Arrays & Maths

Fun

Agenda

1. Majority element
2. Josephus problem

Q1. Majority Element

Given an array nums of size n, return the majority element.

The majority element is the element that appears more than ⌊n / 2⌋ times.

Expected SC: O(1)

floor $(\frac{n}{2})$

Example

ar[6] = 1 2 6 1 1 1

freq(1) = 4 > $\frac{6}{2}$

ans = 1

Example

ar[9] = 3 4 4 8 4 9 4 3 4

freq(4) = 5 > $\frac{9}{2}$

ans = 4

Example **Quiz 1**

arr[11] = 3 3 4 6 1 3 2 5 3 3 3

freq(3) = 6 > $\frac{11}{2}$

ans = 3

Example **Quiz 2**

arr[10] = 4 6 5 3 4 5 6 4 4 4

freq(4) = 5 > $\frac{10}{2}$ No

No majority element

New assumption:

Assume that the array will always contain a majority element.

Solutions

1) For every element, count the frequency.
and check if freq > $\frac{N}{2}$

TC: $O(N^2)$
SC: $O(1)$

2) Sort the array & get the frequency.

Sort the array: $O(n \log n)$
get the frequency: $O(N)$

TC: $O(N \log N)$
SC: $O(1)$

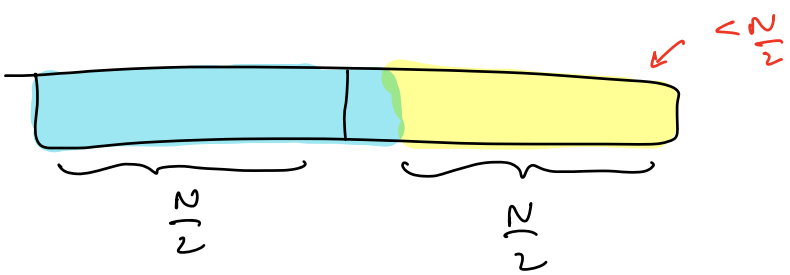
arr[9] = 3 4 4 8 4 9 4 3 4

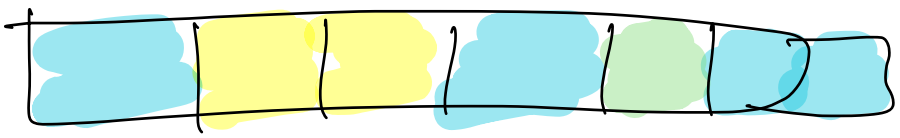
Sorted arr = 3 3 4 4 4 4 4 8 9

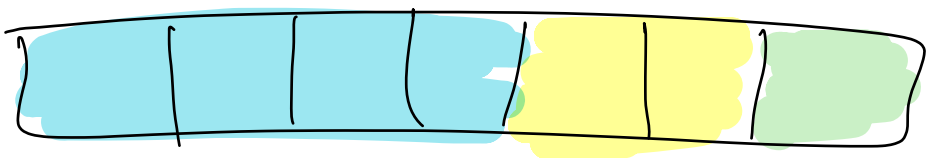
2 5 1 1

Hint **Quiz 3**

Majority element - freq > $\frac{N}{2}$





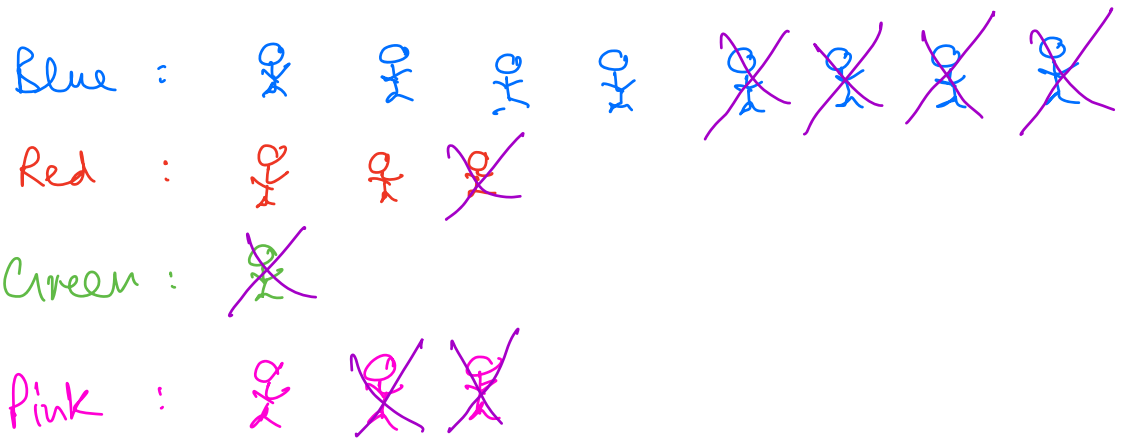


Observations

1. At max we can have one majority element.
2. The frequency of majority element will be greater than all other elements combined.

Elections

Seats = 15



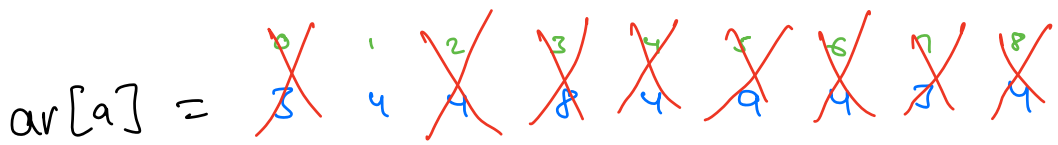
Total	Majority
15	$8 > \frac{15}{2}$
13	$7 > \frac{13}{2}$
