

Todays Content:

→ Searching Basics

→ Why mid at half?

Problems:

- a) Search in sorted arr[]
- b) finding floor in a sorted arr[]
- c) finding 1st occurrence in sorted arr[]
- d) finding local minima

Searching Story:

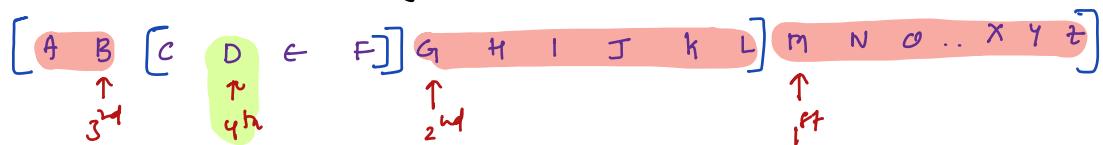
Bro/Sps → Police → {
 → photo / details (What to search) : Target
 → last location (Where to search) : Search Space

Example:

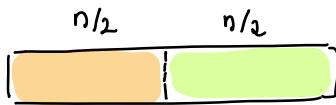
Target	Search Space
Word →	{ Dict / Books / Newspaper }
phonenumber →	{ Contracts / phone books }

Obs: If Search Space is ordered, Searching becomes easier

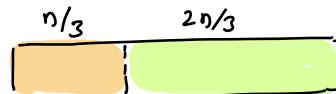
Search Dog in Dictionary:



Why land at mid?



: Always we can discard $n/2$: is better



: Worst Case we can only discard $n/3$

When to apply BS:

→ After dividing search space into 2 parts, if we can somehow discard 1 half of search space using some conditions than we can apply binary search



Note: If we cannot discard one half of search space, we cannot apply BS

Q) Given a sorted arr[N] search if k is present or not?

$$arr[10] = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \\ | 3, 6, 9, 12, 14, 19, 20, 23, 25, 27 \} \quad k=12$$

Idea1: Linear search on arr[] TC: $O(N)$ SC: $O(1)$

Idea2:

Case - I: 
 $arr[mid] == k$: return True

Case - II: 
 $arr[mid] > k$:
discard right
Search left

Case - III: 
 $arr[mid] < k$:
discard left
goto right

$\text{arr}[10] = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$ $k=12$

0	1	2	3	4	5	6	7	8	9
3	6	9	12	14	19	20	23	25	27
↑ m	↑ m	↑↑ l, h	↑ m						

$$\frac{l}{l} \quad h \quad m = \frac{(l+h)}{2}$$

0 9 4 $\text{arr}[4] < 12$: goto right: $h = m-1$

0 3 1 $\text{arr}[1] < 12$: goto right: $l = m+1$

2 3 2 $\text{arr}[2] < 12$: goto right: $l = m+1$

3 3 3 $\text{arr}[3] == 12$: return True

$\text{arr}[10] = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$ $k=21$

0	1	2	3	4	5	19	20	23	25	27
3	6	9	12	14	↑ m	↑ m	↑↑ l, h	↑ m		

$$\frac{l}{l} \quad h \quad m = \frac{(l+h)}{2}$$

0 9 4 : $\text{arr}[4] < 21$: goto right: $l = m+1$

5 9 7 : $\text{arr}[7] > 21$: goto left: $h = m-1$

5 6 5 : $\text{arr}[5] < 21$: goto right: $l = m+1$

6 6 6 : $\text{arr}[6] < 21$: goto right: $l = m+1$

7 6 : $l > h$: break, return false

↳ our search space finished

bool search(int ar[N], int k) Tc: O(log N) Sc: O(1)

$$l = 0, h = N-1$$

while(l <= h) {

$$m = (l + h)/2$$

if (arr[m] == k) {

return True

if ($\text{arr}[m] > k$) {

// goto left

$$h = m - 1$$

class NameRank

// go to right

$$l = m + 1$$

return false

TC:

; : n ele

: $n/2$ After 1 iter

: only after a letter

: n/g After 3 iter

$\frac{1}{n} : n/16$ After 4 iter

二〇一

8

$$n \rightarrow n/2 \rightarrow n/4 \rightarrow n/8 \rightarrow n/16 \rightarrow \dots \rightarrow 1$$

