9-5-8

EE24BTECH11060 - sruthi bijili

Question:
Solve the following differential equation

$$x\frac{dy}{dx} = y - x\sin\frac{y}{x} \tag{1}$$

Theoretical solution:
given equation

$$x\frac{dy}{dx} = y - x\sin\frac{y}{x} \tag{2}$$

$$\implies \frac{dy}{dx} = \frac{y}{x} - \sin\frac{y}{x} \tag{3}$$

$$\tag{4}$$

let y=vx by doing partial differentiation with respect to x

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \tag{5}$$

substitute in given equation

$$v + x\frac{dv}{dx} = v - \sin v \tag{6}$$

$$x\frac{dv}{dx} = -\sin v \tag{7}$$

$$\frac{dv}{\sin v} = -\frac{1}{x}dx \tag{8}$$

$$\tag{9}$$

integrating on both sides

$$\int \csc v dv = -\int \frac{1}{x}dx \tag{10}$$

$$\log\tan\frac{v}{2} = -\log x + \log k \tag{11}$$

$$\log\tan\frac{v}{2} = \log\frac{k}{x} \tag{12}$$

$$\tag{13}$$

let k be any constant

$$\tan\frac{v}{2} = \frac{1}{x} \tag{14}$$

$$\tan\frac{y}{2x} = \frac{1}{x} \tag{15}$$

$$\frac{y}{2x} = tan^{-1}\frac{1}{x} \tag{16}$$

$$y = 2x\tan^{-1}\frac{1}{x} \tag{17}$$

method of finite differences

The derivative of f(x) can be written as

$$\frac{df}{dx} = \frac{f(x+h) - f(x)}{h} \tag{18}$$

$$\implies f(x+h) = f(x) + h\cdot\frac{df}{dx} \tag{19}$$

from the above question

$$\frac{dy}{dx} = \frac{y}{x} - \sin\frac{y}{x} \tag{20}$$

$$\implies y(x+h) = y(x) + h\cdot\frac{y}{x} - \sin\frac{y}{x} \tag{21}$$

for $x \in [x_0, x_n]$ divide into equal parts by difference h

Let us assume that $x_0 = 1$,$y_0 = 1.57$

Let x_1=$x_0 + h$ then

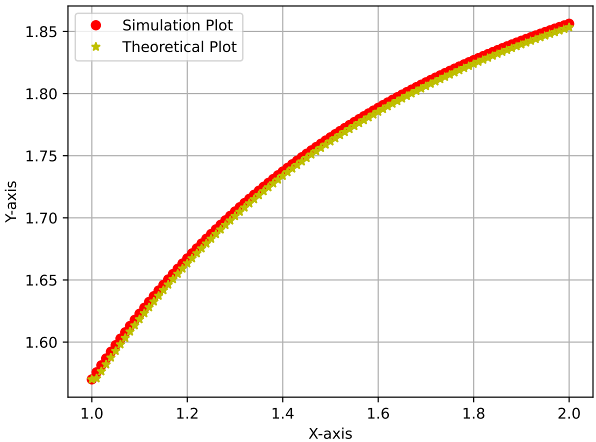
$$y_1 = y_0 + h\cdot\frac{y_0}{x_0} - \sin\frac{y_0}{x_0} \tag{22}$$

To obtain the graph repeat the process until sufficient points to plot the graph and the general equation will be

$$x_{n+1} = x_n + h \tag{23}$$

$$y_{n+1} = y_n + h\cdot\frac{y_n}{x_n} - \sin\frac{y_n}{x_n} \tag{24}$$

The curve generalised using the method of finite differences for the given question taking $x_0 = 1$,$y_0 = 1.57$, $h = 0.1$ and running iterations for 100 times


Simulation Plot
Theoretical Plot
1.85
1.80
1.75
1.70
1.65
1.60
Y-axis
1.0
1.2
1.4
1.6
1.8
2.0
X-axis