

Todays Content

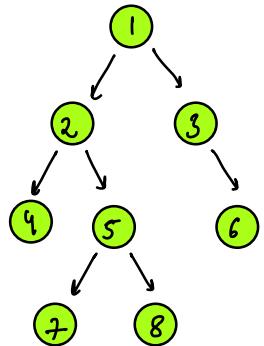
- Graph Intro
- Classification
- Input
- BFS

Introduction to Graphs

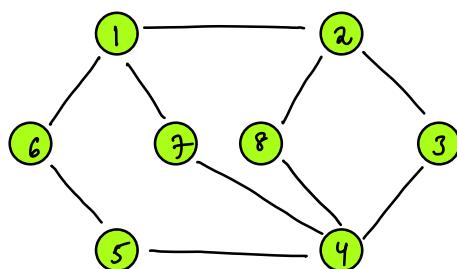
Graph: Connection of nodes w/ edges.

Ex: Tree

$$N = 8$$
$$E = 7$$



Graph: $N = 8$ edges = 10



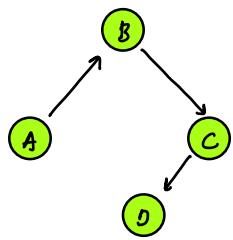
Main diff between Trees & graphs

1. Tree is a hierarchical data structure
2. In Tree: Edges = Nodes - 1

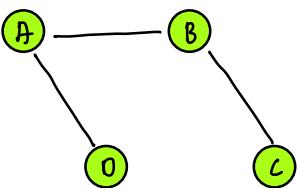
In Graph: above condition may or may not satisfy.

Classification of Graphs

directed graph



Undirected



facebook:

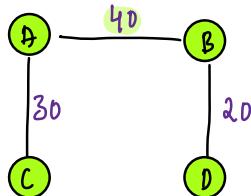


Instagram:



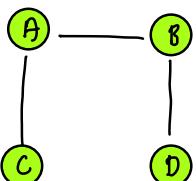
Twitter / LinkedIn

weighted



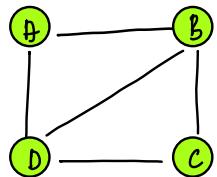
distance
time
mutual friends
Cost
Profit/loss

unweighted

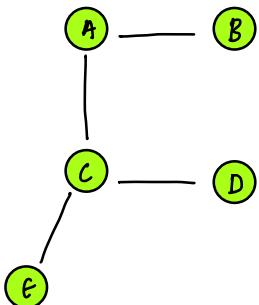


Note: Weights assigned to an edge.

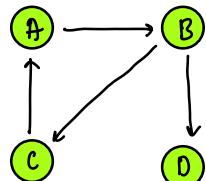
Undirected Cyclic



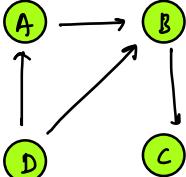
Undirected Acyclic



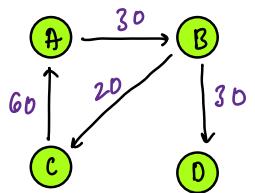
directed cyclic



Directed Acyclic graph: DAG



Note: Graph Can be combination of classification



- directed ✓
- weighted ✓
- Cyclic ✓

How is graph given as input?

: Graph is Collection of Nodes & Edges

↳ E_m: Nodes & Edges of directed Graph.

Input format:

1st line: $N \ E$

[N: no. of Nodes]

[E: no. of Edges]

followed by E line

: u & v

: Indices Edge

Between node

u & v

if weighted edge

: u v w

Edge between

u & v with

weight w

N E

G: nodes 8: edges if directed

Edges: u v u → v

u[8] v[8] w[8]