Log N binary search iterations

Q8) Given a sorted arr[], find floor of given num k

greatest ele r=k in arr[]

$$arr[q] = \{ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ -5 & 2 & 3 & 6 & 9 & 10 & 11 & 14 & 18 \end{matrix} \}$$

k=5 : 3

k=4 : 3

k=10 : 10

k=-7: nothing

↳ INT_MIN

refer ICIPk

k=24: 18

k=8 ans = INT_MIN Idea: Iterate in arr[], check
if arr[i] can be our
ans q, update accordingly
TC: O(N) SC: O(1)

i: arr[i] : ans	
0 : -5 : -5	
1 : 2 : 2	
2 : 3 : 3	
3 : 6 : 6 : return 6	
4 : 9 > 8 : 9 cannot be ans, break because	
5 : 10 > 8 : everything in right also cannot be ans	

Idea2:

Case-I



arr[mid] == k return k

Case-II

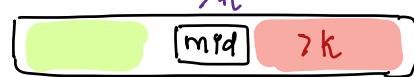


arr[mid] < k :

ans = arr[mid]

goto right

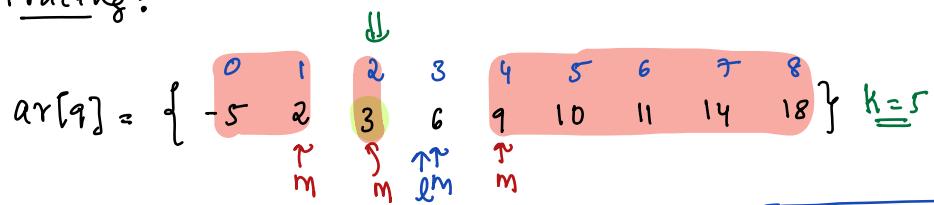
Case-III



arr[mid] > k :

goto left

Tracing:



ans = INT_MIN

<u>l</u>	<u>h</u>	<u>m</u>	
0	8	4	$ar[4] > k$: goto left, $h = m-1$
0	3	1	$ar[1] < k$: goto right, $l = m+1$ ans = $ar[1] = 2$
2	3	2	$ar[2] < k$: goto right, $l = m+1$ ans = $ar[2] = 3$
3	3	3	$ar[3] > k$: goto left, $h = m-1$
3	2		l > h break : return ans = 3

int floor(int arr[], int k) { Tc: O(log N) Sc: O(1)

$l = 0, h = N-1, ans = INT_MIN$

while ($l <= h$) {

$m = (l + h)/2$

if ($arr[m] == k$) {

return arr[m]

}

if ($arr[m] < k$) {

// update ans & goto right

ans = arr[m]

$l = m + 1$

else // $arr[m] > k$

// goto left

$h = m - 1$

return ans

TODO: ceil(k): return
smallest number
 $\geq k$ in arr[]

Q8) Given a sorted arr] find first occurrence index of given element?

$$\text{arr}[] = \{ -5, -5, -3, 0, 0, 1, 1, 5, 5, 5, 5, 5, 5, 8, 10, 10, 15, 15 \}$$

k : first occurrence

5 : 7

-5 : 0

20 : nothing
return -1

Ideas:

kkk k == k kk

Case 1



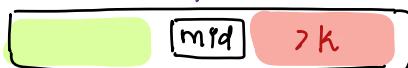
$\text{arr}[\text{mid}] == k$

$\text{ans} = \text{mid}$

goto leftside

$> k$

Case 2

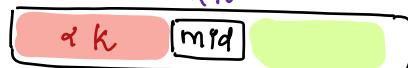


$\text{arr}[\text{mid}] > k :$

goto leftside

$< k$

Case 3



$\text{arr}[\text{mid}] < k :$

goto rightside

$arr[] = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 \}$

K=5

l h m

ans = -1 TODO

int firstOccur(int arr[], int k) { TC: O(log N) SC: O(1)

$l = 0, h = N-1, ans = -1$

while ($l <= h$) {

$m = (l+h)/2$

if ($arr[m] == k$) {

// update q, goto left

$ans = m$ // updating with index, because

$h = m-1$ that's our target

}

else if ($arr[m] > k$) {

// goto left

$h = m-1$

}

else { // $arr[m] < k$

// goto right

$l = m+1$

}

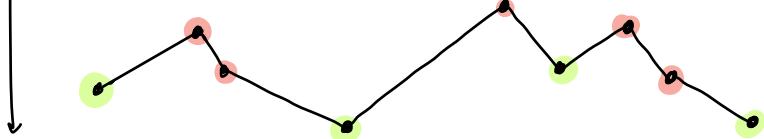
return ans;

