NCERT 9.4.12

EE24BTECH11041 - Mohit

1) Solve the differential equation given below with initial conditions $x = 2$ and $y = 0$ and plot a graph.

$$x(x^2 - 1)\frac{dy}{dx} = 1 \tag{1}$$

Solution:-

a) Rearranging the Equation,

$$dy = \frac{dx}{x(x^2 - 1)} \tag{2}$$

b) **Integration:** Integrating on both sides.

$$\int dy = \int \frac{dx}{x(x^2 - 1)} \tag{3}$$

$$\int dy = \int \frac{dx}{x^3(1 - \frac{1}{x^2})} \tag{4}$$

c) Subsituting ,

$$1 - \frac{1}{x^2} = t \tag{5}$$

d) Differentiating on both side ,

$$\frac{dx}{x^3} = \frac{dt}{2} \tag{6}$$

e) Now integrating,

$$\int dy = \int \frac{dt}{2t} \tag{7}$$

$$y = \frac{1}{2}\ln t + c \tag{8}$$

f) substituting t,

$$y = \frac{1}{2}\left(\ln\left(1 - \frac{1}{x^2}\right)\right) + c \tag{9}$$

g) finding constant by putting x=2 and y=0

$$c = \frac{1}{2} ln \frac{4}{3} \tag{10}$$

h) This leads to:

$$y = \frac{1}{2} \left(\ln \frac{4}{3} \left(1 - \frac{1}{x^2} \right) \right) \tag{11}$$

i) **NOTE:-** We are not using finite difference because the graph is discontinues at x=1 .So, when we find $\frac{dy}{dx}$ its value is becoming too large that we are getting a significant error in calculating $\frac{dy}{dx}$.So,We have to use another method.

j) **CODING LOGIC:** The solution for the differential equation can be graphically solved using coding by using below logic :**Runga-kutta Method**

$$h = 0.001 \tag{12}$$

$$\tag{13}$$

k) Let ,

$$\frac{dy}{dx} = f(x_n) \tag{14}$$

$$k_1 = hf(x_n) \tag{15}$$

$$k_2 = hf(x_n + h/2) \tag{16}$$

$$k_3 = hf(x_n + h/2) \tag{17}$$

$$k_4 = hf(x_n + h) \tag{18}$$

$$k = \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) \tag{19}$$

$$y_{n+1} = y_n + k \tag{20}$$

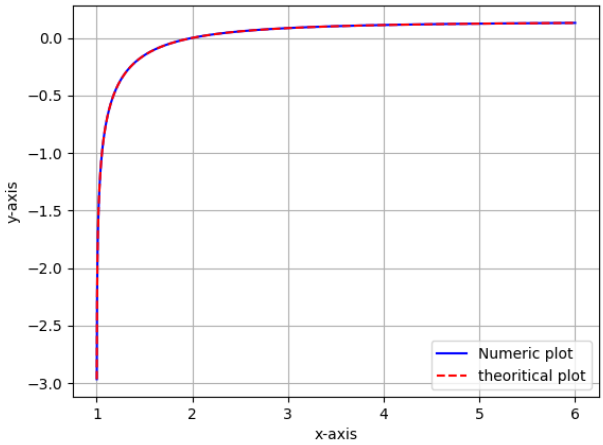

Fig. 1: Plot of functions