Assignment1

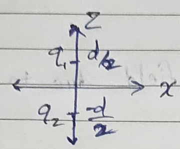
Shivam Shilvant
EE24BTECH11057

March 8, 2025

Assignment 1

Shivam Shilvant
ee24btech11057

(1)



(a) $q_1 = q_2 = q$

$$E_z = E_{q_1} + E_{q_2}$$

$$E_z = \frac{kq}{\left(z-\frac{d}{2}\right)^2}\hat{k} + \frac{kq}{\left(z+\frac{d}{2}\right)^2}\hat{k} \qquad \text{here } k = \frac{1}{4\pi\varepsilon_0}$$

$$E_z = \begin{cases} \dfrac{2kqz}{\left(z^2-\frac{d^2}{4}\right)}\hat{k} & z < d/2 \\[2ex] \dfrac{kq\left(z^2+d^2/4\right)}{\left(z^2-d^2/4\right)} & z > d/2 \end{cases}$$

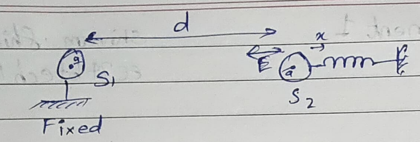
(b) $E_q = E_{q_1} + E_{q_2}$

$$E_x = \frac{2kqx}{\left(x^2+d^2/4\right)}\hat{i} + 0\hat{k}$$

(c) $$E_z = \frac{-kqd}{\left(z^2-d^2/4\right)}\hat{k}$$

$$E_z = 0\hat{i} - \frac{kqd}{\left(z^2+\frac{d^2}{4}\right)^{1/2}}$$

(2)



as a<<d we can assume field due to S_1 at S_2 be ll'el

$$F_{S_2S_1} \cdot q_{S_2} = -kx$$

$$\frac{qq}{4\pi\epsilon_0(d-x)^2} = +kx \qquad \because k \text{ is constant of spring}$$

$$q^2 = k4\pi\epsilon_0(d-x)^2 \cdot x \, c^2$$

$$q = \sqrt{k4\pi\epsilon_0}\,(d-x)\sqrt{x}\,c^2$$

$$\frac{dq}{dx} = k_0\frac{(d-3x)}{2\sqrt{x}} \qquad \because k_0 = \sqrt{4k\pi\epsilon_0 c^2}$$

$$dq/dx = 0 \text{ at } x = d/3$$

$$\frac{d^2q}{dx^2} = k_0\left(\frac{-3}{2\sqrt{x}} - \frac{(d-3x)}{4x^{3/2}}\right)$$

Putting $x = d/3$ we get $\frac{d^2q}{dx^2} < 0$ max^n

$\therefore$ q or Q is maximum at $\boxed{x = d/3}$

$$\boxed{Q_{max}(q_{max}) = \sqrt{4k\pi\epsilon_0 C^2}\,\frac{2d^{3/2}}{3\sqrt{3}}}$$

Put why are we getting this Q_{max} at d=3 if a higher charge is applied then S_2 will cross the $d/3$ mark and the electrostatic force will dominate the spring force & the sphere will meet neutrilising charge.

Here, assumptions made that the sphere 2 (S_2) is moved very slowly to it's equlibrium position or else oscillation ~~he~~ must happen and again the ~~spheer's~~ spheres might meet.

Now let's forset this assumption

Charges are dropped instantly and the sphere 2 (S_2) gaing kinetic enegry in middle and the stops at extremum $x = x_0$, momentailly and oscillates back.

Initial Potential Energy = Final Potential Energy

$$(V_{S_2S_1}\, q_{S_2})_i = (V_{S_2S_1}\, q_{S_2})_f + \frac{1}{2} k x_0^2$$

$$(V_i - V_f)\, q_{S_2} = \frac{1}{2} k x_0^2$$

$$\cancel{V = \frac{kq_1}{4\pi}} \qquad V = -\frac{q_1}{4\pi\varepsilon_0 r}$$

$$\therefore \frac{Q^2}{C^2 4\pi\varepsilon_0}\left(\frac{1}{x_0} - \frac{1}{d}\right) = \frac{1}{2} k x_0^2$$

$$Q^2 = \frac{1}{2} k \frac{x_0^3\, d\, c^2\, 4\pi\varepsilon_0}{d - x}$$

$$Q = \sqrt{\frac{1}{2} k c^2 4\pi\varepsilon_0 d}\ \sqrt{\frac{x_0^3}{d - x_0}}$$

