Question-9.4.9

EE24BTECH11038 - MALAKALA BALA SUBRAHMANYA ARAVIND

Question: $\frac{dy}{dx} = \sin^{-1} x$

Solution:
Integrate on both sides

$$\int dy = \int \sin^{-1} x \, dx \tag{0.1}$$

Using integration by parts

$$y = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \, dx \tag{0.2}$$

$$t = \sqrt{1-x^2} \tag{0.3}$$

$$dt = \frac{-2x}{\sqrt{1-x^2}} \, dx \tag{0.4}$$

$$y = x \sin^{-1} x + \int \frac{dt}{2\sqrt{t}} \tag{0.5}$$

$$y = x \sin^{-1} x + \sqrt{t} + c \tag{0.6}$$

substituting value of t gives

$$y = x \sin^{-1} x + \sqrt{1-x^2} + c \tag{0.7}$$

Let the intital conditions be $X_0 = 0, Y_0 = 1$

$$1 = 0 + 1 + c \tag{0.8}$$

$$c = 0 \tag{0.9}$$

Final equation of the curve

$$Y = x \sin^{-1} x + \sqrt{1-x^2} \tag{0.10}$$

Now let us this computationally from the definition of $\frac{dy}{dx}$

$$Y_{n+1} = Y_n + \frac{dy}{dx}.h \tag{0.11}$$

From the differential equation

$$\frac{dy}{dx} = \frac{y_n - x_n}{y_n + x_n}.h \tag{0.12}$$

$$y_{n+1} = y_n + \left(\frac{y_n - x_n}{y_n + x_n}\right).h \tag{0.13}$$

BY taking x_1=0 and y_1=1 and h=0.01 going till x=1 by iterating through the loop and finding $y_2, y_3, y_4, \cdots$ and plotting the graph. we can verify the function we got by solving the differential equation mathematically

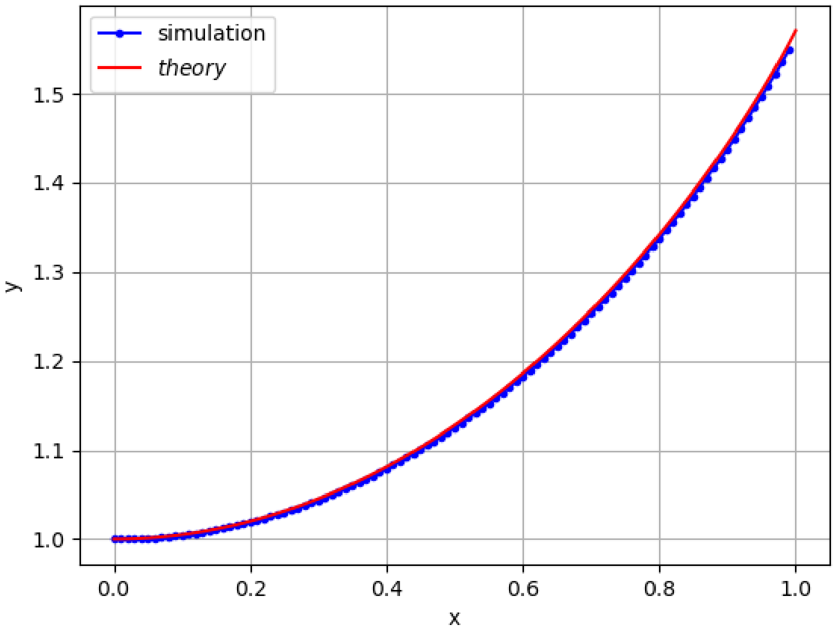

Fig. 0.1