11	$6 > \frac{11}{2}$
9	$5 > \frac{9}{2}$
7	$4 > \frac{7}{2}$

IAS Officer : Singham

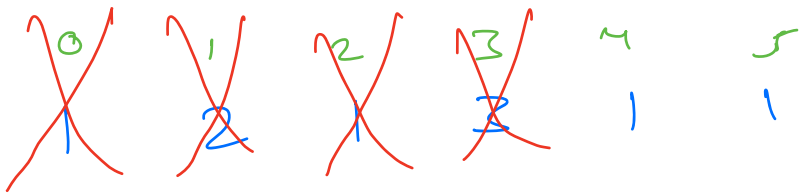
Observation 3

If we remove 2 different elements, majority still remains the same.

Dry run with example



N	freq(4) > N/2	Majority
9	5 > 9/2	4
7	4 > 7/2	4
5	3 > 5/2	4
3	2 > 3/2	4
1	1 > 1/2	4



N	freq(i) > N/2	Majority
6	4 > 6/2	1
4	3 > 4/2	1
2	2 > 2/2	1

Remove an element = Reduce its frequency

Moore's Voting Algorithm

Keep only one element (majority element candidate)
and its frequency

→ ele
→ freq

if (same)
freq ++
else
freq --

	0	1	2	3	4	5	6	7	8
	3	4	4	8	4	9	4	7	4
ele	3		4		4		4		4
freq	1	0	1	0	1	0	1	0	1

	0	1	2	3	4	5	6	7	8	9	10
	3	3	4	6	1	3	2	5	3	3	3
ele	3	3	3	_	1	_	2	_	3	3	3
freq	1	2	1	0	1	0	1	0	1	2	3

	0	1	2	3	4
	9	9	2	9	2
ele	9	9	9	9	9
freq	1	2	1	2	1

Pseudocode

```
// Initialisation
ele = A[0]
freq = 1

// Loop
for (i=1; i<N; i++) {
    if (freq == 0) {
        ele = A[i]
        freq = 1
    }
    else if (ele == A[i]) {
        freq ++
    }
    else {
        freq--
    }
}
return ele
```

TC: O(N)
SC: O(1)

← In case, majority element is not assured, check if A.count(ele) > $\frac{N}{2}$

Java

```java
int majorityElement(int[] nums) {
    int ele = nums[0];
    int freq = 1;

    for (int i = 1; i < nums.length; i++) {
        if (freq == 0) {
            ele = nums[i];
            freq = 1;
        } else if (ele != nums[i]) {
            freq--;
        } else {
            freq++;
        }
    }
    return ele;
}
```

