NCERT-9.3.11

EE24BTECH11023 - RASAGNA

Question:
Find the solution of the following differential equation, Given that $x = 0$ when $y = 1$.

$$(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$$

Solution: From the question, after simplification:

$$\frac{dy}{dx} = \frac{2x^2 + x}{(1 + x)(1 + x^2)} \tag{0.1}$$

Let,

$$\frac{2x^2 + x}{(1 + x)(1 + x^2)} = \frac{A}{1 + x} + \frac{Bx + C}{1 + x^2} \tag{0.2}$$

When $x = 0$:

$$A + C = 0 \tag{0.3}$$

When $x = 1$:

$$A + B + C = \frac{3}{2} \tag{0.4}$$

When $x = \frac{-1}{2}$:

$$5A - B + 2C = 0 \tag{0.5}$$

Solving these equations,

$$A = \frac{1}{2}, \quad B = \frac{3}{2}, \quad C = -\frac{1}{2} \tag{0.6}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2(1 + x)} + \frac{3x - 1}{2(1 + x^2)} \tag{0.7}$$

Integrating both sides:

$$y = \int \left(\frac{1}{2(1 + x)} + \frac{3x}{2(1 + x^2)} - \frac{1}{2(1 + x^2)} \right) dx \tag{0.8}$$

The solution is,

$$y = \frac{1}{2}\ln(1 + x) + \frac{3}{4}\ln(1 + x^2) - \frac{1}{2}\tan^{-1}(x) + C \tag{0.9}$$

Given $x = 0$ when $y = 1$, solve for C;

$$C = 1 - \left(\frac{1}{2}\ln(1+0) + \frac{3}{4}\ln(1+0^2) - \frac{1}{2}\tan^{-1}(0)\right) \tag{0.10}$$

$$\therefore C = 1 \tag{0.11}$$

Thus, the final required equation is,

$$y = \frac{1}{2}\ln(1+x) + \frac{3}{4}\ln(1+x^2) - \frac{1}{2}\tan^{-1}(x) + 1 \tag{0.12}$$

Logic for writing the code(method of finite difference)
The slope of a tangent at a point (x_0, y_0) on the curve is:

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} \tag{0.13}$$

By moving infinitesimally small distance(h) along the tangent , we get another point (x_1, y_1) .the value of (x_1, y_1)

$$x_1 = x_0 + h \tag{0.14}$$

$$y_1 = y_0 + \frac{dy}{dx}h \tag{0.15}$$

Here,

$$\frac{dy}{dx} = \frac{2{x_0}^2 + x_0}{(1+x_0)(1+{x_0}^2)}. \tag{0.16}$$

On substituting the value of $\frac{dy}{dx}$ in equation (0.15) we get,

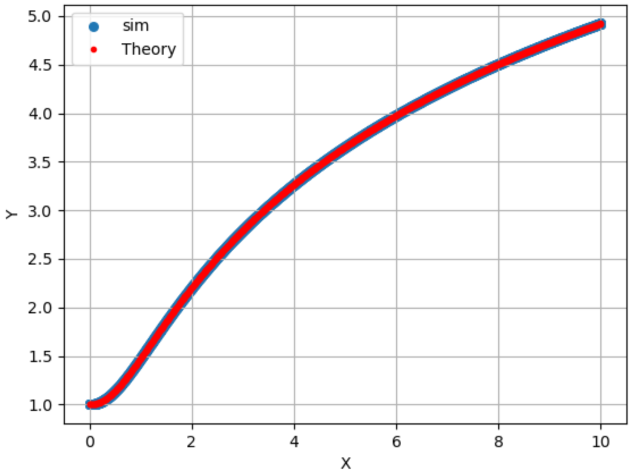
$$y_1 = y_0 + \frac{2{x_0}^2 + x_0}{(1+x_0)(1+{x_0}^2)}h \tag{0.17}$$

Similarly we can obtain n number of points where

$$x_n = x_{n-1} + h \tag{0.18}$$

$$y_n = y_{n-1} + \frac{2{x_{n-1}}^2 + x_{n-1}}{(1+x_{n-1})(1+{x_{n-1}}^2)}h \tag{0.19}$$

Together these points form the curve representing one of the general solutions of the given Differential Equation. The plot is generated by choosing a known point (x_0, y_0) which satisfies the equation.The value of h is taken to be very small.We generate a large number of points and then plot them.



sim
Theory
Y
X