

Today's Content:

→ Quick sort

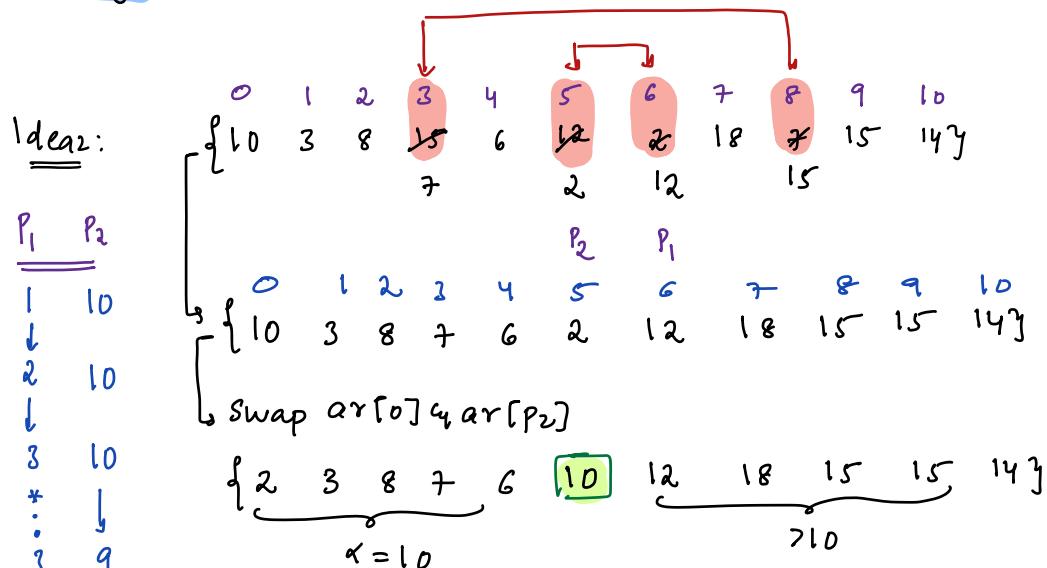
→ Countsort

Q] Given $\text{ar}[N]$ elements, re-arrange the array such that

- $\text{ar}[0]$ should go to correct sorted position } Expected SC: $O(1)$
- All elements $\leq \text{ar}[0]$ leftside of $\text{ar}[0]$
- All elements $> \text{ar}[0]$ rightside of $\text{ar}[0]$

$$\text{ar}[11] = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \\ 10, 3, 8, 15, 6, 12, 2, 18, 7, 15, 14 \}$$

Ideal:
Sort $\text{ar}[]$ { 2, 3, 6, 7, 8, **10**, 12, 14, 15, 15, 18 }
 $\text{TC: } N \log N$



3 8 : Swap P_1++ P_2--

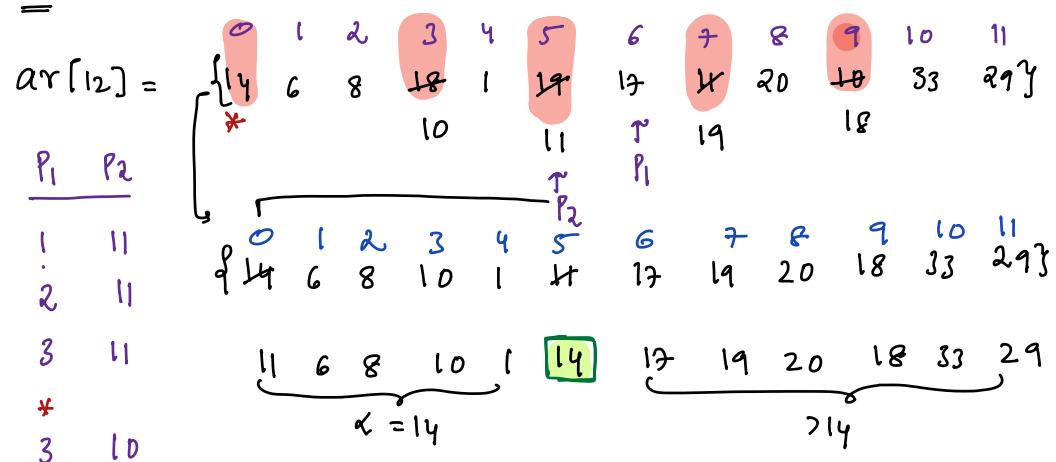
↓
4 ?
↓
5 ?
↓

5 6 : swap P_1++ , P_2--

↓
6 ?