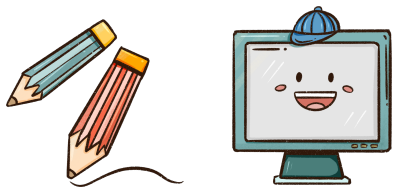
Python

```python
def majorityElement(nums):
    ele = nums[0]
    freq = 1

    for i in range(1, len(nums)):
        if freq == 0:
            ele = nums[i]
            freq = 1
        elif ele != nums[i]:
            freq -= 1
        else:
            freq += 1

    return ele
```

Time - O( )

Space - O( )

Try it out



Majority Element 2

Given an integer array of size n, find all elements that appear more than ⌊ n/3 ⌋ times.

N = 7

5 6 5 5 6 3 6

5, 6

N = 7

$> \frac{7}{3} = 2$

2 majority elements

2 frequencies

$\frac{N}{4}$ → 3 elements, 3 frequencies } 12-13 conditions

Never be asked.

Break till 10:25 PM

Q2. Given N, how many perfect squares are there in [1, N].

$N = 25 \rightarrow [1, 4, 9, 16, 25] \rightarrow 5$

$1^2, 2^2, 3^2, 4^2, 5^2$

$N = 50 \rightarrow [1, 4, 9, 16, 25, 36, 49] \rightarrow 7$

$6^2, 7^2$

$i^2 \leq N$

$\Rightarrow i \leq \sqrt{N}$

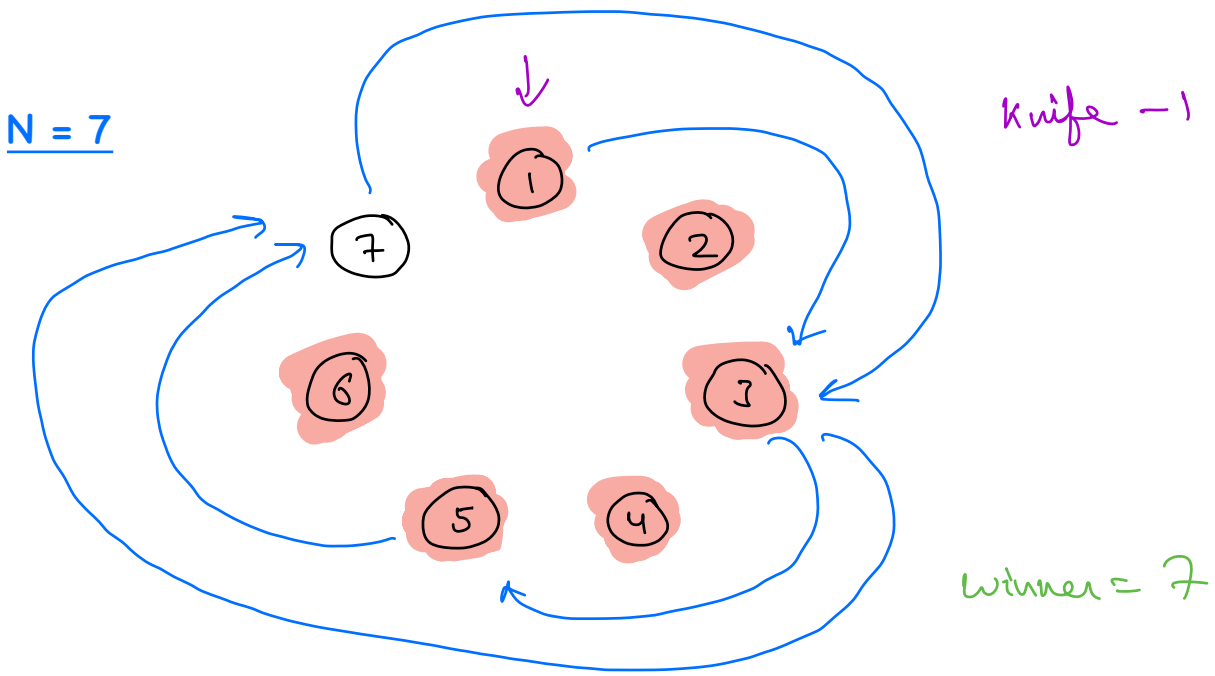
Count of perfect squares till N $= \sqrt{N}$

```
i=1; i*i<=N; i++
print(i*i)
```

