text{ways to arrange } r \text{ items from } N \text{ items} = \frac{N!}{(N-r)!} = {}^{N}P_{r}}$$

↳ order matters

Selection: #ways to select → {order won't matter}

$\{ \underbrace{(i, j) \quad (j, i)}_{\text{Same items}} \}$ Both are same

$\boxed{\begin{matrix} 1 \text{ Red C} \\ 1 \text{ Bl C} \end{matrix}}$

bag

Say 4 people, how many ways we can select 2 people?

$P_1 \; P_2 \; P_3 \; P_4$ = 6 selections

$P_1 P_2 \quad P_2 P_3 \quad P_3 P_4$

$P_1 P_3 \quad P_2 P_4$

$P_1 P_4$

Say 4 people, how many ways we can select 3 people

$P_1 \; P_2 \; P_3 \; P_4$:

$P_1 P_2 P_3$	$P_2 P_3 P_4$	$P_1 P_3 P_4$	$P_1 P_2 P_4$
$P_1 P_3 P_2$	$P_2 P_4 P_3$	$P_1 P_4 P_3$	$P_1 P_4 P_2$
$P_2 P_1 P_3$	$P_3 P_2 P_4$	$P_3 P_1 P_4$	$P_2 P_1 P_4$
$P_2 P_3 P_1$	$P_3 P_4 P_2$	$P_3 P_4 P_1$	$P_2 P_4 P_1$
$P_3 P_1 P_2$	$P_4 P_2 P_3$	$P_4 P_1 P_3$	$P_4 P_1 P_2$
$P_3 P_2 P_1$	$P_4 P_3 P_2$	$P_4 P_3 P_1$	$P_4 P_2 P_1$
$\{P_1 \; P_2 \; P_3\}$	$\{P_2 \; P_3 \; P_4\}$	$\{P_1 \; P_3 \; P_4\}$	$\{P_1 \; P_2 \; P_4\}$

arrange — Selections

$3! = 1$

$24 = x$

$x * 3! = 24$

$x * 6 = 24$

$x = 24/6 = 4$

// Number of ways to <u>select</u> r from N items

↳ order not matters :

	arrage	selection
r items →	$r!$	1
# we can arrage r items from N →	$^N P_r$	n

$n * r! = {}^N P_r$

$n = \frac{^N P_r}{r!}$

$$\#\text{ways to select } R \text{ from } N = \frac{N!}{(N-r)!\, r!} = {}^N C_R$$

$$\#\text{ways to arrage } R \text{ from } N = \frac{N!}{(N-r)!} = {}^N P_r$$

Q) How may ways we can selct 2 items from $N = {}^N C_2$

$$= \frac{N!}{2!\,(N-2)!}$$

$$= \frac{(N)(N-1)\cancel{(N-2)!}}{2!\,\cancel{(N-2)!}}$$

$$= \frac{(N)(N-1)}{2}$$

Properties: $^{N}C_{R} = \frac{N!}{(N-R)!\,R!}$

$\boxed{0! = 1}$

$^{N}C_{0} = 1$

$^{N}C_{N} = 1$

$^{0}C_{0} = 1$

Q) 5 people, #ways to select 2 people

P_1 P_2 P_3 P_4 P_5
$P_1\ P_2 \rightarrow P_3\ P_4\ P_5$
$P_1\ P_3 \rightarrow P_2\ P_4\ P_5$
$P_1\ P_4 \rightarrow P_2\ P_3\ P_5$
$P_1\ P_5 \rightarrow P_2\ P_3\ P_4$
$P_2\ P_3 \rightarrow P_1\ P_4\ P_5$
$P_2\ P_4 \rightarrow P_1\ P_3\ P_5$
$P_2\ P_5 \rightarrow P_1\ P_3\ P_4$
$P_3\ P_4 \rightarrow P_1\ P_2\ P_5$
$P_3\ P_5 \rightarrow P_1\ P_2\ P_4$
$P_4\ P_5 \rightarrow P_1\ P_2\ P_3$

obs: Given N items

select R items = $^{N}C_{R}$

reject N-R items = $^{N}C_{N-R}$

$$^{N}C_{R} = {}^{N}C_{N-R}$$

$$\frac{N!}{(N-R)!\,R!} = \frac{N!}{(\cancel{N}-(\cancel{N}-R))!\,(N-R)!}$$

$$\boxed{^{N}C_{R} = \frac{N!}{R!\,(N-R)!}}$$

$$\boxed{{}^{N}C_{R} = {}^{N-1}C_{R} + {}^{N-1}C_{R-1}}$$

$$R! = R * (R-1)!$$

$$(N-R)! = (N-R) * (N-R-1)!$$

$$= \frac{(N-1)!}{(N-R-1)!\, R!} + \frac{(N-1)!}{(N-R)!\, (R-1)!}$$

$$= \frac{(N-1)!}{(N-R-1)!\, R * (R-1)!} + \frac{(N-1)!}{(N-R) * (N-R-1)! * (R-1)!}$$

$$= \frac{(N-1)!}{(N-R-1)!\, (R-1)!} \left[\frac{1}{R} + \frac{1}{N-R} \right]$$

$$= \frac{(N-1)!}{(N-R-1)!\, (R-1)!} \left[\frac{N-\cancel{R} + \cancel{R}}{(R)(N-R)} \right]$$

$$= \frac{(N)!}{(N-R)!\, R!}$$

$$= {}^{N}C_{R}$$

// Given N & R calculate $^{N}C_{R}$, given $^{N}C_{R}$ won't Overflow

Constraints

$1 <= N <= 20$

$1 <= R <= N$

```
__ func(N, R){                 // implement facts
|   return [ fact(N) / (fact(N-R) * fact(R)) ]
}
```