6 5 : $P_1 > P_2$ break : swap ($\text{ar}[0]$ & $\text{ar}[P_2]$)

E_{n2}:



3 9 : Swap ($ar[3]$ & $ar[9]$) P_1++, P_2--

* *

4 8

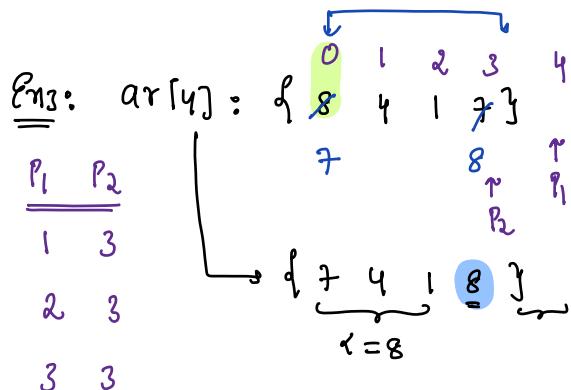
5 8

5 7 : Swap ($ar[5]$ & $ar[7]$) P_1++, P_2--

6 6

*

6 5 : $P_1 > P_2$ break : Swap ($ar[0]$ & $ar[P_2]$)



4 3 : $P_1 > P_2$ break, Swap ($ar[0]$ & $ar[P_2]$)

```
void re-arrange(int ar[N]) { Tc:O(N) Sc:O(1)
```

```
// ref = ar[0]
```

```
P1 = 1, P2 = N-1
```

```
while (P1 <= P2) {
```

```
    if (ar[P1] <= ar[0]) {
```

```
        // P1 is correct pos
```

```
        P1 = P1 + 1
```

```
    } else if (ar[P2] > ar[0]) {
```

```
        // P2 is correct pos
```

```
        P2 = P2 - 1
```

```
    } else { // Both P1 & P2 are wrong
```

```
        swap(ar[P1], ar[P2])
```

```
        P1++, P2--
```

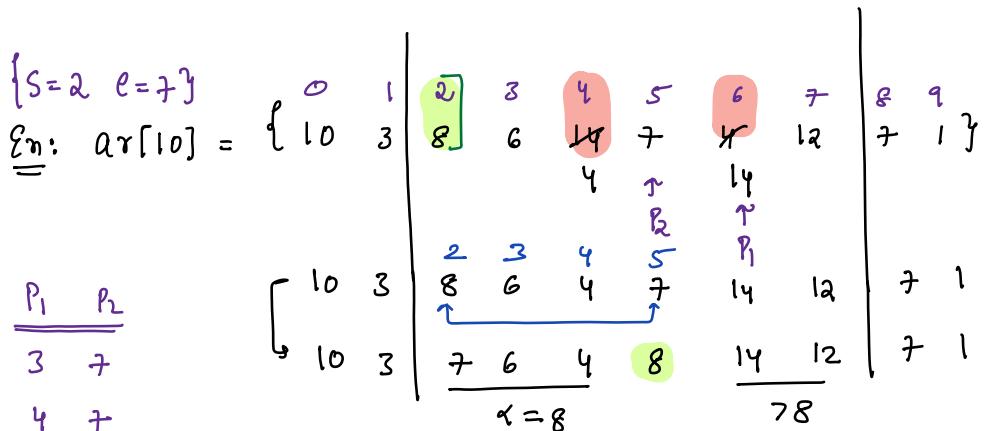
```
}
```

```
swap(ar[0] & ar[P2])
```

```
}
```

Q8)

- Given $\text{ar}[N]$ & subarray $[s, e]$ Re-arrange sub $[s - e]$ such that
- $\text{ar}[s]$ should come to correct pos in $[s, e]$
 - all elements $\leq \text{ar}[s]$ should left
 - all elements $> \text{ar}[s]$ should right
 - & finally return correct pos of $\text{ar}[s]$

