NCERT-9.5.17.2

EE24BTECH11065 - Spoorthi yellamanchali

Question:
Find the solution of the following differential equation:

$$xy\,(dx) = \left(x^3 + y^3\right)(dy)$$

Solution :
From the questions the expression for $\frac{dy}{dx}$ obtained is:

$$\frac{dy}{dx} = \frac{xy}{x^3 + y^3} \tag{0.1}$$

The plot of the curve can be obtained by the finite difference method which is a numerical technique for solving complex differential equations by approximating derivatives with differences.
The first forward difference approximation of the derivative of $f(x)$ at x is given by:

$$\frac{dy}{dx} = \frac{f(x+h) - f(x)}{h} \tag{0.2}$$

From equation (0.2), $f\,(x+h)$ can be written as:

$$f(x+h) = h\left(\frac{dy}{dx}\right) + f(x) \tag{0.3}$$

Using this method and on assuming initial conditions (x_0, y_0) and on the curve,we can get expressions for (x_1, y_1) as

$$x_1 = x_0 + h; \tag{0.4}$$

And from the equation (0.3),we can get

$$y_1 = y_0 + h\left(\frac{dy}{dx}|_{x=x_0}\right) \tag{0.5}$$

similarly the expressions for (x_n, y_n) can be given by,

$$x_n = x_{n-1} + h; \tag{0.6}$$

$$y_n = y_{n-1} + h\left(\frac{dy}{dx}|_{x=x_{n-1},y=y_{n-1}}\right) \tag{0.7}$$

On substituting our expression of $\frac{dy}{dx}$ in equation (0.7), we get the difference equation for the curve which is,

$$y_n = y_{n-1} + h\left[\frac{x_{n-1}y_{n-1}}{{x_{n-1}}^3 + {y_{n-1}}^3}\right] \tag{0.8}$$

On assuming a value for h which is close to zero, we can get the values of (x_1, y_1).
For our plot,let

$$h = 0.01 \tag{0.9}$$
$$x_0 = 0.5 \tag{0.10}$$
$$y_0 = 2 \tag{0.11}$$

then, on substituting equations $(0.9), (0.10), (0.11)$, in the equations $(0.4), (0.5)$, we get , the values of (x_1, y_1) to be $(0.51, 2.123)$,
what we have essentially done above is, obtaining a point which is very close to the initial point along the direction of derivative at that point.
On substituting the values of h,$n = 2$ in the equations (0.6) and (0.8), we will get the values of (x_2, y_2).
Similarly on substituting $n = 3$, we get (x_3, y_3)
In the same way, by substituting different n values, we can obtain different points on the curve.
$\therefore$ we can plot the curve by the points obtained.