9.5.17.1

EE24BTECH11003 - Akshara Sarma Chennubhatla

Question: Solve the differential equation $(4x+6y+5)\,dy-(3y+2x+4)\,dx=0$, with initial conditions $x_0=0, y_0=0$

Solution:
Theoretical Solution:

$$(3y+2x+4)\,dx = (4x+6y+5)\,dy \tag{1}$$
$$\frac{dy}{dx} = \frac{(3y+2x+4)}{(4x+6y+5)} \tag{2}$$
$$\tag{3}$$

Taking $2x+3y$ as t

$$2+3\frac{dy}{dx} = \frac{dt}{dx} \tag{4}$$
$$\frac{t+4}{2t+5} = \frac{1}{3}\left(\frac{dt}{dx}-2\right) \tag{5}$$
$$\frac{7t+22}{2t+5} = \frac{dt}{dx} \tag{6}$$
$$\tag{7}$$

Integrating on both sides,

$$\int dx = \int \frac{2t+5}{7t+22}dt \tag{8}$$
$$x = \frac{2}{7}\left(t-\frac{9}{14}\log\left(14t+44\right)\right)+C \tag{9}$$
$$x = \frac{2}{7}\left(2x+3y-\frac{9}{14}\log\left(28x+42y+44\right)\right)+C \tag{10}$$
$$x_0 = 0, y_0 = 0, \tag{11}$$
$$3x = 6y-\frac{9}{7}\log\left(\frac{28x+42y+44}{44}\right) \tag{12}$$
$$\tag{13}$$

Simulated Solution:

By first principle of derivatives,

$$y'(x) = \lim_{h \to 0} \frac{y(x+h) - y(x)}{h} \tag{14}$$

$$y(x+h) = y(x) + hy'(x) \tag{15}$$

Given differential equation can be written as,

$$y\prime = \frac{3y + 2x + 4}{4x + 6y + 5} \tag{16}$$

So, by using the method of finite diffferences,

$$y_1 = y_0 + h\left(\frac{3y_0 + 2x_0 + 4}{4x_0 + 6y_0 + 5}\right) \tag{17}$$

Similarly, by iterating for $y_2, y_3 ...$, The general difference equation is:

$$y_{n+1} = y_n + h\left(\frac{3y_n + 2x_n + 4}{4x_n + 6y_n + 5}\right) \tag{18}$$

Below is the simulated plot and the theoretical plot for given curve based on initial conditions, obtained by iterating through the values of x with step size of h

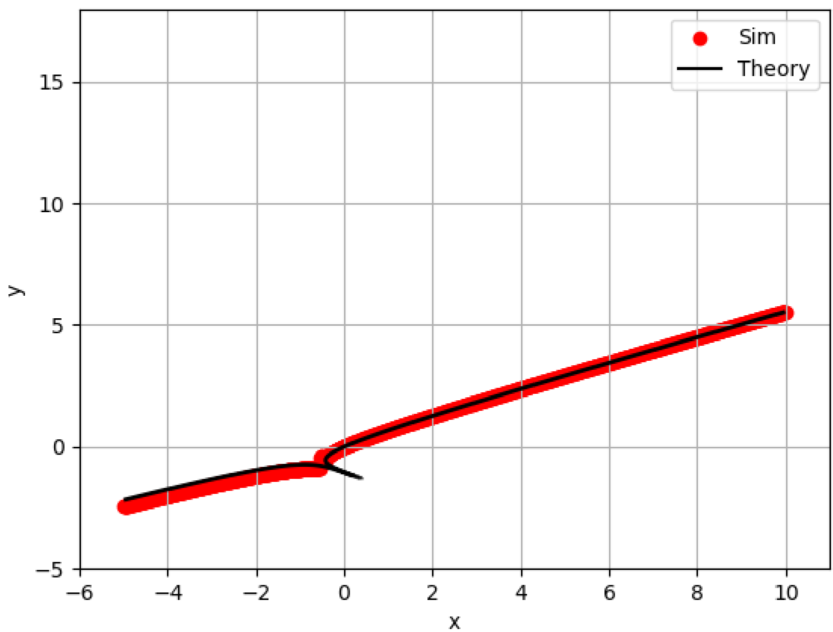

Fig. 1: Plot of the solution of $(4x + 6y + 5)\,dy - (3y + 2x + 4)\,dx = 0$