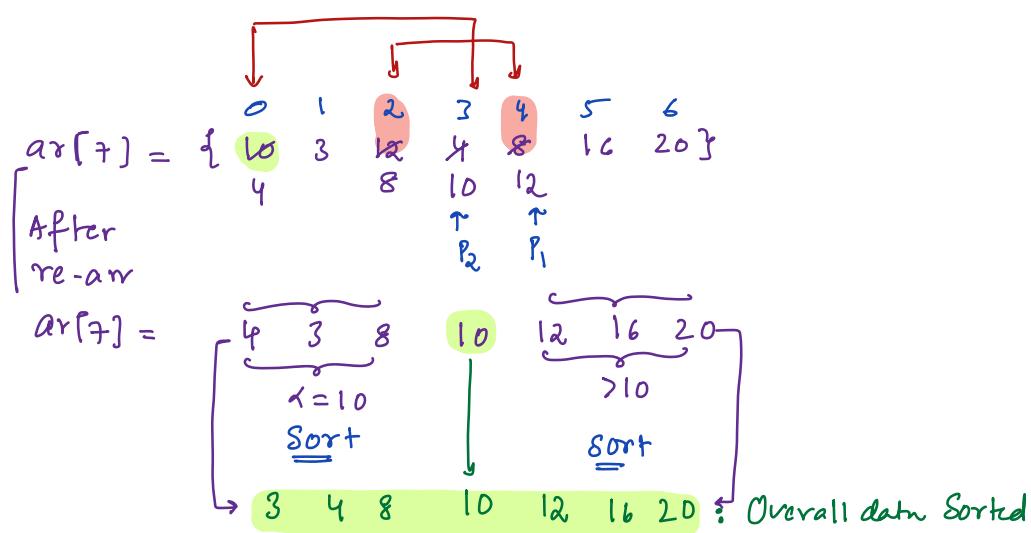


* $4, 6$: Swap $\text{ar}[4]$ & $\text{ar}[6]$ P_1+1, P_2--

$5, 5$

$6, 5$: $P_1 > P_2$ break : swap ($\text{ar}[P_2]$ & $\text{ar}[s]$)

QuickSort Idea :



`int re-arrange(int ar[N], int s, int e) { TC: O(N) SC: O(1)`

// ref = ar[s]

10:20 → 10:30pm

$P_1 = s+1, P_2 = e$

while ($P_1 <= P_2$) {

if ($ar[P_1] \leq ar[s]$) { // P_1 is correct pos

$P_1 = P_1 + 1$

else if ($ar[P_2] > ar[s]$) { // P_2 is correct pos

$P_2 = P_2 - 1$

else { // Both P_1 & P_2 are wrong

 swap(ar[P1], ar[P2])

P_1++, P_2--

}

swap(ar[s] & ar[P2])

