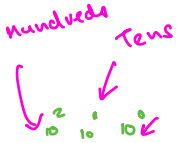
Bit Manipulation 1

Agenda

- Number System Basics
- Binary to Decimal
- Decimal to Binary
- Adding 2 binary numbers
- Bitwise Operators
    - Basic Properties
    - Basic Problems

Number System Basics



7 3 4 : $700 + 30 + 4 = 7 \times 100 + 3 \times 10 + 4$

$= 7 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$

6 5 9 4 : $6000 + 500 + 90 + 4 = 6 \times 10^3 + 5 \times 10^2 + 9 \times 10 + 4 \times 10^0$

2 4 5 : $200 + 40 + 5 = 2 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

Digits - 0 to 9 - 10 digits

Base Power = 10

} Decimal Number System

Other Number Systems

- → Binary - 2
- → Octal - 8
- → Hexa - 16

Octal - $(125)_8$

→ Digits - 0 to 7

→ Power - 8

Quiz!

1 2 5

8^2 8^1 8^0

$= 1 \times 8^2 + 2 \times 8^1 + 5 \times 8^0$

$= 64 + 16 + 5$

$= 85$

Binary → Digits - 0 and 1

Base = 2 → Power = 2

Quiz 2

1) $(1\ 0\ 1\ 1\ 0)_2$ (powers: $2^4\ 2^3\ 2^2\ 2^1\ 2^0$)

$= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 16 + 0 + 4 + 2 + 0$

$= 22$

2) $(1\ 0\ 1\ 0\ 0)_2$ (powers: $2^4\ 2^3\ 2^2\ 2^1\ 2^0$)

$= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$

$= 1 \times 2^4 + 1 \times 2^2$

$= 16 + 4 = 20$

3) $(1\ 2\ 0)_2$ (powers: $2^2\ 2^1\ 2^0$) $\rightarrow$ Invalid Input

Digits in Binary = 0 and 1

Decimal to Binary

2	37
2	18
2	9
2	4
2	2
2	1
	0

1 0 1 0 0 1

$(100101)_2$

Quiz 3

2	25
2	12
2	6
2	3
2	1
	0

1 0 0 1 1

$(11001)_2$

1) Repeatedly divide N by 2, till you get 0, note down the remainders.

