Laplace Transform Formulae

1) L$\{1\}=\frac{1}{s}$
2) L$\{e^{at}\} = \frac{1}{(s-a)}$
3) L$\{e^{-at}\} = \frac{1}{(s+a)}$
4) L$\{\sin at\}=\frac{a}{(s^2+a^2)}$
5) L$\{\sinh at\}=\frac{a}{(s^2-a^2)}$
6) L$\{\cos at\} =\frac{s}{(s^2+a^2)}$
7) L$\{\cosh at\} = \frac{s}{(s^2-a^2)}$
8) L$\{t^n\} = \frac{n!}{s^{n+1}}$ or $\frac{\lceil n+1}{s^{n+1}}$
9) L$\{c^{at}\}=\frac{1}{(s-a\ log c)}$

Trigonometric Formulae

1) $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$
2) $\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$
3) $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$
4) $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
5) $\sin^2 x = \frac{1-cos2x}{2}$
6) $\cos^2 x =\frac{1+cos2x}{2}$
7) $\sin^3 x= \frac{1}{4}[3\sin x - \sin 3x]$
8) $\cos^3 x = \frac{1}{4}[\ 3\cos x + \cos 3x]$
9) $\sin(-x) = -\sin x$
10) $\cos(-x) = \cos x$

Properties

1) First Shifting property
   a. L$\{e^{at}$ f(t)$\}$ = F$(s-a)$
   b. L$\{e^{-at}$ f(t)$\}$ = F$(s+a)$
2) Particular value
   $\int_0^\infty e^{-st}\,dt$ (Family)
3) Multiplication by t
   L$[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}$ F(s)
4) Division by t
   L$[\frac{F(t)}{t}] = \int_s^\infty F(s)ds$
5) Laplace t of Integral
   L$[\int_0^t f(u)du] = \frac{1}{s}$ F(s)
6) Periodic Function
   $F(t+p) = F(t) \therefore p\ is\ period$
   L[F(t)] = $\frac{1}{1-e^{-ps}}\int_0^p e^{-st} f(t)$

7) Change of scale property $[f(\boldsymbol{a}t)\ (bodyguard)]$

L$[f(at)] = \frac{1}{a}\ f\left(\frac{s}{a}\right)$

8) Laplace T of derivative

$$L\left[\frac{d}{dt}\ f(t)\right] = s^1 F(s) - s^0\ f(0)$$

9) Special formula

a. $\int e^{Ax} \sin Bx\ \ dx = \frac{e^{ax}}{A^2 + B^2}[A \sin Bx - B \cos Bx]$

b. $\int e^{Ax} \cos Bx\ \ dx = \frac{e^{ax}}{A^2 + B^2}[A \cos Bx + B \sin Bx]$

For Division by t

L$\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)ds$

$\int \frac{f^`(x)}{f(x)} dx = \log f(x)$

$\int \frac{1\, dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

Timepass

1. $\log m^n = n \log m$
   $n \log m = \log m^n$
2. $\log a - \log b = \log \frac{a}{b}$
3. $-\log \frac{a}{b} = +\log \frac{b}{a}$
4. $\left[\log \frac{a}{b}\right]^\infty = 0$
5. $\tan^{-1} \infty = \frac{\pi}{2}$
6. $\frac{\pi}{2} - \tan^{-1}\frac{A}{B} = \tan^{-1}\frac{B}{A} = \cot^{-1}\frac{A}{B}$
7. $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

Inverse Laplace Transform

1. $L^{-1}\left\{\frac{1}{s}\right\} = 1$
2. $L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$
3. $L^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$
4. $L^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at$
5. $L^{-1}\left\{\frac{a}{s^2-a^2}\right\} = \sinh at$
6. $L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$
7. $L^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at$
8. $L^{-1}\left\{\frac{1}{s^n}\right\} = \frac{t^{n-1}}{(n-1)!}$ Or $\frac{t^{n-1}}{\lfloor\overline{n}}$

Properties

1. Partial fraction

2. Convolution Theorem

$L^{-1}[F_1(s)F_2(s)] = \int_0^t F_1(u)F_2(t-u)du$

3. Log and Inverse

$L^{-1}[F(s)] = \frac{-1}{t}L^{-1}[F'(s)]$

4. Division by s

$L^{-1}\left\{\frac{1}{s}\ F(s)\right\} = \int_0^t f(u)du$

5. Heaviside
$L[H(t-a)] = \frac{e^{-as}}{s}$
$L[f(t-a)H(t-a)] = e^{-as}\ F(s)$

6. Dirac Delta

$\mathrm{L}\{d(t-a)\ \} = e^{-as}$
$\mathrm{L}\{f(t-a)f(t-a)\ \} = e^{-a}\ \ F(s)$

7. Application Laplace
Assume $L(y) = y(s)y(0)y'(0)y''(0)$
$L(y') = +s^1y(s) - s^0y(0)$
$L(y'') = +s^2y(s) - s^1y(0) - s^0y'(0)$
$L(y''') = +s^3y(s) - s^2y(0) - s^1y'(0) - s^0y''(0)$
First sign alsways positive (+), remaining negative (-)