return P_2 // correct index position of $ar[s]$

}

Sort entire subarray

Ass: Given subarray $[s - e]$ in $ar[]$ sort the subarray $[s - e]$

`void Quicksort(int ar[], int s, int e) {`

`if (e == s) { return }`

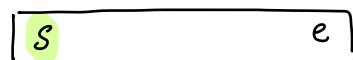
$P = \text{re-arrange}(ar, s, e)$

// correct index pos of $ar[s]$

with in Subarray $[s - e]$

`Quicksort(ar, s, P-1)`

`Quicksort(ar, P+1, e)`



// re-arrange sub $[s - e]$

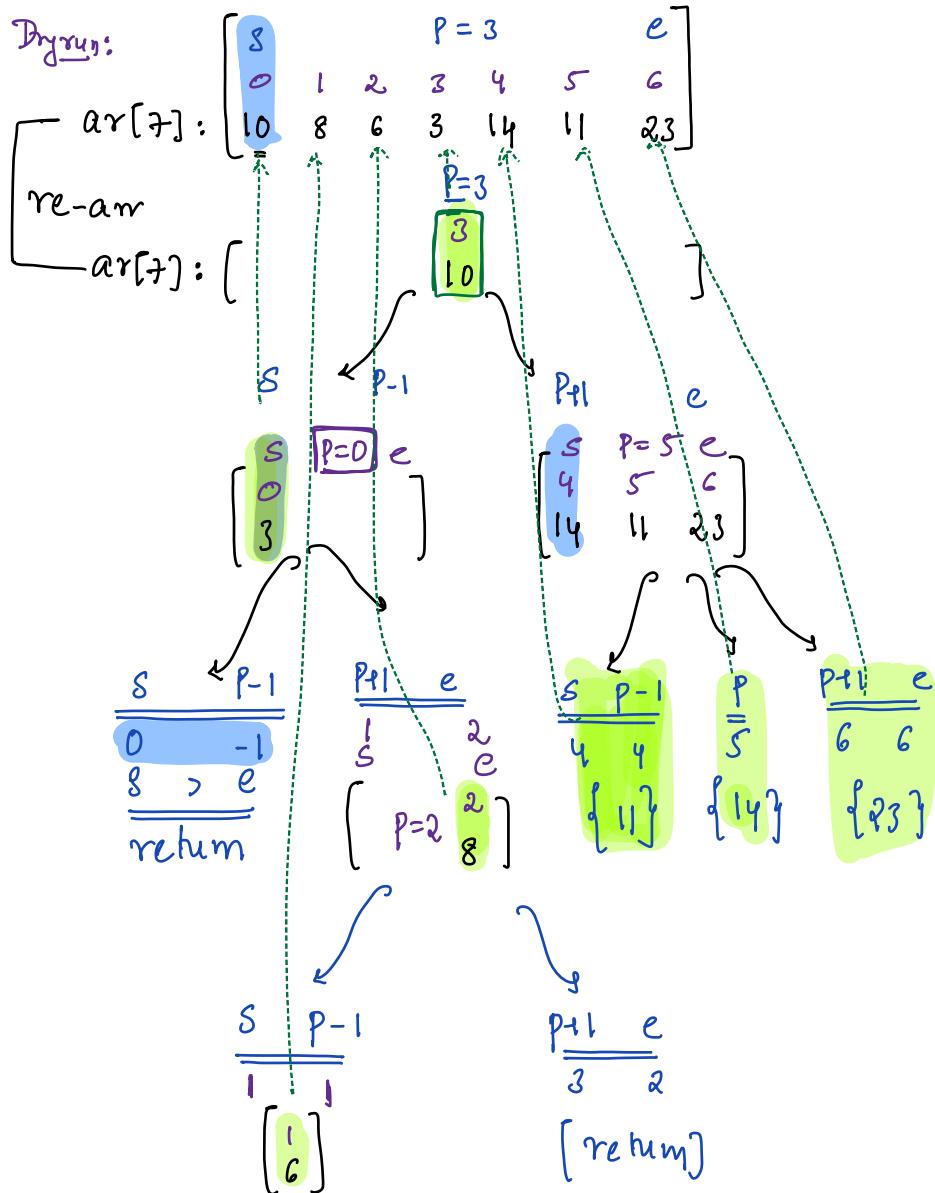
such $ar[s]$ goes to cor pos = P



$\xrightarrow{\text{Sort}} ar[P]$

$\xrightarrow{\text{Sort}} ar[P]$

Sorted $\xrightarrow{\text{Sort}} ar[P]$ & sorted $\xrightarrow{\text{Sort}}$



$\text{ar}[] = \{ 3, 6, 8, 10, 11, 14, 23 \}$ Sorted data

Time Complexity Quick Sort

```
void Quicksort(int arr[], int s, int e) {
```

if($e \leq s$) return;

$p = \text{re-arrange}(arr, s, e)$
// correct index pos of arr[s]
with in Subarray [s-e]

SC: $O(1)$:

Recursive Call Stack Size

$\approx O(\log N)$

unstable : TODO

```
Quicksort(arr, s, p-1) }
```

```
Quicksort(arr, p+1, e) }
```

// Say we sort N elements .

