

# NCERT - 9.4.15

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## I. DIFFERENTIAL EQUATIONS

**Question:**  $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$ ;  $y = 2$  when  $x = 1$

**Solution: (Theoretical Solution)** The given differential equation can be written as

$$\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{2x^2} \quad (1)$$

To solve it, we make the substitution

$$y = xt \quad (2)$$

Differentiating equation (2) with respect to  $x$ , we get

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \quad (3)$$

Substituting the value of  $y$  and  $\frac{dy}{dx}$  in equation (1), we get

$$t + \frac{t^2}{2} = t + x \frac{dt}{dx} \quad (4)$$

$$\frac{t^2}{2} = x \frac{dt}{dx} \quad (5)$$

$$\frac{dx}{2x} = \frac{dt}{t^2} \quad (6)$$

$$\int \frac{dx}{2x} = \int \frac{dt}{t^2} \quad (7)$$

$$\frac{\ln x}{2} = -\frac{1}{t} + c \quad (8)$$

Replacing  $t$ , we have

$$\frac{\ln x}{2} = -\frac{x}{y} + c \quad (9)$$

Substituting  $x = 1$ ,  $y = 2$  to find  $c$ , we get

$$\frac{\ln 1}{2} = -\frac{1}{2} + c \quad (10)$$

$$c = \frac{1}{2} \quad (11)$$

By solving, we obtain  $y$  as

$$y = \frac{2x}{1 - \ln x} \quad (12)$$

**Solution by the method of finite differences:**

$$\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{2x^2} \quad (13)$$

Using the method of finite differences, we approximate the derivative as

$$\frac{dy}{dx} \approx \frac{y_{n+1} - y_n}{h} \quad (14)$$

Substitute equation(14) in equation(13)

$$\frac{y_{n+1} - y_n}{h} = \frac{y_n}{x_n} + \frac{y_n^2}{2x_n^2} \quad (15)$$

$$y_{n+1} - y_n = h\left(\frac{y_n}{x_n} + \frac{y_n^2}{2x_n^2}\right) \quad (16)$$

$$y_{n+1} = y_n + h\left(\frac{y_n}{x_n} + \frac{y_n^2}{2x_n^2}\right) \quad (17)$$

The initial conditions are given as:  $x_0 = 1$ ,  $y_0 = 2$ ,  $h = 0.005$ . Using the recurrence relation (17), we compute values of  $x_n$  and  $y_n$ . These values can be used to approximate the solution numerically for a given range of  $x$ .

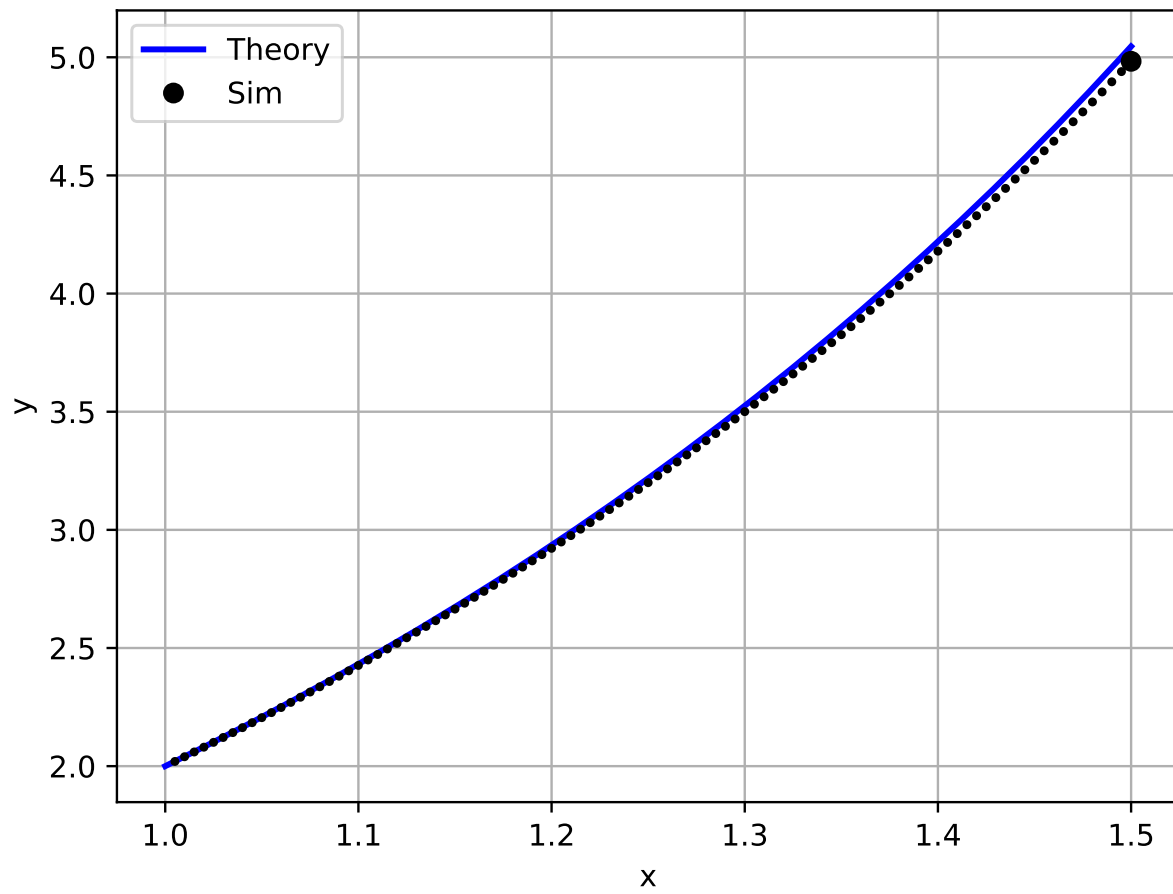


Fig. 0. Solution of given DE