TODO: find last occurrence
index of k

10:45 → 10:55 break

Q8) Given $\underline{N \geq 2}$ unsorted arr[N] distinct elements return any local minima

An element is said to be local minima, if its less than <
than its adjacent elements



Adjacent nodes

$$\left[\begin{array}{l} \text{arr}[i-1] > \text{arr}[i] < \text{arr}[i+1] \\ \text{arr}[i] < \text{arr}[i+1] \\ \text{arr}[n-2] > \text{arr}[n-1] \end{array} \right]$$

Eg: $\text{arr}[8] = \{ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 9 & 8 & 7 & 3 & 6 & 4 & 1 & 5 & 2 \end{matrix} \}$ return any one $\{3, 12\}$

Idea: Iterate on arr[], check if a element $\text{arr}[i]$ is
local minima or not, but comparing with adj elements

TC: $O(N)$ SC: $O(1)$

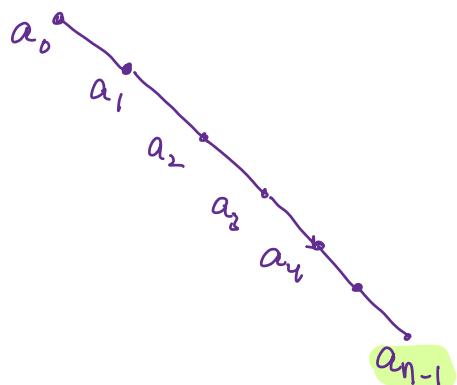
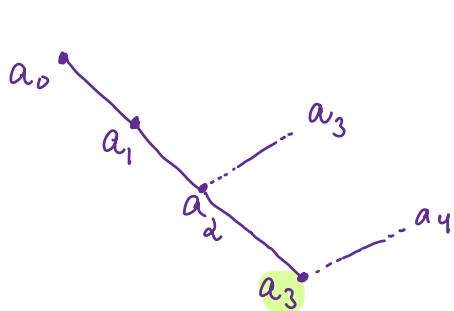
Will we always have local minima? We will have atleast 1 local minima

