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EE24BTECH11057 - SHIVAM SHILVANT*

1) A fair die is rolled two times independently. Given that the outcome on the first roll is 1, the expected value of the sum of the two outcomes is:

a) 4 b) 4.5 c) 3 d) 5.5

2) The dimension of the vector space of 7×7 real symmetric matrices with trace zero and the sum of the off-diagonal elements zero is: .

a) 47 b) 28 c) 27 d) 26

3) Let A be a 6×6 complex matrix with $A^3 \neq 0$ and $A^4 = 0$. Then the number of Jordan blocks of A is:

a) 1 or 6 b) 2 or 3 c) 4 d) 5

4) Let $X_1, ..., X_n$ be a random sample from a uniform distribution defined over $(0, \theta)$, where $\theta > 0$ and $n \geq 2$. Let X(1)=min$\{X_1, ..., X_n\}$ and X(n)=max$\{X_1, ..., X_n\}$. Then the covariance between X(n) and X(1)/X(n) is:

a) 0 b) $n(n+1)\theta$ c) $n\theta$ d) $n^2(n+1)\theta$

5) Let $X_1, ..., X_n$be a random sample drawn from a population with probability density function f$(x;\theta)$=$\theta x^{\theta-1}, 0 \leq x \leq 1, \theta > 0$. Then the maximum likelihood estimator of θ is:

a) $\frac{-n}{\sum_{i=1}^{n} \log_e X_i}$ b) $\frac{-\sum_{i=1}^{n} \log_e X_i}{n}$ c) $\left(\Pi_{i=1}^{n} X_i\right)^{1/n}$ d) $\frac{(\Pi_{i=0}^{n} X_i)}{n}$

6) Let $Y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i$, for $i = 1, ..., 10$, where $x'_{1i}s$ and $x'_{2i}s$ are fixed covariates and ϵ_i's are uncorrelated random variables with mean 0 and unknown variance σ^2. Here $\beta_0,\ \beta_1,$ and β_2 rare unknown parameteres. Further, define $\hat{Y}_i = \hat{\beta_0}\ + \hat{\beta}_1 x_{1i}\ + \hat{\beta}_2 x_{2i}$, where $\left(\hat{\beta_0}, \hat{\beta_1}, \hat{\beta_2}\right)$ is the unbiased least square estimator of $(\beta_0, \beta_1, \beta_2)$. Then an unbiased estimator of σ^2 is

a) $\frac{\sum_{i=1}^{1} 0\left(Y_i - \hat{Y}_i\right)^2}{10}$ b) $\frac{\sum_{i=1}^{1} 0\left(Y_i - \hat{Y}_i\right)^2}{7}$ c) $\frac{\sum_{i=1}^{1} 0\left(Y_i - \hat{Y}_i\right)^2}{8}$ d) $\frac{\sum_{i=1}^{1} 0\left(Y_i - \hat{Y}_i\right)^2}{9}$

7) For $i = 1, 2, 3$, let $Y_i = \alpha + \beta x_i + \epsilon_i$, where x_i's are fixed covariates, and ϵ_i's are independent and identically distributed standard normal random variables. Here, α and β are unknown parameters. Given the following observation,
the best linear unbiased estimate of $\alpha + \beta$ is equal to

Y_i	0.5	2.5	0.5
x_i	1	1	-2

a) 1.5 b) 1 c) 1.8 d) 2.1

8) Consider a discrete time Markov chain on the space $\{1, 2, 3\}$ with one-step transition probablity matrix. Which of the following statements is true ?

$$\begin{array}{c|ccc|} & \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \hline \mathbf{1} & \mathbf{0.7} & \mathbf{0.3} & \mathbf{0} \\ \mathbf{2} & \mathbf{0} & \mathbf{0.6} & \mathbf{0.4} \\ \mathbf{3} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \hline \end{array}$$

Fig. 8.1: Probablity matrix

a) States 1, 3 are recurrent and state 2 is transient.
b) State 3 is recurrent and states 1, 2 are transient.
c) States 1, 2, 3 are recurrent.
d) States 1, 2 are recurrent and state 3 is transient.

9) The minimal polynomial of the matrix $\begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$ is

a) $(x-1)(x-2)$ b) $(x-1)^2(x-2)$ c) $(x-1)(x-2)^2$ d) $(x-1)^2(x-2)^2$

10) Let (X_1, X_2, X_3) be a trivariate normal random vector with mean vector $(-3, 1, 4)$ and variance-covariance matrix $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 3 & -3 \\ 0 & -3 & 4 \end{pmatrix}$. Which of the following statements are true ?

a) X_2 and X_3 are independent.
b) $X_1 + X_3$ and X_2 are independent.
c) (X_2, X_3) and X_1 are independent.
d) $\frac{1}{2}(X_2 + X_3)$ and X_1 are independent.

a) (i) and (iii)
b) (ii) and (iii)
c) (i) and (iv)
d) (iii) and (iv)

11) A 2^3 factorial experiment with factors A, B and C is arranged in two blocks of four plots each as follows: (Below, (1) denotes the treatment in which A, B, and C are at the lower level, ac denotes th
The treatment contrast that is confounded with the blocks is

Block 1	(1)	ab	ac	bc
Block 2	a	b	c	abc

a) BC
b) AC
c) AB
d) ABC

12) Consider a fixed effects two-way analysis of variance model $Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$, where $i = 1, ..., a; j = 1, ..., b; k = 1, .., r$ and ϵ_{ijk} 's are independent and identically distributed normal random variables with zero mean and constant variance. Then the degrees of freedom available to estimate the error variance is zero when

a) $a = 1$
b) $b = 1$
c) $r = 1$
d) None of the above.

13) For $k = 1, 2, ..., 10$, let the probability density function of the random variable X_k be

$$f_{X_k} = \begin{cases} \frac{e^{-\frac{x}{k}}}{k}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Then $E\left(\Sigma_{k=1}^{10} kX_k\right)$ is equal to ...