: assume time taken to sort N elements is $f(N)$

Best Case :

: Equal division into 2 sides

Avg Time

$\approx O(N \log N)$

$$f(N) = N + 2f(N/2)$$

$$f(N) = N \log N$$

Worst Case :

: All $N-1$ elements going
to 1 side

$$f(N) = N + f(N-1)$$

$$f(N) = O(N^2)$$

Inbuilt :

Java → Tim Sort : MergeSort + QuickSort {2 references / pivot}

C++ → introSort : QuickSort + HeapSort

python → Tim Sort : Insertion, Merge ↴ {of future}

→ pick k min : Selection

→ pick man in stable : Bubble

→ Insert in sortedT : Insertion

→ Sort entire data : Inbuilt sort ↴ $O(N \log N)$ SC: O(1)
↳ in your language of choice inbuilt method

CountSort / Freq Sort /

// Given $\text{arr}[N]$, all elements in range $[1-5]$, sort $\text{arr}[]$

$$\text{arr}[] = \{ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 1 & 5 & 5 & 2 & 1 & 3 & 2 & 2 & 1 \end{matrix} \}$$

Idea1: Sort $\text{arr}[]$ TC: $O(N \log N)$

Idea2: Store all freq in hashmap

$$\left\{ \begin{matrix} \langle 3, 3 \rangle \\ \langle 1, 3 \rangle \\ \langle 5, 2 \rangle \\ \langle 2, 2 \rangle \end{matrix} \right\}$$

Iterate from 1-5:

$$1 : \text{freq}[1] = 3$$

$$2 : \text{freq}[2] = 2$$

$$3 : \text{freq}[3] = 3$$

$$4 : \text{freq}[4] = 0$$

$$5 : \text{freq}[5] = 2$$

Output:

$$\{ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 1 & 1 & 2 & 2 & 3 & 3 & 3 & 2 & 1 \end{matrix} \}$$

// data sorted

PseudoCode:

1) Insert all $\text{arr}[] \rightarrow \text{hm}$: $O(N)$

// Iterate in given data range

$k=0$

$i=1 ; i \leq 5 ; i++ \{$

// Freq of i get from hashmap hm

$n = \text{hm}[i] // \text{freq of } i$

// If i is not there, assign $n=0$

$j=0 ; j < n ; j++ \} k$

$\text{arr}[k] = i , k++$

3

SCS = Data in range $[s \rightarrow e]$? $TC: O(N + N + e - s + 1) \Rightarrow$

i) Insert all $arr[] \rightarrow hm: O(N)$

// Iterate in given data range

$k=0$

$i=s; j=c; i++ \} \rightarrow TC: N + C - S + 1$

// Freq of i get from hashmap hm

$n = hm[i] // Freq of i$

// If i is not there, assign n=0

$j=0; j < n; j++ \}$

$arr[k] = i, k++$

3

Ex: $arr[10]$ all elements in range $[1 \rightarrow 100]$

$arr[10] = \{ 2 2 3 1 1 4 1 3 4 100 \}$

// Insert in hashmap

$hm: \{ 2: 2 3: 2 1: 3 4: 2 100: 1 \}$

i	j: freq of i times	Total iterations	
1	$freq[1] = 3$	3	sum of all freq = N
2	$freq[2] = 2$	2	sum of is \approx no of elem in range
3	$freq[3] = 2$	2	$\approx [s \rightarrow e]$
4	$freq[4] = 2$	2	$\rightarrow e - s + 1$
5	$freq[5] = 0$	1	
6	$freq[6] = 0$	1	
7	$freq[7] = 0$	1	
:			
100	$freq[100] = 1$	1	

Total iterations = $N + e - s + 1$

Starting all

$arr[]$ ele

in hashmap

$SC: O(N)$

Range of data
 $// e - s + 1 = R$

$TC: O(N + R)$

// $\alpha[N]$: CountSort : $\Theta(N + R)$ // $R = \text{dataRange}$, $N = \text{no. of array elements}$

$\underline{\underline{R \approx N}}$	$\underline{\underline{R \geq N \log N}}$
$\text{MergeSort} : \underline{\underline{N \log N}}$	$\underline{\underline{N \log N}}$
$\text{CountSort} : = \underline{\underline{N + R}}$ $= \underline{\underline{N + N}}$	$\underline{\underline{N + R \geq N \log N}}$
<u><u>CountSort</u></u> $= \underline{\underline{O(N)}}$	<u><u>MergeSort</u></u>

[TODO: CountSort only works on Integers]