u[0]..v[0] 2 3 30

u[1]..v[1] 3 5 40

u[2]..v[2] 1 4 20

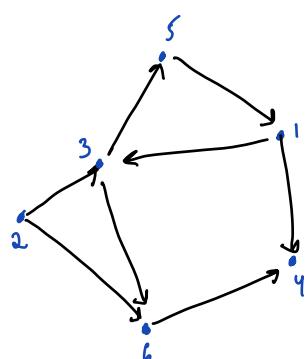
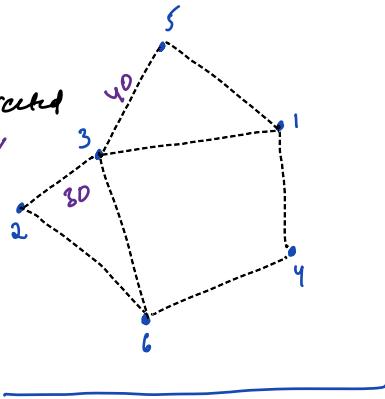
u[3]..v[3] 6 9 10

u[4]..v[4] 2 6 6

u[5]..v[5] 5 1 80

u[6]..v[6] 1 3 20

u[7]..v[7] 3 6 50

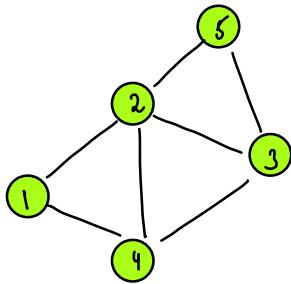


Question: Following inf is given in Question

a) Directed / Undirected

b) Weighted / Unweighted

Storing Undirected Graph:



Input:

$N \quad E$
 $5 \quad 7$

$u : v$

1 4 ✓
2 5 ✓
3 2 ✓
4 3 ✓
2 4 ✓
3 5 ✓
1 2 ✓

Way 1: Adj mat:

int mat[G][G] = 0

	0	1	2	3	4	5
0	0	1	1	1	1	1
1	1	0	1	0	2	0
2	1	1	0	1	1	1
3	0	1	1	0	1	1
4	1	1	1	0	0	0
5	0	1	1	0	0	0

// Say N Nodes & E Edges : $\text{mat}[N+1][N+1]$

	unweighted	Weighted u v w :
undirected	$u \text{ --- } v$ $\text{mat}[u][v] = 1$ $\text{mat}[v][u] = 1$	$u \xrightarrow{w} v$ $\text{mat}[u][v] = w$ $\text{mat}[v][u] = w$
directed	$u \xrightarrow{} v$ $\text{mat}[u][v] = 1$	$u \xrightarrow{w} v$ $\text{mat}[u][v] = w$

TC for adj matrix:

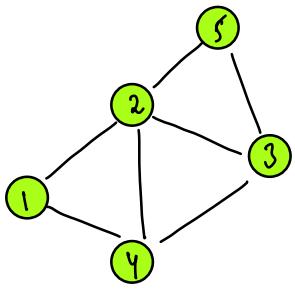
TC: $O(E)$

SC: $(N+1)(N+1) \approx O(N^2)$

Disadvantages:

: To much Space Wastage.

: To avoid Space Wastage we go to adj list



list<int>, l[3]; array of lists of 10

l[0]	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td></td></tr></table>		3 5 : 2 =
l[1]	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td></td></tr></table>		6 7 8 : 3
l[2]	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td></td></tr></table>		10 2 9 2 5 : 5

Input: Way2: Adj List:

N E
5 6
list<int> g[6]

u : v	g[u]	Empty
1 - 4 ✓	g[1]	4 → : 1
2 - 5 ✓	g[2]	5 3 4 : 3
3 - 2 ✓	g[3]	2 4 5 : 3
4 - 3 ✓	g[4]	1 3 2 : 3
2 - 4 ✓	g[5]	2 3 : 2
3 - 5		Sum of sizes of = 12 = 6 × 2

Note: array of lists: TODO

Check declaration in your language

Input: Directed Weighted Graph

list<pair<int, int>> g[4]

N E	u v w	g[0]	v w
3 5			
u v w	1 - 2 : 5	g[1]	2 5 ↗ 13, 27
1 - 3 : 2	g[2]	3, 4 ↗ 11, 67	
2 - 3 : 4	g[3]	1, 7 ↗	
2 1 6			
3 1 +			