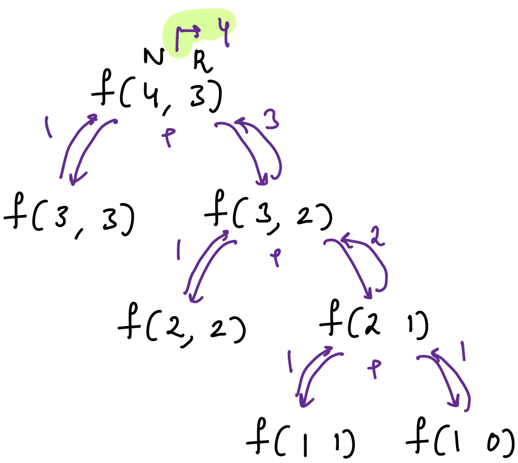
Input

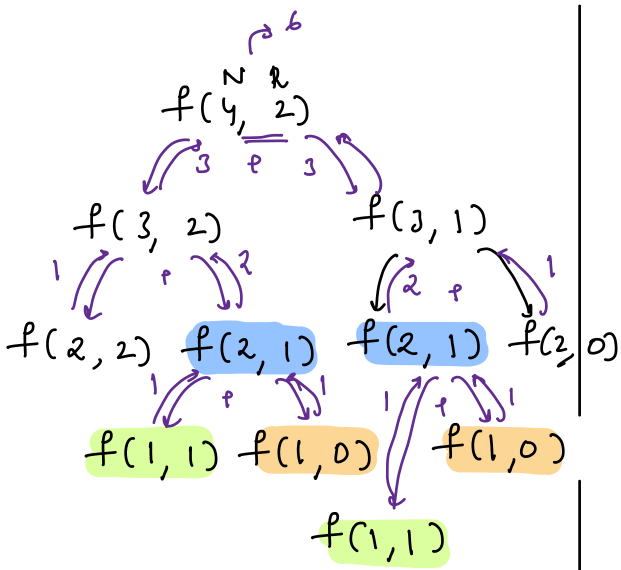
$N = 20, r = 10 \longrightarrow \dfrac{(20)!}{10! * 10!}$ // data might overflow

$$\boxed{{}^{N}C_{R} = {}^{N-1}C_{R} + {}^{N-1}C_{R-1}}$$

$f(N, R) = f(N-1, R) + f(N-1, R-1)$ } recursion

```
int f(N, R){   ≈ TC : O(2^N) ?
    if(R==0 || N==R) { return 1}
    return f(N-1, R) + f(N-1, R-1)
}
```





obs:

In above recursive code, same subproblems called again & again, Calculate it once, store it & re-use it

// $f(\overset{N}{4}, \overset{R}{3}) \rightarrow$ mat[5, 4] $\rightarrow$

	0	1	2	3
0	1	0	0	0
1	1	1	0	0
2	1	2	1	0
3	1	3	3	1
4	1	4	6	$\boxed{4}$

Final ans

mat[4, 3] = ${}^{4}C_{3}$

$${}^{N}C_{R} = {}^{N-1}C_{R} + {}^{N-1}C_{R-1}$$

$${}^{1}C_{1} = {}^{0}C_{1} + {}^{0}C_{0}$$
$${}^{2}C_{1} = {}^{1}C_{1} + {}^{1}C_{0}$$
$${}^{2}C_{2} = {}^{1}C_{2} + {}^{1}C_{1}$$
$${}^{3}C_{1} = {}^{2}C_{1} + {}^{2}C_{0}$$
$${}^{3}C_{2} = {}^{2}C_{2} + {}^{2}C_{1}$$
$${}^{3}C_{3} = {}^{2}C_{3} + {}^{2}C_{2}$$

$${}^{4}C_{1} = {}^{3}C_{1} + {}^{3}C_{0}$$
$${}^{4}C_{2} = {}^{3}C_{2} + {}^{3}C_{1}$$
$${}^{4}C_{3} = {}^{3}C_{3} + {}^{3}C_{2}$$

Note: Filled data row by row

```
cal(N, R){    TC: (N*R)  SC: O(N*R)

    mat[N+1][R+1] = 0 → {entire data already initialized to 0}

    i=0; i<=N; i++){
        mat[i][0] = 1 // 0th col data = 1
    }
    i=1; i<=N; i++){
        j=1; j<=R; j++){
            if(i>=j){
                mat[i][j] = mat[i-1][j] + mat[i-1][j-1]
            }
        }    else{ not needed }
    }
    return mat[N][R]
}
```

$$\underbrace{\text{mat}[i][j]}_{{}^{i}C_{j}} = \underbrace{\text{mat}[i-1][j]}_{{}^{i-1}C_{j}} + \underbrace{\text{mat}[i-1][j-1]}_{{}^{i-1}C_{j-1}}$$

Q) Given string with all distinct characters find rank?

Ex: tac → re-arrange given characters, words in lexicographic order / sorting order

act
atc
cat
cta
tac → 5th rank

// divya → rank of word divya?

order of characters

a
~~d~~
~~i~~
~~v~~
~~y~~

a 4 more char re-arrange = 4! = 24

d a 3 more char re-arr = 3! = 6

d i a 2 more re-arr = 2! = 2

d i v a 1 more char = 1! = 1

Before divya there are 33 words

d i v y a → 34th rank

// find rank for kumar

m o h a n

order:

~~a~~
~~h~~
~~m~~
n
~~o~~

a re-4 char = 4! = 24

h re-4 char = 4! = 24

m a re-3cha = 3! = 6

m h re-3cha = 3! = 6

m n re-3cha = 3! = 6

m o a re-2 = 2! = 2

Before mohan no. of words = 68

m o h a n = 69