



**DALHOUSIE
UNIVERSITY**

**Network Security
(CSCI-6708)**

Assignment - 7

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Question 1.

a) $p = 7$
 $q = 11$
 $M = 6$

Finding e :

$$n = pq$$
$$n = 11 \times 7 = 77$$

$$p-1 = 6$$
$$q-1 = 10$$
$$p-1 \times q-1 = 60$$

here 1,2,3,4,5,6 are factors of 60.

$$e = 7$$

Finding d :

$$ed \bmod (p-1)(q-1) = 1$$

$$7d \bmod 60 = 1$$

$$7d = 61 \text{ NO}$$

$$7d = 121 \text{ NO}$$

$$7d = 181 \text{ NO}$$

$$7d = 241 \text{ NO}$$

$$7d = 301 \text{ YES}$$

$$d = 43$$

Public Key $K_1 = (7, 77)$

Private key $K_2 = (43, 77)$

Encrypted message:

$$C = M^e \bmod n$$

$$C = 6^7 \bmod 77$$

$$C = (6^2 \cdot 6^2 \cdot 6^2 \cdot 6) \bmod 77$$

$$C = (36 \cdot 36 \cdot 36 \cdot 6) \bmod 77$$

$$C = (1296 \cdot 216) \bmod 77$$

$$C = (64 \cdot 62) \bmod 77$$

$$C = 3968 \bmod 77$$

$$\mathbf{C = 41}$$

b) $p=11$

$q=13$

$M=9$

$n = 11 \times 13 = 143$

$n=143$

$p-1 \times q-1 = 10 \times 12 = 120$

$(p-1) \cdot (q-1) = 120$

here 1,2,3,4,5,6 are factors of 120.

$e = 7$

$ed \bmod (p-1)(q-1) = 1$

$7d \bmod 120 = 1$

$7d = 121$ NO

$7d = 241$ NO

$7d = 361$ NO

$7d = 481$ NO

$7d = 601$ NO

$7d = 721$ YES

$d=103$

Public key = (7,143)

Private key = (103,143)

Encrypted message:

$C = M^e \bmod n$

$C = 9^7 \bmod 143$

$C = 9^2 \cdot 9^2 \cdot 9^2 \cdot 9 \bmod 143$

$C = 81 \cdot 81 \cdot 81 \cdot 9 \bmod 143$

$C = 6561 \cdot 729 \bmod 143$

$C = 126 \cdot 14 \bmod 143$

$C = 1764 \bmod 143$

C = 48

c) $p=17$
 $q=31$
 $M=5$
 $n=17 \times 31=527$

$$(p-1) \cdot (q-1) = 16 \times 30 = 480$$

Here 1,2,3,4,5,6 are factor of 480
 $e=7$

$$ed \bmod (p-1)(q-1) = 1$$

$$7d \bmod 480 = 1$$

$$7d = 481 \text{ NO}$$

$$7d = 961 \text{ NO}$$

$$7d = 1441 \text{ NO}$$

$$7d = 1921 \text{ NO}$$

$$7d = 2401 \text{ YES}$$

$$d = 343$$

public key = (7,527)

private key = (343,527)

Encrypted message:

$$C = M^e \bmod n$$

$$C = 5^7 \bmod 527$$

$$C = 5^2 \cdot 5^2 \cdot 5^{2.5} \bmod 527$$

$$C = 25 \cdot 25 \cdot 5 \bmod 527$$

$$C = 625 \cdot 125 \bmod 527$$

$$C = 98.125 \bmod 527$$

$$C = 12250 \bmod 527$$

$$\mathbf{C = 129}$$

Question 2: C=10

Public key: (5,35)

$$n=pq$$

$$35=pq$$

$$e=5$$

Let's get the factors of 35: 7×5 , 35×1 .

Case 1:

$$P=7$$

$Q=5$ or Vice Versa $p=5$ and $q=7$

$$p-1 \times q-1 = 6 \times 4 = 24$$

1,2,3,4, are factors of 24.

$e=5$ which is already given (matched)

$$ed \bmod (p-1)(q-1)=1$$

$$5d \bmod 24=1$$

$$5d=25 \text{ yes}$$

$$d=5$$

$$M = C^d \bmod n$$

$$M = 10^5 \bmod 35$$

$$M = 100.100.10 \bmod 35$$

$$M = 30.30.10 \bmod 35$$

$$M = 900.10 \bmod 35$$

$$M = 25.10 \bmod 35$$

$$\mathbf{M = 5}$$

Case 2:

$p=35$

$q=1$

$n=35$

$p-1 \times q-1 = 34 \times 0 = 0$

we can't get factors of 0, so this case is invalid.

Out of 2 cases, case 2 is invalid, so Case 1 is correct, which means:

$M=5$, the plain text message is 5.

Yes, we can determine the plaintext message M by trial and error or brute force method as I did above.