// Given N & E edges

a) Unweighted graph: list<int> g[N+1]

b) Weighted graph: list<pair<int, int>> g[N+1]

	unweighted	weighted
undirected	$u \rightarrow v$ $g[u].add(v)$ $g[v].add(u)$	$u \xrightarrow{w} v$ $g[u].add(pair(v, w))$ $g[v].add(pair(u, w))$
directed	$g[u].add(v)$	$u \xrightarrow{w} v$ $g[u].add(pair(v, w))$

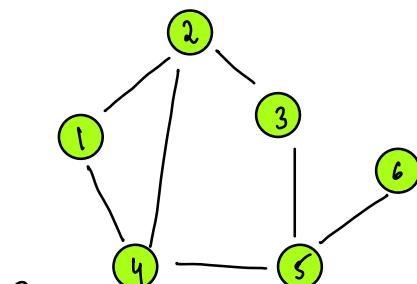
TC: $O(E)$: To add 1 element in list: $O(1)$, E edges: $O(CE)$

SC: In Undirected: For every edge: 2 values = $2E + N = O(N+E)$

In Directed: For every edge: 1 value = $E + N = O(N+E)$

Q: Given a undirected graph w/ Source Node & Dest Node
Check if node can be visited from Source Node?

Graph: $\frac{S}{1} \quad \frac{D}{6}$:



Input:

N	E	Construct Adj List	<u>S = 1</u> <u>D = 6</u>	Check path?:
6	7	list(e_{ij} , $g[v_i]$)		Note: Each node should be added only once
$u[v_j]$	$v[v_j]$	$g[0] = []$		↓ Check it using bool $ch[]$
1	2	$g[1] = [2, 4]$		
1	4	$g[2] = [1, 4, 3]$		
2	4	$g[3] = [2, 5]$		
2	3	$g[4] = [1, 2, 5]$		Ideal: bool $ch[v_j] = [F, F, F, T, F, F, F]$
3	5	$g[5] = [3, 6, 4]$		$ch[6] = T$: Visited.
5	6	$g[6] = [5]$		Datastruct: Queue
4	5			

Idea: BFS : Breath First Search?

1. Delete front ele from Queue: u
2. Iterate on nodes connected to u ?
 1. Iterate on $g[u]$ to get nodes connected to u
 2. Add a particular node in Queue if it's not visited, mark as visited.
3. Repeat till Queue is Empty
4. Check visited of destination is True / False

Param: Nodes N, Edges E, Edges: $u[E] \leftrightarrow v[E]$ S D
 bool bfs (int N, int E, int u[E], int v[E], int S, int D) {

Step1: Created adj list TC: $O(E)$ SC: $O(N+E)$

```

list<int> g[N+1]
i=0; i < E; i++ {
    // if edge: u[i] —> v[i]
    int a = u[i] int b = v[i]
    g[a].add(b)
    g[b].add(a) // if directed remove
}
    
```

TC: $O(N+E)$
 SC: $O(N+E)$

Step2: Apply BFS TC: $O(N+E)$ SC: $O(N+N)$

bool vis[N+1] = False; int l[N+1] = -1; int p[N+1] = -1

Queue q

q.insert(s) vis[s] = True l[s] = 0 p[s] = -1

while(q.size() > 0) {

int u = q.front();

q.delete() // delete front ele que

// Get nodes connected to u?

// Iteration list g[u]

i=0; i < g[u].size(); i++ {

int v = g[u][i] // u —> v

if (vis[v] == false) {

q.insert(v)

vis[v] = True l[v] = l[u]+1 p[v] = u

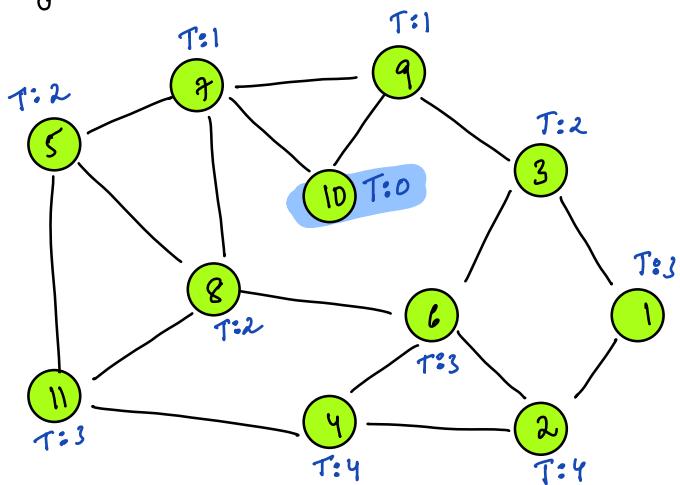
return vis[D] // l[D] // using p[] get path S → D //

