

NCERT 12.8.3.1.1

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EE24BTECH11057 - Shivam Shilvant

Question: Find the area of the region bounded by the curve $y = x^2$ and lines $x = 1$, $x = 2$ and x axis.

Solution:

Theoretical logic:

1) Set up the integral:

The area under the curve can be calculated as:

$$\text{Area} = \int_a^b f(x)dx \quad (1)$$

Here:

$$f(x) = x^2, \quad x_1 = 1, \quad x_2 = 2 \quad (2)$$

Thus, the integral becomes:

$$\text{Area} = \int_1^2 x^2 dx \quad (3)$$

2) Compute the integral:

The integral of x^2 is:

$$\int x^2 dx = \frac{x^3}{3} \quad (4)$$

3) Evaluate the definite integral:

Substitute the limits of integration:

$$\text{Area} = \left[\frac{x^3}{3} \right]_1^2 \quad (5)$$

First, calculate $\frac{x^3}{3}$ at $x = 2$:

$$\frac{2^3}{3} = \frac{8}{3} \quad (6)$$

Next, calculate $\frac{x^3}{3}$ at $x = 1$:

$$\frac{1^3}{3} = \frac{1}{3} \quad (7)$$

Subtract the two results:

$$\text{Area} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3} \quad (8)$$

4) Final result:

area of the region bounded by the curve $y = x^2$ and lines $x = 1$, $x = 2$ and x axis is:

$$\frac{7}{3} \text{square units i.e. } 2.3333333333461046 \text{ square units} \quad (9)$$

Computational Logic: Using the trapezoidal rule to get the area. The trapezoidal rule is as follows.

$$\int_a^b f(x) dx \approx \sum_{k=1}^N \frac{f(x_{k+1}) + f(x_k)}{2} h \quad (10)$$

where

$$h = \frac{b-a}{N} \quad (11)$$

∴ The difference equation obtained is

$$\int_1^2 f(x) dx = \sum_{k=1}^N \frac{x_{k+1}^2 + x_k^2}{2} h \quad (12)$$

$$h = 0.00001 \quad (13)$$

$$N = 300000 \quad (14)$$

Using the code answer obtained is 2.3333333333461046