2) Take the remainders in reverse

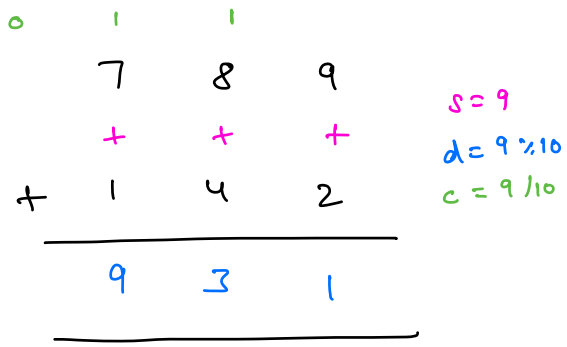
2	19
2	9
2	4
2	2
2	1
	0

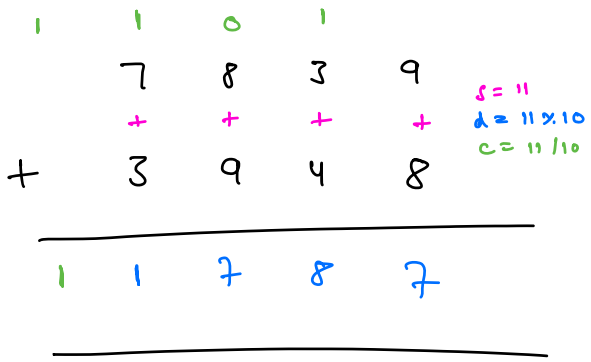
1 0 0 1 1

Quiz 4

$(10011)_2$

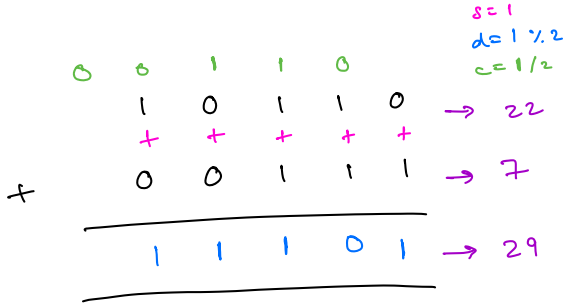
Adding 2 decimal numbers

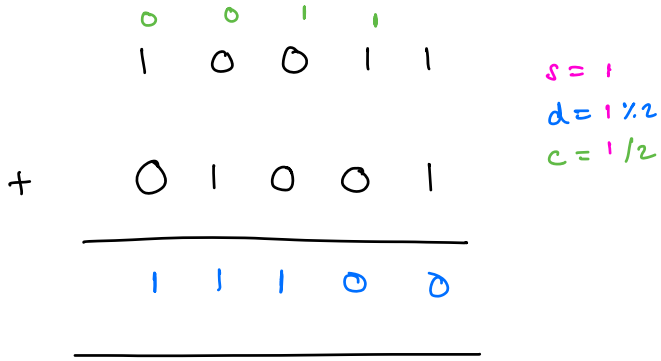




Adding 2 binary numbers

Quiz 5

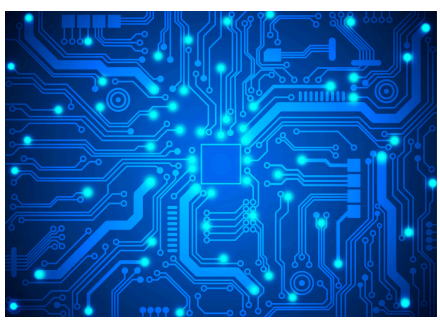




Why Binary ?

We humans use a decimal, or base-10, numbering system, presumably because people have 10 fingers

Early computers were designed around the decimal numbering system. This approach made the creation of computer logic capabilities unnecessarily complex and did not make efficient use of resources. (For example, 10 vacuum tubes were needed to represent one decimal digit.)



To deal with the basic electronic states of on and off, Von Neumann suggested using the binary numbering system

ON or OFF

1 0

Bitwise Operators

AND	OR	XOR	Inverse (Not)	Left shift	Right shift
&	|	^	~	<<	>>

Truth Table

if both are 1 then 1 else 0

1 if either is 1 else 0

Addition without carry

Same same, puppy shame

a	b	a & b	a | b	a ^ b	~a	~b
0	0	0	0	0	1	1
0	1	0	1	1	1	0
1	0	0	1	1	0	1
1	1	1	1	0	0	0

```
  0      0      1      1
+ 0    + 1    + 0    + 1
---    ---    ---    ---
  0      1      1     10
```

