

Solving differential equation

NCERT-9.5.4

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simulation method

Given:

$$\frac{dy}{dx} + \sec(x) \cdot y = \tan(x) \quad (1)$$

Rewriting:

$$\frac{dy}{dx} = \tan(x) - y \cdot \sec(x) \quad (2)$$

Using the definition of derivative:

$$\lim_{h \rightarrow 0} \frac{y(x+h) - y(x)}{h} = \tan(x) - y \cdot \sec(x) \quad (3)$$

Approximating for small h :

$$\frac{y_{n+1} - y_n}{h} \approx \tan(x) - y \cdot \sec(x) \quad (4)$$

Reorganizing:

$$y_{n+1} - y_n = h \cdot \frac{dy}{dx} \quad (5)$$

Therefore:

$$y_{n+1} = y_n + h \cdot \frac{dy}{dx} \quad (6)$$

$$y_{n+1} = y_n + h \cdot (\tan(x_n) - y_n \cdot \sec(x_n)) \quad (7)$$

By initializing the values of x and y and iterating the process several times and plotting them gives the curve for solution of the differential equation

Theoretical solution:

Consider the differential equation:

$$\frac{dy}{dx} + \sec(x)y = \tan(x) \quad (8)$$

where $0 \leq x < \frac{\pi}{2}$.

This is a first-order linear differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x) \quad (9)$$

where $P(x) = \sec(x)$ and $Q(x) = \tan(x)$.

The integrating factor (IF) is given by:

$$I(x) = e^{\int P(x) dx} = e^{\int \sec(x) dx}. \quad (10)$$

The integral of $\sec(x)$ is:

$$\int \sec(x) dx = \ln |\sec(x) + \tan(x)|. \quad (11)$$

Thus, the integrating factor becomes:

$$I(x) = e^{\ln |\sec(x) + \tan(x)|} = |\sec(x) + \tan(x)|. \quad (12)$$

Applying the integrating factor to the differential equation:

$$\frac{d}{dx} (y \cdot |\sec(x) + \tan(x)|) = \tan(x) |\sec(x) + \tan(x)|. \quad (13)$$

Integrate both sides:

$$y \cdot |\sec(x) + \tan(x)| = \int \tan(x) |\sec(x) + \tan(x)| dx. \quad (14)$$

$$y \cdot |\sec(x) + \tan(x)| = \int \tan(x) \cdot \sec(x) + \tan^2(x) \quad (15)$$

$$y \cdot |\sec(x) + \tan(x)| = \sec(x) + \tan(x) - x + C. \quad (16)$$

Therefore, the general solution is:

$$y = 1 - \frac{C \cdot x}{\sec(x) + \tan(x)} \quad (17)$$

where C is the constant of integration.

by assuming C as 1 we get a solution for this differential equation as

$$y = 1 - \frac{x}{\sec(x) + \tan(x)} \quad (18)$$

