

9.4.18

EE24BTECH11047 - Niketh Prakash Achanta

Question: At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point $(-4, -3)$. Find the equation of the curve given that it passes through $(-2, 1)$.

Solution

Slope of tangent = $\frac{dy}{dx}$

Based on the given information, we can form the following differential equation:

$$\frac{dy}{dx} = 2 \left(\frac{y + 3}{x + 4} \right)$$

Then, cross-multiply

$$\frac{dy}{y + 3} = 2 \frac{dx}{x + 4}$$

Integrate

$$\int \frac{dy}{y + 3} = 2 \int \frac{dx}{x + 4}$$

After integration

$$\ln(y + 3) = 2 \ln(x + 4) + k$$

Where k is an arbitrary constant of integration.

Now, rewrite $2 \ln(x + 4)$ as $\ln(x + 4)^2$ and take that term to LHS

$$\begin{aligned} \ln \frac{(y + 3)}{(x + 4)^2} &= k \\ \frac{(y + 3)}{(x + 4)^2} &= e^k \end{aligned}$$

Replace e^k with another constant "c"

$$y = c(x + 4)^2 - 3$$

Given that the curve passes through $(-2, 1)$ substitute $x = -2, y = 1$ in the current equation

Thus, $c = 1$

∴ Final Equation of the Curve: $y = (x + 4)^2 - 3$

Additionally, a numerical solution is generated using the method of finite differences:

The derivative is approximated using finite differences:

$$\frac{dy}{dx} = \frac{y(x+h) - y(x)}{h}$$

$$y(x+h) = y(x) + h \left(\frac{dy}{dx} \right)$$

h is a value close to zero, that can be chosen accordingly while creating the program. So we iterate as follows:

$$y_{n+1} = y_n + h \left(\frac{dy}{dx} \right)$$

$$x_{n+1} = x_n + h$$

$$\text{Here, } \frac{dy}{dx} = 2 \frac{(y+3)}{(x+4)}$$

We start with given point $(-2, 1)$. $x_0 = -2$ and $y_0 = 1$. We iterate for a suitable number of times in order to get a solution that is approximately equal to the theoretical solution.

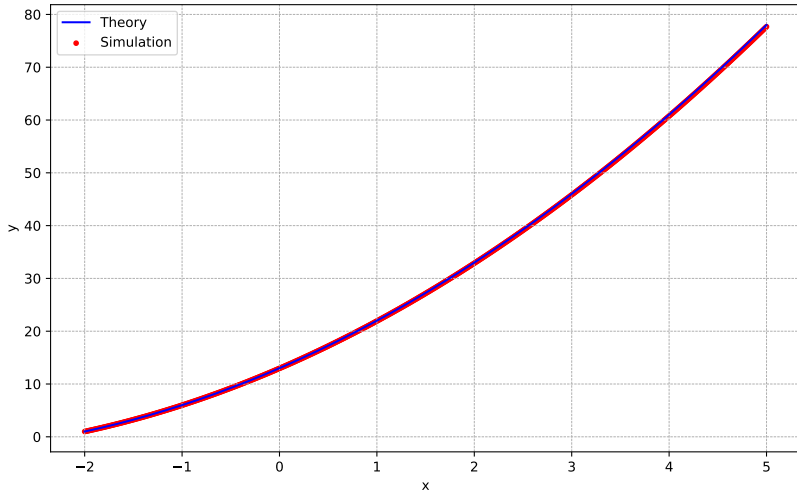


Fig. 0