Question-9.6.13

EE24BTECH11048-NITHIN.K

Question:
Find a particular solution for the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$; $y = 0$ when $x = \frac{\pi}{3}$
Solution:
The original Differential Equation is

$$\frac{dy}{dx} + 2y\tan x = \sin x \tag{0.1}$$

$$\frac{dy}{dx} = \sin x - 2y\tan x \tag{0.2}$$

By Euler's approximation

$$\frac{y_{n+1} - y_n}{h} = \sin x_n - 2y_n \tan x_n \tag{0.3}$$

Hence the difference equation is

$$y_{n+1} = y_n + h\left(\sin x_n - 2y_n \tan x_n\right) \tag{0.4}$$

Solution for the First Order Differential Equation:

$$\frac{dy}{dx} + 2y\tan x = \sin x \tag{0.5}$$

$$\frac{dy}{dx} + P(x)\, y = Q(x) \tag{0.6}$$

$$P(x) = 2\tan x, Q(x) = \sin x \tag{0.7}$$

$$IF = e^{\int 2\tan x dx} = e^{2\ln|secx|} = (\sec x)^2 \tag{0.8}$$

By multiplying the Integrating Factor on both sides

$$(\sec x)^2 \frac{dy}{dx} + 2y(\sec x)^2 \tan(x) = (\sec x)^2 \sin x \tag{0.9}$$

$$\frac{d}{dx}\left[(\sec x)^2 y\right] = (\sec x)^2 \sin x \tag{0.10}$$

and Integrating on both sides

$$(\sec x)^2 y = \int (\sec x)^2 \sin(x) dx \tag{0.11}$$

$$y = \cos x - 2\cos x^2 \tag{0.12}$$

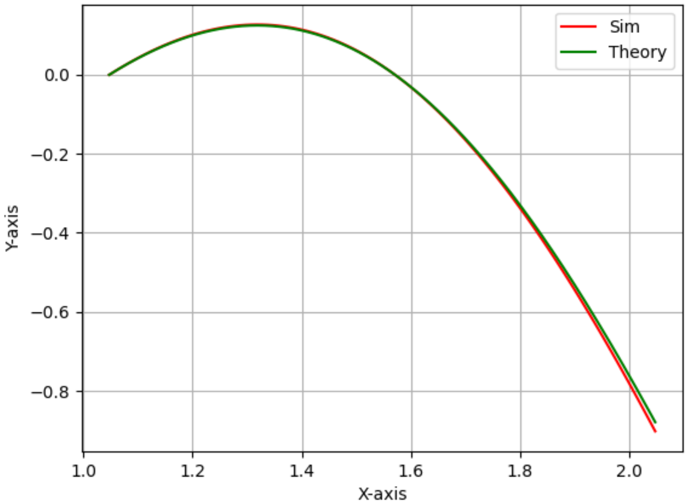

Fig. 0.1: Simulation VS Theoretical Plot.