Q3. Josephus Problem

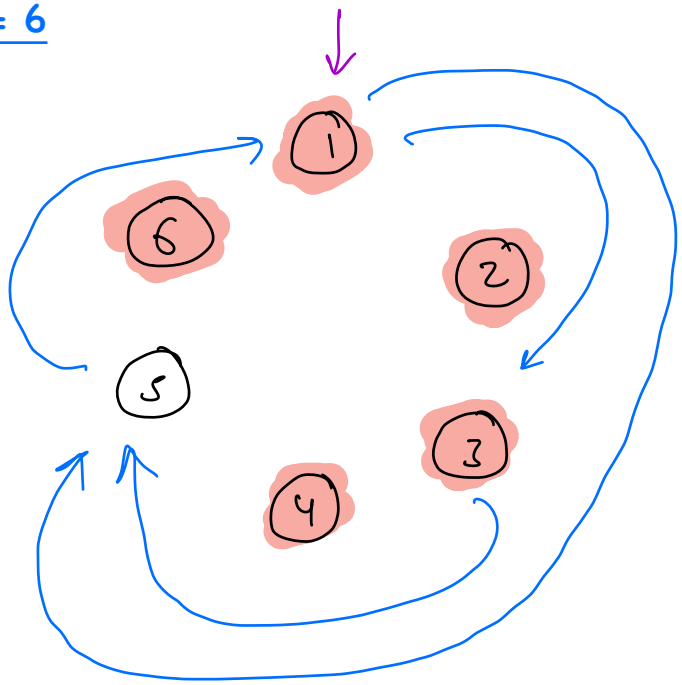
Puzzle

Initially N people are standing in a circle. Person 1 has a knife. He kills his adjacent clockwise person, and pass on the knife to next adjacent clockwise person. Repeat until only a single person stands. Who is this last remaining person ?



N = 6

Knife - 1



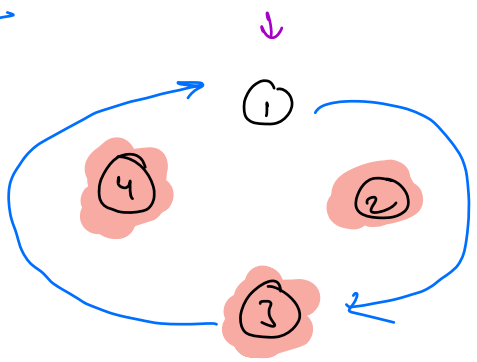
Winner = 5

N=7 → 7

N=6 → 5

if (even)
    winner = N-1
else
    winner = N

N=4



Knife - 1
Alive - 4

Winner = 1

Figure it out with examples

N = 1

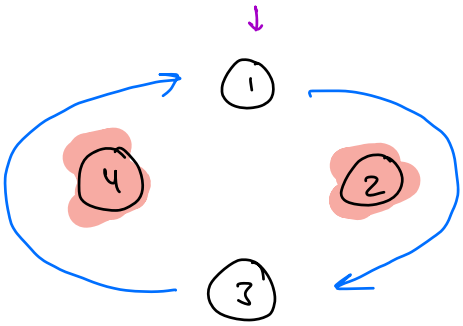
ans = 1

N = 2

knife - 1
People - 2

ans = 1

N = 4



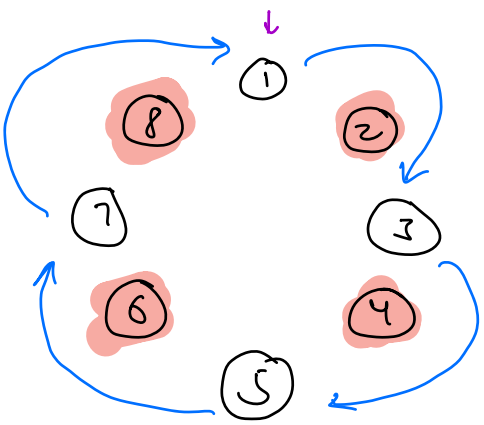
After 1 round, knife is back to 1 and there are 2 people alive. This is same as N=2.

ans = 1

knife - 1
People - 4

N = 8

Quiz 4



After 1 round knife → 1, 4 people alive.

