NCERT 12.8.3.1.1

EE24BTECH11057 - Shivam Shilvant

Question: Find the area of the region bounded by the curve $y = x^2$ and lines $x = 1$, $x = 2$ and x axis.

Solution:

Therotical logic:

1) Set up the integral:
   The area under the curve can be calculated as:
$$\text{Area} = \int_a^b f(x)dx \tag{1}$$
   Here:
$$f(x) = x^2, \quad x_1 = 1, \quad x_2 = 2 \tag{2}$$
   Thus, the integral becomes:
$$\text{Area} = \int_1^2 x^2\, dx \tag{3}$$
2) Compute the integral:
   The integral of x^2 is:
$$\int x^2\, dx = \frac{x^3}{3} \tag{4}$$
3) Evaluate the definite integral:
   Substitute the limits of integration:
$$\text{Area} = \left[\frac{x^3}{3}\right]_1^2 \tag{5}$$
   First, calculate $\frac{x^3}{3}$ at $x = 2$:
$$\frac{2^3}{3} = \frac{8}{3} \tag{6}$$
   Next, calculate $\frac{x^3}{3}$ at $x = 1$:
$$\frac{1^3}{3} = \frac{1}{3} \tag{7}$$
   Subtract the two results:
$$\text{Area} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3} \tag{8}$$

4) Final result:
area of the region bounded by the curve $y = x^2$ and lines $x = 1$, $x = 2$ and x axis is:

$$\frac{7}{3}\text{square units i.e. } 2.3333333333461046 \text{ square units} \tag{9}$$

Computational Logic: Using the trapezoidal rule to get the area. The trapezoidal rule is as follows.

$$\int_a^b f(x)\,dx \approx \sum_{k=1}^{N} \frac{f(x_{k+1}) + f(x_k)}{2} h \tag{10}$$

where

$$h = \frac{b-a}{N} \tag{11}$$

$\therefore$The difference equation obtained is

$$A = \int_a^b f(x)\ dx \approx h\left(\frac{1}{2}f(a) + f(x_1) + f(x_2)\cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right) \tag{12}$$

$$h = \frac{b-a}{n} \tag{13}$$

$$A = j_n,\ \text{where, } j_{i+1} = j_i + h\frac{f(x_{i+1}) + f(x_i)}{2} \tag{14}$$

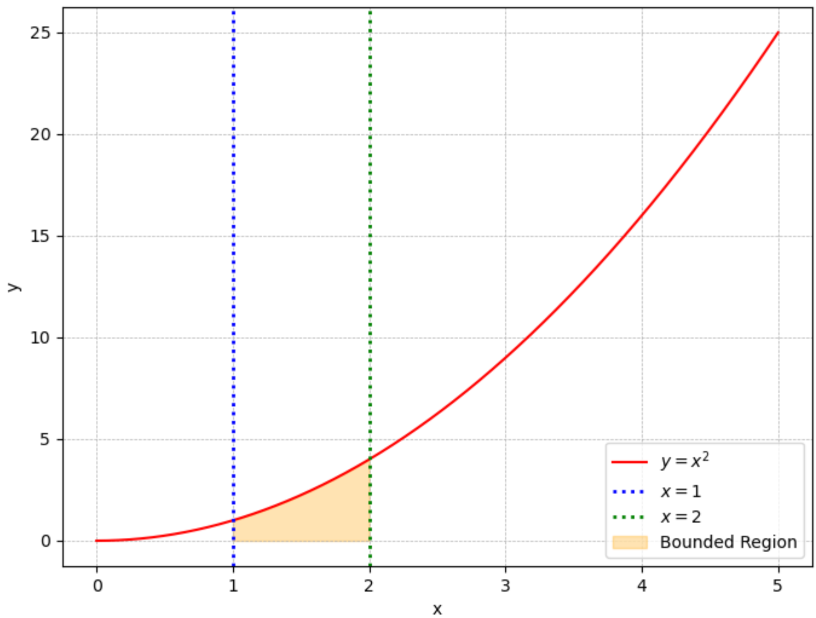
$$\rightarrow j_{i+1} = j_i + h\left(x_{i+1}^2 + x_i^2\right) \tag{15}$$

$$x_{i+1} = x_i + h \tag{16}$$

$$h = 0.00001 \tag{17}$$

$$n = 300000 \tag{18}$$

Using the code answer obtained is 2.3333333333461046



25
20
15
10
5
0
y
0
1
2
3
4
5
x
$y = x^2$
$x = 1$
$x = 2$
Bounded Region