

Arrays: Prefix Sum 2

No HW or Assn.

AGENDA:

- Queries on pfeven
- Special index

pfeven

Q4 Given N array elements & Q queries, for each query calculate sum of all even indices in given range.

$ar[8] :$

0	1	2	3	4	5	6	7
3	4	-2	8	6	2	1	3

$pf_{even}[8] :$

3	→ 3	+	1	→ 1	+	7	→ 7	+	8	→ 8
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Queries

$$\begin{aligned}
 [2 \quad 6] : \quad a_2 + a_4 + a_6 &= -2 + 6 + 1 \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 & pf_{even}[6] - pf_{even}[1] \\
 &= 8 - 3 = 5
 \end{aligned}$$

$$\begin{aligned}
 [3 \quad 7] : \quad a_4 + a_6 &= 6 + 1 \\
 &= 7
 \end{aligned}$$

$$\begin{aligned}
 & pf_{even}[7] - pf_{even}[3-1] \\
 &= 8 - 1 = 7
 \end{aligned}$$

$$[i][j] : \begin{cases} \text{if } i > 0, & p_{\text{even}}[j] - p_{\text{even}}[i-1] \\ \text{if } i == 0, & p_{\text{even}}[j] \end{cases}$$

length = N

$\rightarrow \frac{N}{2}$

$O(N)$

$\rightarrow O(N)$

Time

Space

Simplicity

Say we delete 4th index. How will the array look like ?

ar[10] =

0	1	2	3	4	5	6	7	8	9
-1	3	2	6	4	2	7	3	2	10

Q5 Special Index

HARD



An index is said to be special index, if after deleting it

Sum of all
even index

=

Sum of all
odd index

Count how many special index are there ?

$N=6$

A = $\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 2 & 7 & 6 & -2 \end{matrix}$

Delete Index 0

A = $\begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 3 & 2 & 7 & 6 & -2 \end{matrix}$

$S_{\text{even}} = 8$ $S_{\text{odd}} = 8$

Special Index

Delete Index 1

A = $\begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 4 & 2 & 7 & 6 & -2 \end{matrix}$

$S_{\text{even}} = 9$ $S_{\text{odd}} = 8$

Delete Index 3

A = $\begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 6 & -2 \end{matrix}$

$S_{\text{even}} = 4$ $S_{\text{odd}} = 9$

Not a special index

Delete Index 4

A = $\begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 7 & -2 \end{matrix}$

$S_{\text{even}} = 4$ $S_{\text{odd}} = 10$

Not a special index

Delete index 2

A = ⁰4 ¹3 ²7 ³6 ⁴-2

$$S_{\text{even}} = 9 \quad S_{\text{odd}} = 9$$

Special index

Not a special index

Delete index 5

A = ⁰4 ¹3 ²2 ³7 ⁴6

$$S_{\text{even}} = 12 \quad S_{\text{odd}} = 10$$

Not a special index

Count = 2

Brute Force

```
int count = 0
```

```
for (i=0; i < N; i++) {  
    // Need to check if i is  
    a special index
```

```
    int [] copy = deleteIndex(arr, i)
```

```
    int se = 0, so = 0
```

```
    for (j=0; j < (N-1); j++) {
```

```
        if (j is even)  
            se += copy[j]
```

```
        else  
            so += copy[j]
```

```
    }
```

```
    if (se == so)  
        count ++
```

```
}
```

```
return count;
```

It will create &
return a new array
with ith index
deleted.

Implement
on
your
own.

Time - $O(N^2)$

Space - $O(N)$

Quiz 1

Delete index 2. Sum of all odd indices

0	1	2	3	4	5
4	1	5	3	7	10

$$\begin{aligned} S_{\text{odd}} &= 1 + 7 = 8 \\ &= a_1 + a_4 \\ &= \text{sumodd}[0, 1] + \text{sumeven}[3, 5] \end{aligned}$$

Quiz 2

Delete index = 3. Sum of all even indices.

0	1	2	3	4	5	6	7	8	9
2	3	1	4	0	-1	2	-2	10	8

$$\begin{aligned} S_{\text{even}} &= 2 + 1 + (-1) + (-2) + 8 = 8 \\ &= a_0 + a_2 + a_5 + a_7 + a_9 \\ &= \text{sumeven}[0, 2] + \text{sumodd}[4, 9] \end{aligned}$$

Generalisation

$$A = a_0 \ a_1 \ a_2 \ \dots \ a_{i-1} \ a_i \ a_{i+1} \ \dots \ a_{n-1}$$

$$S_{\text{even}} = \text{sumeven}[0 \ i-1] + \text{sumodd}[i+1 \ n-1]$$

$$S_{\text{odd}} = \text{sumodd}[0 \ i-1] + \text{sumeven}[i+1 \ n-1]$$

1) Construct $\text{pfeven}[n]$ & $\text{pfodd}[n]$

2)

$$\text{sumeven}[0 \ i-1] = \text{pfeven}[i-1]$$

$$\text{sumodd}[i+1 \ n-1] = \text{pfodd}[n-1] - \text{pfodd}[i]$$

$$\text{sumodd}[0 \ i-1] = \text{pfodd}[i-1]$$

$$\text{sumeven}[i+1 \ n-1] = \text{pfeven}[n-1] - \text{pfeven}[i]$$

Generic Formula

$$[l \quad r] = \begin{cases} \text{if } l > 0, & pf[r] - pf[l-1] \\ \text{if } l = 0, & pf[r] \end{cases}$$