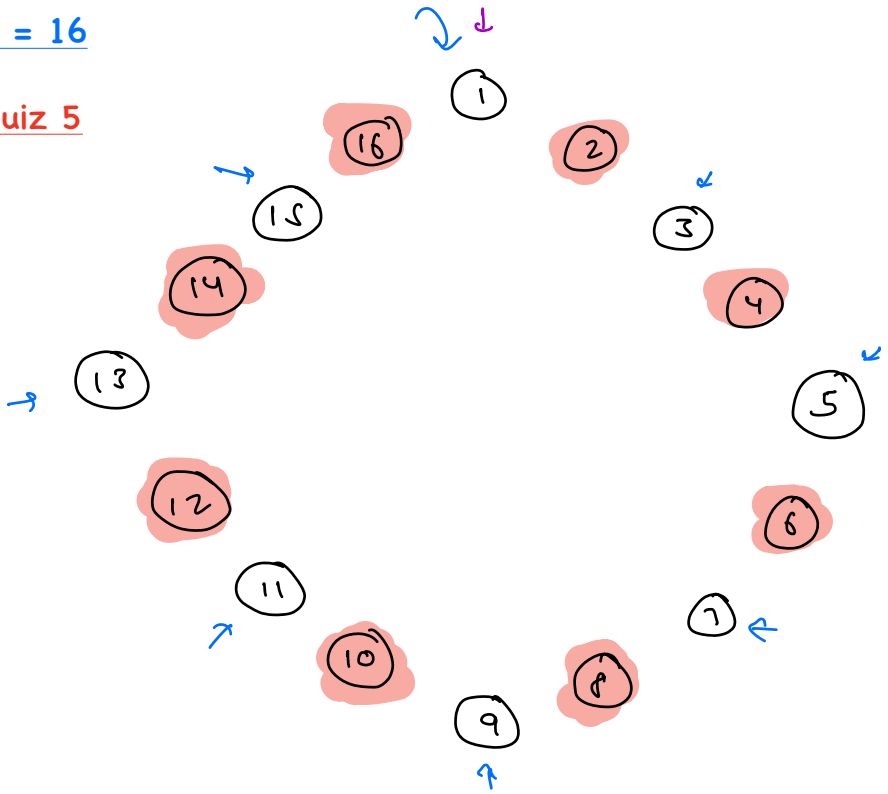
Same as case when N = 4

ans = 1

knife - 1
People - 8

N = 16

Quiz 5



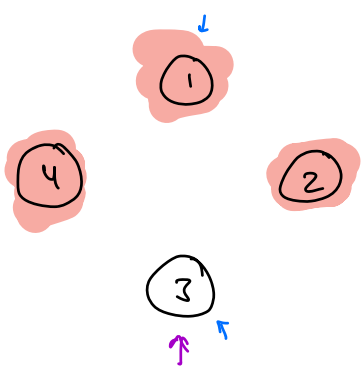
Knife - 1
People - 16

After 1 round,
Knife → 1
People = 8

Same as N=8

ans = 1

N = 4 What if, initially the knife is with 3 ?



ans = 3

N = 8 What if, initially the knife is with 4 ?

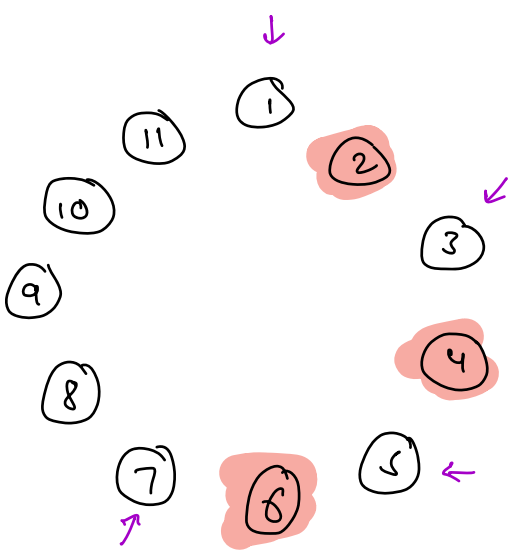
ans = 4

Observation

If N is a power of 2, and we start from x, winner is x.