Break till 10 : 12 PM

Basic Problems on Bitwise Operators

a = 29, b = 19

a : 1 1 1 0 1

b : 1 0 0 1 1

a & b : 1 0 0 0 1

a | b : 1 1 1 1 1

a ^ b : 0 1 1 1 0

a = 13, b = 10

a : 1 1 0 1

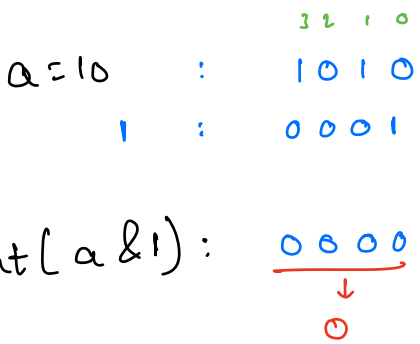
b : 1 0 1 0

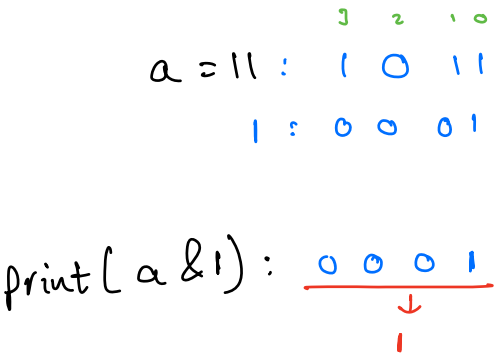
Quiz 6 a & b : 1 0 0 0 ⇒ 8

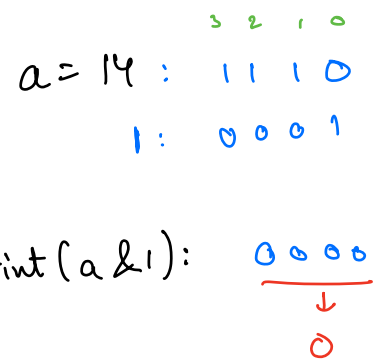
Quiz 7 a | b : 1 1 1 1 ⇒ 15

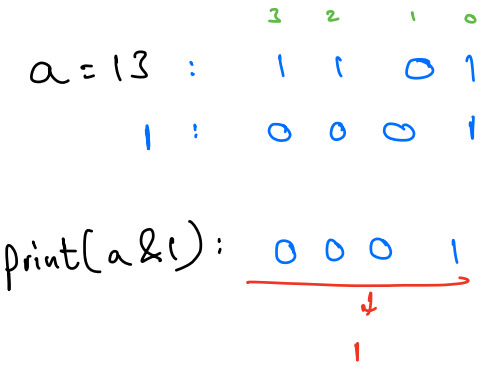
Quiz 8 a ^ b : 0 1 1 1 ⇒ 7

Properties of Bitwise Operators









Observations

- ✓ 10 & 1 → 0
- ✓ 11 & 1 → 1
- ✓ 14 & 1 → 0
- ✓ 13 & 1 → 1

Even → 0

odd → 1

% 2 == 0

Faster

```
if ( a & 1 == 1 )
    a is odd
else
    a is even
```

Few more properties

1) $a \,\&\, 0 = 0$

```
a: 1011
& 0: 0000
-----------
   0000
```

2) $a \,\&\, a = a$

```
a: 1011
& a: 1011
-----------
   1011
```

3) $a \mid 0 = a$

```
a: 1011
| 0: 0000
-----------
   1011
```

4) $a \mid a = a$

```
 1011
or 1011
-----------
 1011
```

5) $a \wedge 0 = a$

Important

```
a: 1011
^ 0: 0000
-----------
  1011
```

6) $a \wedge a = 0$

Important

```
a: 1011
^ a: 1011
-----------
  0000
```

7) $a \wedge 1$ → a is even → a+1 ; → a is odd → a-1

```
a: 11   a: 1011
        1: 0001
        ---------
10 ← 1010
```

```
a=10
a: 1010
1: 0001
---------
11 ← 1011
```

8) $a \mid 1$ → TODO

Just a bit more ...

$a + b = b + a$

$a * b = b * a$

$$a \,\&\, b = b \,\&\, a$$
$$a \mid b = b \mid a$$
$$a \wedge b = b \wedge a$$

} Commutative Property

$$a \,\&\, b \,\&\, c = c \,\&\, b \,\&\, a$$

$a \,\&\, b \,\&\, c$

↓

$a \,\&\, (b \,\&\, c)$

↓

$a \,\&\, (c \,\&\, b) \rightarrow (c \,\&\, b) \,\&\, a$

$$a \mid b \mid c = c \mid b \mid a$$
$$a \wedge b \wedge c = c \wedge b \wedge a$$

} Associative Property

Q. What is the value of

$$a \wedge b \wedge a \wedge d \wedge b$$

$$= a \wedge a \wedge b \wedge b \wedge d$$

$$= 0 \wedge 0 \wedge d$$

$$= 0 \wedge d$$

$$= d$$

Q. What is the value of

$$c \wedge f \wedge a \wedge f \wedge c \wedge g \wedge a$$

$$= g$$

$a \wedge a = 0$

$0 \wedge a = a$

Q1 Single Number

Uber amazon Microsoft Bloomberg

Given N array elements, every element repeats twice except 1. Find the unique element.

ar[5] = 6 9 6 10 9

↳ 10

ar[7] = 12 9 12 8 7 9 8

↳ 7

ar[5] = 2 9 7 2 7

↳ 9

Brute Force Idea

1) for every element, count its frequency

2) If freq == 1

ans = a[i]

```
for (i=0; i<N; i++) {
    for( j=i+1 ; j<n ; j++)
    ==
}
```

TC: $O(N^2)$

Optimised Idea

Take XOR of all elements

1) Take ans = 0 initially

2) Iterate & take

ans = ans ^ A[i]

