ASSIGNMENT-1

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9) The slope of tangent to curve y=$f(x)$ at $[x, f(x)]$ is $2x+1$. If the curve passes through the point $(1,2)$, then the area bounded by curve, the x axis and the line $x = 1$ is (1995S)

   a) $\frac{5}{6}$ b) $\frac{6}{5}$ c) $\frac{1}{6}$ d) $\frac{6}{1}$

10) If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \leq 1$, then in this interval $(1997 - 2Marks)$
   a) both $f(x)$ and $g(x)$ are increasing functions.
   b) both $f(x)$ and $g(x)$ are decreasing functions.
   c) $f(x)$ is an increasing function.
   d) $g(x)$ is an increasing function.

11) The function $f(x) = \sin^4 x + \cos^4 x$ increases if $(1999 - 2Marks)$

   a) $0 < x < \frac{\pi}{8}$ b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$ c) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

12) Consider the following statements in S and R (2000S) S: Both $\sin x$ and $\cos x$ are decreasing functions in the interval $\left(\frac{\pi}{2}, \pi\right)$
   R: If a differentiable function decreases in a interval (a, b),then its derivative also decreases in (a, b).
   Which of the following is true ?
   a) Both S and R are wrong.
   b) Both S and R are correct, but R is not the correct explanation of S.
   c) S is correct and R is correct explanation for S.
   d) S is correct and R is wrong.

13) Let $f(x) = \int e^x (x-1)(x-2)\, dx$. Then f decreases in the interval (2000S)

   a) $(-\infty, -2)$ b) $(-2, -1)$ c) $(1, 2)$ d) $(2, +\infty)$

14) If normal to curve $y = f(x)$ at the point $(3, 4)$ makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3) =$ (2000S)

   a) -1 b) -$\frac{3}{4}$ c) $\frac{4}{3}$ d) 1

15) Let $f(x)= \begin{cases} |x|, & \text{for } 0 < |x| \leq 2 \\ 1, & \text{for } x = 0 \end{cases}$ then at $x = 0$, f has (2000S)

a) a local maximum
b) no local maximum
c) a local minimum
d) no extremum

16) For all $x \in (0, 1)$ (2000S)

a) $e^x < 1 + x$ b) $\log_e(1 + x) < x$ c) $\sin x > x$ d) $\log_e x > x$

17) If $f(x) = xe^{x(1-x)}$,then $f(x)$ is (2001S)

a) increasing on $\left[-\frac{1}{2}, 1\right]$
b) decreasing on $\mathbb{R}$
c) increasing on $\mathbb{R}$
d) decreasing on $\left[-\frac{1}{2}, 1\right]$

18) The triangle formed by tangent to curve $f(x) = x^2 + bx - b$ at the point $(1, 1)$ and the coordinate axes, lies in the first quadrant. If its area is 2, then the value of b is (2001S)

a) -1 b) 3 c) -3 d) 1

19) Let $f(x) = \left(1 + b^2\right)x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is (2001S)

a) $[0, 1]$ b) $\left(0, \frac{1}{2}\right]$ c) $\left[\frac{1}{2}, 1\right]$ d) $(0, 1]$

20) The length of a longest interval in which the function $3\sin x - 4\sin^3 x$ is increasing, is (2002S)

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{3\pi}{2}$ d) π

21) The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is(are) (2002S)

a) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ b) $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$ c) $(0, 0)$ d) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

22) In $[0, 1]$ Lagranges Mean Value theorem is NOT applicable to (2003S)

a) $f(x) = \begin{cases} \frac{1}{2} - x, x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, x \geq \frac{1}{2} \end{cases}$

b) $f(x) = \begin{cases} \frac{\sin x}{x}, x \neq 0 \\ 1, x = 0 \end{cases}$

c) $f(x) = x|x|$

d) $f(x) = |x|$

23) Tangent is drawn to ellipse $\frac{x^2}{27} + y^2 = 1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$(where $\theta \in \left(0, \frac{\pi}{2}\right)$). Then the value of θ such that sum of intercepts on axes made by this tangent is minimum, is (2003S)

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{8}$ d) $\frac{\pi}{4}$