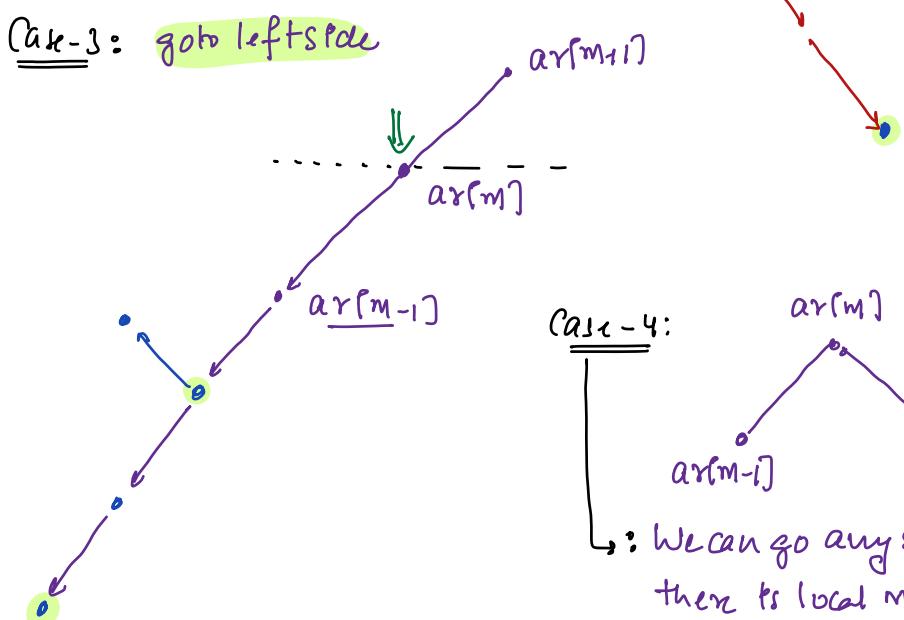
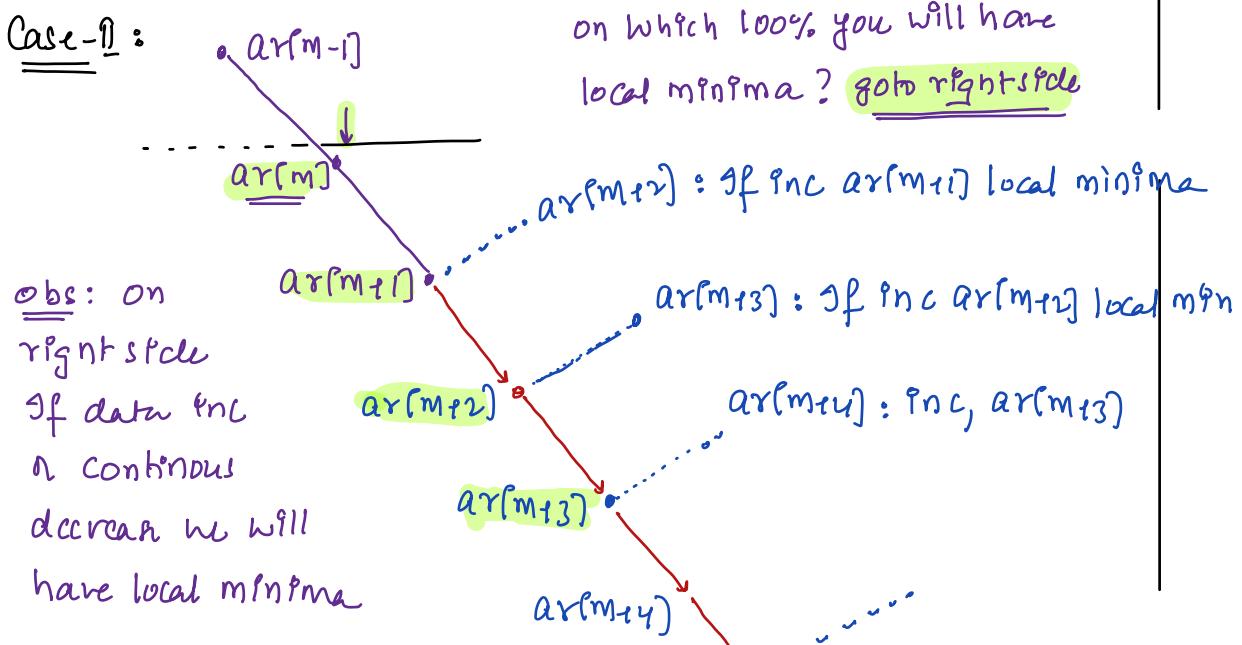
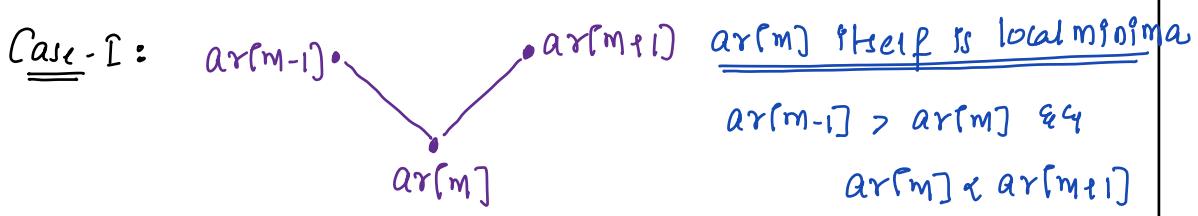
Case-1:



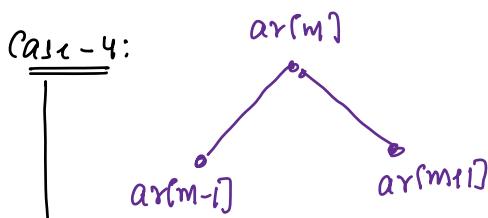
Case-2: $\left\{ \begin{array}{l} \text{obs1: Say if data increases even at one index pos} \\ \text{that itself is local minima} \end{array} \right.$