a^0 = a

a^a = 0

arr[5] = 2 9 7 2 7

0 → 2 → 11 → 12 → 14 → 9

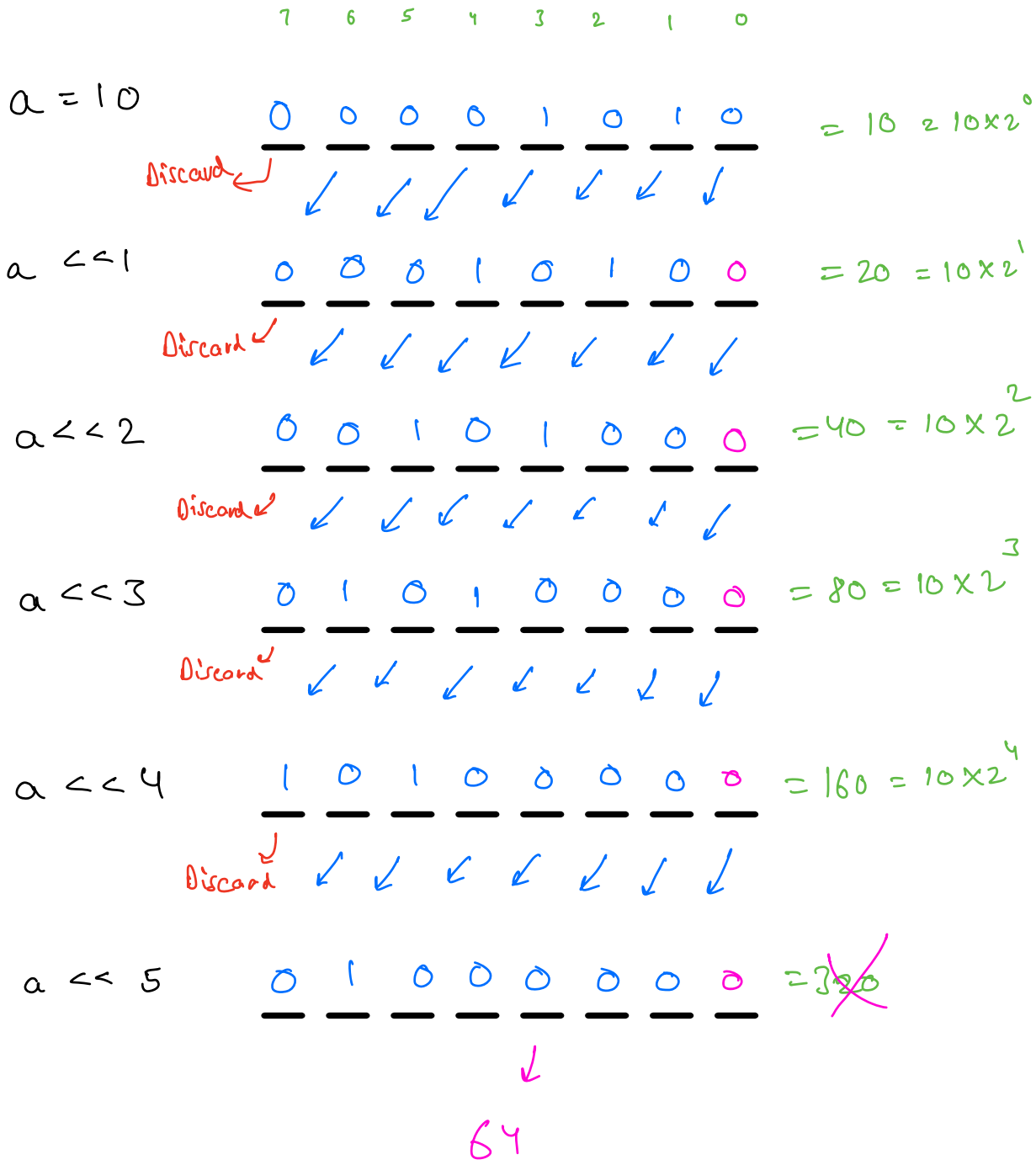
2 : 0 0 1 0
9 : 1 0 0 1
---
11 : 1 0 1 1
7 : 0 1 1 1
---
12 : 1 1 0 0
2 : 0 0 1 0
---
14 : 1 1 1 0
7 : 0 1 1 1
---
9 : 1 0 0 1

```
singleNumber(int arr[]) {
        n = arr.length
        ans = 0
        for ( i=0 ; i<n ; i++ ) {
                ans = ans ^ arr[i]
        }
        return ans
}
```

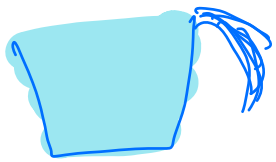
Time - O(N)
Space - O(1)

Left shift operator <<

8 bits



Overflow



Exceeding your capacity

Generalisation (No overflow)

$a << 1 = a \times 2^{1}$

$a << 2 = a \times 2^{2}$

$a << 3 = a \times 2^{3}$

$a << 4 = a \times 2^{4}$

$a << N = a \times 2^{N}$

Important Result

$1 << 1 = 1 \times 2^1$

$1 << 2 = 1 \times 2^2$

$1 << 3 = 1 \times 2^3$

$1 << N = 1 \times 2^N = 2^N$ ← O(1)