Worst Case: We delete every node.

u	$g[u]$	Total Iter	(Total Adj List Elements)
1	$\text{delete}(1) : 1 + \text{iteration } g[1] :$	$1 + g[1].\text{size}$	$\overbrace{\quad}^{\text{undirected}} \quad \mathcal{O}(E)$
2	$\text{delete}(2) : 1 + \text{iteration } g[2] :$	$1 + g[2].\text{size}$	$\text{directed} : E$
3	$\text{delete}(3) : 1 + \text{iteration } g[3] :$	$1 + g[3].\text{size}$	$: \mathcal{O}(E)$
\vdots	\vdots		
n	$\text{delete}(n) : 1 + \text{iteration } g[n] :$	$1 + g[n].\text{size}$	
			$n + \mathcal{O}(E) = \mathcal{O}(N+E)$

len of Shortest Path:

Tracing $S=10 D=2$:

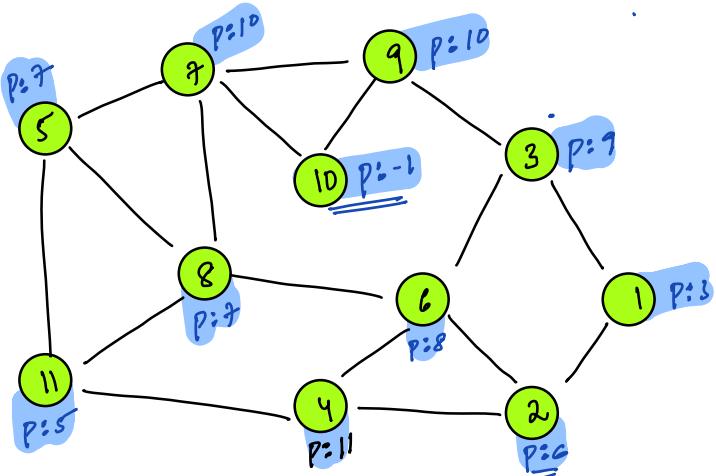


level: $0 \quad 1 \quad 2 \quad 3 \quad 4$

$10 | \alpha | \beta | \gamma | \delta | \epsilon | \zeta$

Note: Using BFS we can get length of shortest path from $S \rightarrow D$

Tracing $S=10$ $D=2$:



10 | 7 | 9 | 5 | 8 | 11 | 6 | 4 | 2 | 1 | 3 | 10 | 11

Note: For every node, store its parent node.

$\text{par}[12] = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ -1 & 3 & 6 & 9 & 11 & 7 & 8 & 10 & 7 & 10 & -1 & 5 \end{matrix}$

$S=10$ $D=2$ $\text{par}[2]$ $\text{par}[6]$ $\text{par}[8]$ $\text{par}[7]$ $\text{par}[10]$
 $\therefore D=2 \rightarrow 6 \rightarrow 8 \rightarrow 7 \rightarrow 10 \rightarrow -1$ Stop

$S=10$ $D=11$ $\text{par}[11]$ $\text{par}[5]$ $\text{par}[7]$ $\text{par}[10]$
 $D=11 \rightarrow 5 \rightarrow 7 \rightarrow 10 \rightarrow -1$ Stop.

II(a) Given S & D // get shortest path from $S \rightarrow D$

i) Fill $\text{par}[n]$ —

ii) List $\{int\}$ path

```

 $n = D$ 
while( $n \neq -1$ ) {
    path.insert(n)
    n = par[n]
}
return path;
  
```