$\left. \begin{array}{l} \text{obs2: If data continuously decreases, last element is} \\ \text{local minima} \end{array} \right.$



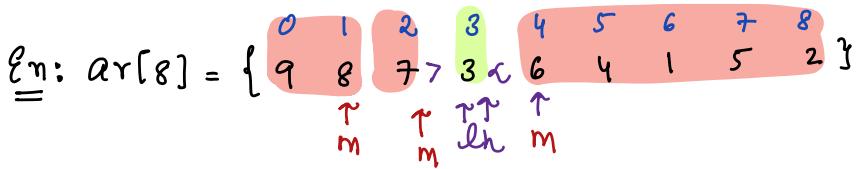


Final: Which ever side data is decreasing we goto that side



: We can go any side because there is local minima in both sides

: goto left side



l h m : date dec

0 8 4 : $ar[m-1] < ar[m]$: left : goto left : $h = m-1$

0 3 1 : $ar[m] > ar[m+1]$: right : goto right = $l = m+1$

2 3 2 : $ar[m] > ar[m+1]$: right : goto right = $l = m+1$

3 3 3 : $ar[m]$ is local minima return $ar[m]$

int localmin (int ar[N]) { T(: O(log N) SC: O(1)

↳ $N \geq 2$

if ($ar[0] < ar[1]$) { return $ar[0]$ }] // Edge Case

if ($ar[n-1] < ar[n-2]$) { return $ar[n-1]$ }]

$l = 1, h = N-2$ // update to avoid edge cases
Edge Cases:

while ($l <= h$) {

$m = (l+h)/2$

→ : $m = 0$: $ar[-1] > ar[0]$: error
→ : $m = n-1$: $ar[n-1] < ar[n]$: error

if ($ar[m-1] > ar[m]$ && $ar[m] < ar[m+1]$) {

return $ar[m]$

}

if ($ar[m-1] < ar[m]$) {] // going to solve which

// goto left

$h = m-1$

}

else { // $ar[m] > ar[m+1]$ }] // going to solve which

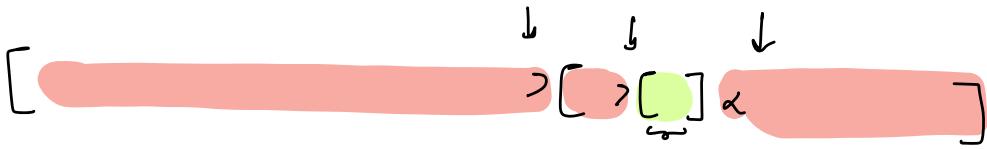
// goto right

$l = m+1$

}

decrease, because
we are 100% it
will contain local
minima

→ Mallis:



→ $\binom{31}{2}$

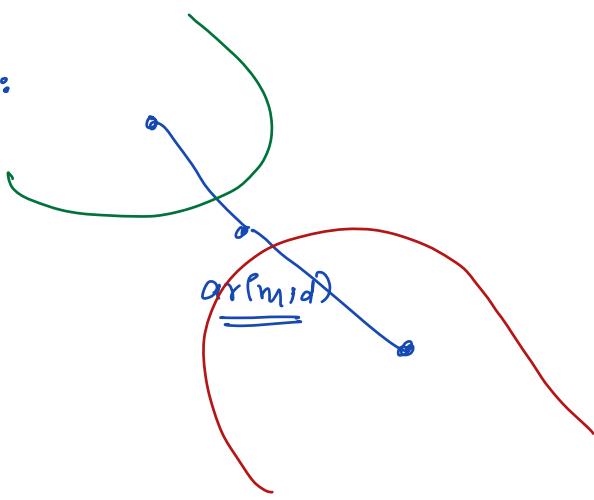
$$m = \frac{l+h}{2} \quad \text{or} \quad m = \frac{l + \frac{h-l}{2}}{2}$$

$$m = \frac{l+h}{2} \quad l=s \quad h=r \rightarrow m=4$$

$\frac{l}{2} + \frac{h}{2} \rightarrow \frac{l}{2} + \frac{r}{2} \rightarrow \binom{3}{2}$

→ any minima

→ 1st minima:



During prep

→ Before Inter:

→ previous asked question

→ review notes

→ Topic:

→ Lecture

→ Easy / medium / Above med / hard : 45 min she stop by
think / you think / understand