N = 11

Quiz 6



situation

Knife – 7

People – 8

ans = 7

N=11 —3 kills, 3 passes→ 8

winner = 2(3) + 1

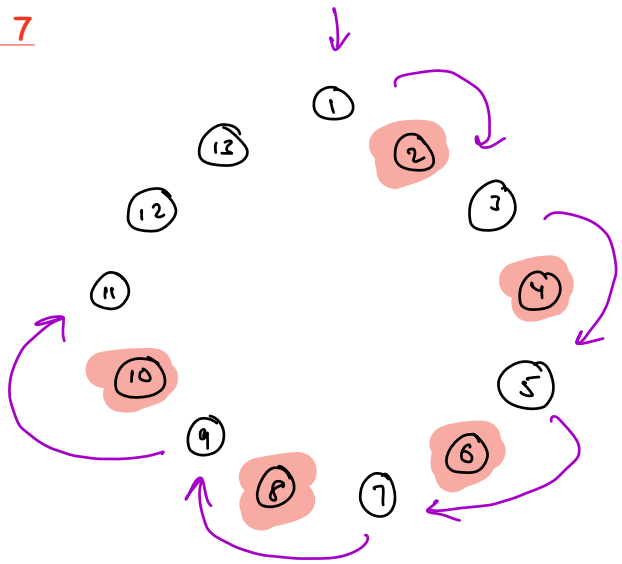
3 ← kill

2 ← size of each pass

1 ← starting point

N = 13

Quiz 7



Situation

Knife – 11

People – 8

ans = 11

$2(5)+1 = 11$

N=13 —(5 kills, 5 passes)→ 8

N = 9

N=9 —(1 kill, 1 pass)→ 8

ans = 3

$2(1)+1 = 3$

N = 100

$$100 \xrightarrow[36 \text{ passes}]{36 \text{ kill}} 64$$

$$2(36) + 1 = 73$$

N = 16

$$16 \xrightarrow[0 \text{ passes}]{0 \text{ kills}} 16$$

$$2(0) + 1 = 1$$

Pseudocode

Hint: Focus on bits

N people ← Input

TODO: $O(\log_2 N)$

p = Nearest power of 2 <= N

kills = N - p

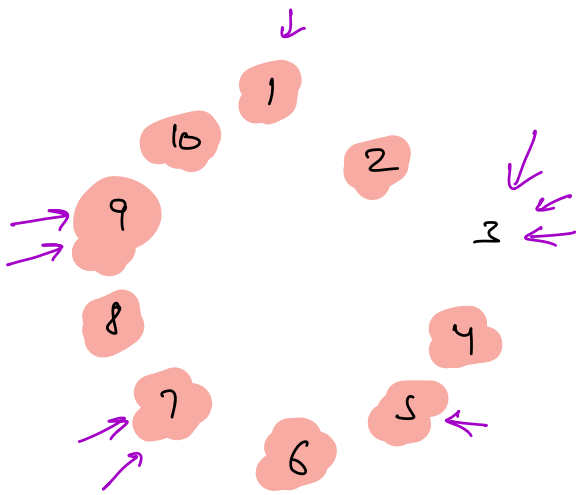
ans = 2 (kills) + 1

TC: $O(\log_2 N)$
SC: $O(1)$

Doubts

Thank
You

N=10 → Start from 9



10 —(2 kills, 2 passes)→ 8

%. N

(2(2) + 9) %. 10

= (4 + 9) %. 10

= (13) %. 10

Generic formula

N people,

Starting point = 8

p = Nearest power of 2 $\leq N$

kills = $N - p$

ans = $(2(\text{kills}) + 8) \% N$

Good Night

Thank You

Wednesday