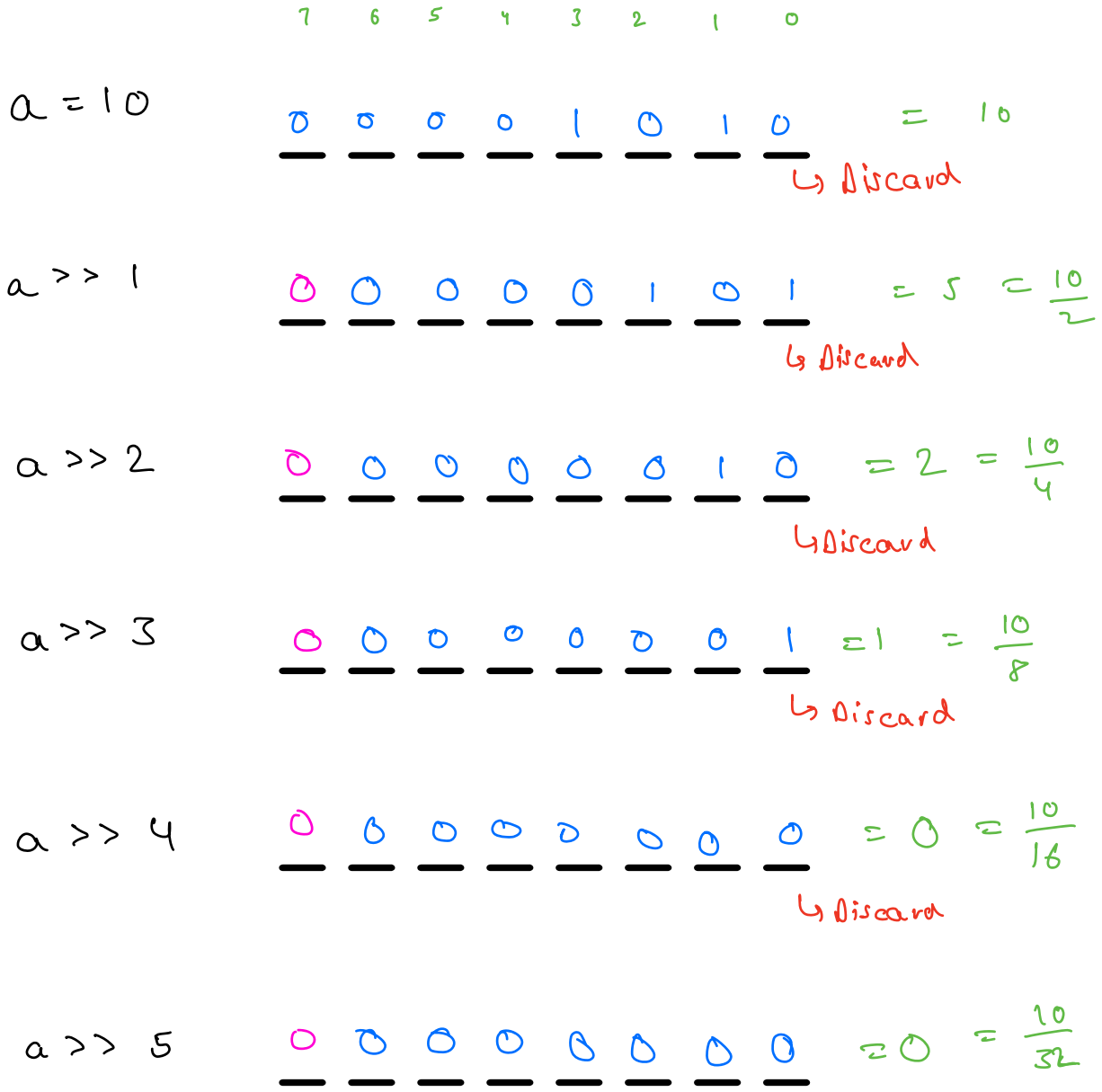
```
ans = 1
for (i=1; i<=n; i++)        ← O(N)
    ans = ans * 2
```

```
2 ** N
```

```
import math
math.pow(2, N)
```

→ $O(\log_2 N)$

Right Shift Operator >>



Generalisation

1/2

Integer Division

$a >> 1 = \frac{a}{2}$

$a >> 2 = \frac{a}{2^2}$

$a >> 3 = \frac{a}{2^3}$

$a >> 4 = \frac{a}{2^4}$

$a >> i = \frac{a}{2^i}$

Doubts

Thank You

Revise Today's Notes before the next lecture.

Binary to Decimal

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

1 0 1 0 1

$= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 16 + 0 + 4 + 0 + 1$

$= 21$

$0 \wedge 0 = 0$

$a \wedge 0 = a$

$a \wedge a = 0$

For more questions — Only for Array problems

- Leetcode — Easy
- Interviewbit — Medium

Good Night

Thank You

Monday