Pseudocode

```
int specialIndex(int arr[]) {
```

```
    n = arr.length
```

```
    int pfodd[N], pfeven[N]
```

```
    // Fill these prefix sum arrays. - O(N) time
```

```
    Count = 0
```

```
    for (i = 0; i < N; i++) { → O(N) time
```

```
        // check if i is a special index
```

$$\begin{aligned} s_{\text{even}} &= \text{sumeven}[0 \dots i-1] + \text{sumodd}[i+1 \dots n-1] \\ &= \underbrace{\text{pfeven}[i-1]}_{\text{if } i=0: 0} + \text{pfodd}[n-1] - \text{pfodd}[i] \end{aligned}$$

$$\begin{aligned} s_{\text{odd}} &= \text{sumodd}[0 \dots i-1] + \text{sumeven}[i+1 \dots n-1] \\ &= \underbrace{\text{pfodd}[i-1]}_{\text{if } i=0: 0} + \text{pfeven}[n-1] - \text{pfeven}[i] \end{aligned}$$

```
    if (seven == sodd)  
        Count ++
```

```
    }
```

```
    return Count
```

```
}
```

$$\text{Time} = O(N)$$

$$\text{Space} = O(N)$$

Doubts

Thank
You

Revise Prefin Sum notes

before next session.

If you have solved a problem

→ Check out the editorial.

List

→ Mutable

$a = [1, 5, 9]$

$a[2] = 10$ ✓

Tuple

→ Immutable

$a = (1, 5, 9)$

$a[2] = 10$ ← Error

→ Slower in performance

→ faster to process

Typecasting

a = [1, 10, 2]

b = tuple(a) # (1, 10, 2)

c = (2, 9, 6)

d = list(c) # [2, 9, 6]

Dynamic Arrays

List in Python

$$b = 100$$

$$a = [1, 2, 3]$$

$a.append(4)$

Avg Case Time - $O(1)$

Worst - $O(N)$

Contiguous Memory.

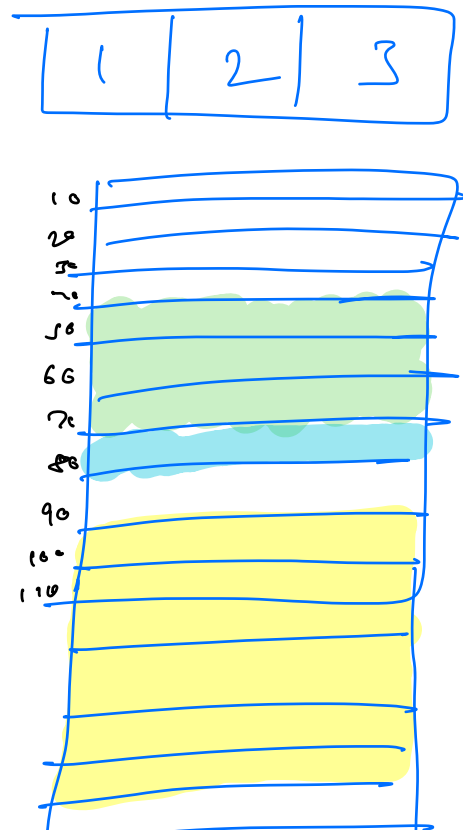
Internally,

1) Python will create

a new array

with size atleast 4.

New Size = 6



2) Copy data from

old array to $O(N)$

new array



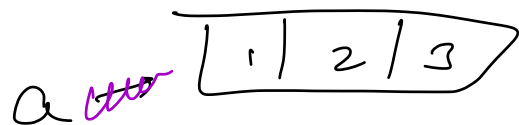
RAM

3) Then append

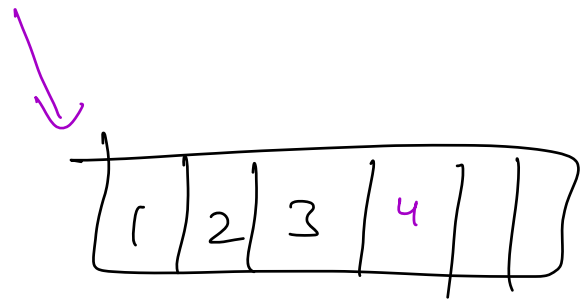
new data

4) Change the

references



5) Garbage collector
will free memory
of old array.



C++ - vector < >

Java - ArrayList

←
Scaler
Topics

Good
Night

Thank
you

wednesday