NCERT 9.4.7

EE24BTECH11036 - Krishna Patil

Question: Solve the differential equation given below with initial conditions $x = 1$ and $y = e$.

$$y \ln y \, dx - x \, dy = 0 \tag{1}$$

Solution:

1) **Rearranging the Equation:** First, we rewrite the equation in a more convenient form:

$$y \ln y \, dx = x \, dy \tag{2}$$

Next, we divide both sides by $x \ln(y)$:

$$\frac{dy}{dx} = \frac{y \ln y}{x} \tag{3}$$

2) **Separation of Variables:** Now, we separate the variables to prepare for integration:

$$\frac{dy}{y \ln y} = \frac{dx}{x} \tag{4}$$

3) **Integration:** We now integrate both sides.

$$u = \ln y \implies du = \frac{1}{y} dy \tag{5}$$

$$\therefore \int \frac{1}{u} \, du = \int \frac{1}{x} \, dx \tag{6}$$

This leads to:

$$\ln |u| = \ln |x| + C \tag{7}$$

Substituting $u = \ln y$, we have:

$$\ln |\ln y| = \ln |x| + C \tag{8}$$

Exponentiating both sides:

$$|\ln y| = e^C \, |x| \tag{9}$$

Let $C' = e^C$, so:

$$|\ln y| = C' |x| \tag{10}$$

Now solving for $\ln y$, we get:

$$\ln y = C' x \tag{11}$$

Exponentiating again:

$$y = e^{C' x} \tag{12}$$

4) **Applying the Initial Conditions:** To determine the constant C', we apply the initial condition $x = 1$ and $y = e$:

$$e = e^{C' \cdot 1} \tag{13}$$

Thus, $C' = 1$, and the solution to the differential equation is:

$$y = e^x \tag{14}$$

5) **CODING LOGIC:** The solution for the differential equation can be graphically solved using coding by using below logic :

$$x_0 = 1 \tag{15}$$
$$y_0 = e \tag{16}$$
$$h = 0.1 \tag{17}$$
$$y_{n+1} = y_n + h \cdot \left(\frac{y_n \ln y_n}{x_n} \right) \tag{18}$$
$$x_{n+1} = x_n + h \tag{19}$$

Below is verification 5 :

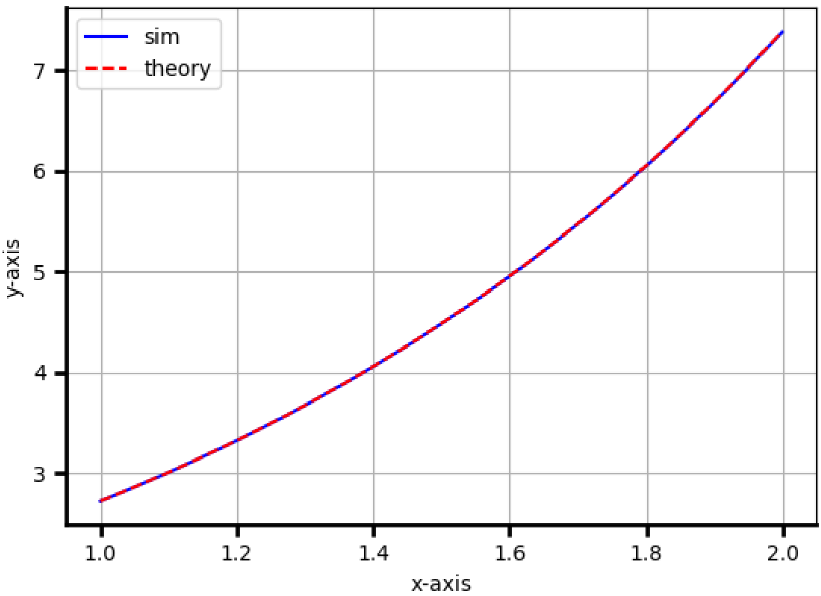

Fig. 5: Verification