$$Q = k_0 \frac{x^{3/2}}{\sqrt{d-x}} \qquad k_0 = \sqrt{\tfrac{1}{2} k c^2 4\pi\varepsilon_0 d}$$

$$\frac{dQ}{dx} = k_0\left(\frac{3}{2}\frac{x_0^{1/2}}{\sqrt{d-x_0}} + \frac{1}{2}\frac{x_0^{3/2}}{(d-x_0)^{3/2}}\right) = 0$$

$$\Downarrow$$

$$\boxed{x_0 = \frac{3d}{4}}$$

$$Q_{max} = k_0\sqrt{\frac{x^3}{d-x}} = k_0\sqrt{\frac{27 d^3 \times 3}{64\,(2d)}} = \sqrt{kc^2 4\pi\varepsilon_0 d}\left(\frac{9d}{16}\right)$$

again if more than this charge is applied, then spheres will meet.

(4)

(3) $F_1 = 5a_z$

i) $x^2+y^2+z^2=a^2$, $\theta \in (0, \pi/2)$, $\phi \in (0, 2\pi)$

$$\text{Flux} = \iint_S \vec{F_1} \cdot \hat{n} \, ds$$

$$\hat{n} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{a}$$

$$\text{Flux} = \iint_S (5\hat{k}) \cdot \frac{x\hat{i}+y\hat{j}+z\hat{k}}{a} \, ds$$

$$= \iint_S \frac{5z}{a} \, ds$$

let $z = a\cos\theta$, $ds = a^2 \sin\theta \, d\theta \, d\phi$

$$\text{Flux} = 5a^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta\cos\theta \, d\theta \, d\phi$$

$$= \frac{5a^2}{2} \int_0^{2\pi} d\phi$$

$$\underline{\underline{\text{Flux} = 5\pi a^2}}$$

now, trying symmetrical

$x = \rho\sin\theta\cos\phi$, $y = \rho\sin\theta\sin\phi$, $z = \rho\cos\theta$

$\hat{n}\,ds = a^2 \sin\theta \, d\theta \, d\phi$

$F_1 = 5\cos\theta \, \hat{r} - 5\sin\theta \, \hat{\theta}$

$$\text{Flux} = \iint F_1 \cdot \hat{n} \, ds$$

$$= \iint (5\cos\theta \, \hat{r} - 5\sin\theta \, \hat{\theta})(\hat{r})(a^2 \sin\theta \, d\theta \, d\phi)$$

$$= \iint 5\cos\theta \sin\theta \, a^2 \, d\theta \, d\phi$$

$$= 5\pi a^2$$

ii) $F_2 = 5z\hat{k}$

$\hat{n} = (x\hat{i} + y\hat{j} + z\hat{k})/a$

Flux $= \iint 5z^2 \, \frac{1}{a} \, ds$

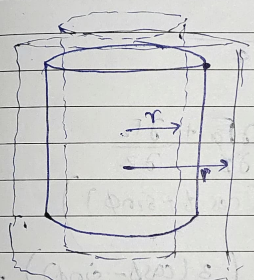
$= 5a^3 \int_0^{2\pi}\int_0^{\pi} \cos^2\theta \sin\theta \, d\theta \, d\phi$

$= \frac{10 a^3 \pi}{3}$

Flux $= \iiint \nabla F_2 \, dv$

$= \int_0^{a}\int_0^{2\pi}\int_0^{\pi} 5 \, d\theta \, d\phi \, d\rho = \frac{10}{3} a^3 \pi$

4)



$\rho_r = ar^2$

Let a guassian surface inside cylinder with same axis using guass law

$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$

$E(2\pi r L) = \frac{1}{\varepsilon_0} \int_0^{r} (2\pi r)(ar^2) \, dr$

$E = \frac{r^3 a}{4\varepsilon_0}$

outside cylinder

$\oint E \cdot dA = \frac{1}{\varepsilon_0} \int_0^{b} 2\pi r L a r^2 \, dr$

$E(2\pi r L) = \frac{1}{4\varepsilon_0} \, 2\pi \, b^4 a L$

$E = \frac{b^4 a}{4 r \varepsilon_0}$

$$E = \begin{cases} r^3/4\varepsilon_0 & r \le b \\ \dfrac{b^4 a}{4r\varepsilon_0} & r > b \end{cases}$$

(5) If $\vec{F} = a\,\hat{s} + b\hat{\phi} + c\hat{z}$
then, $|\vec{F}| = \sqrt{a^2 + c^2}$.

(a) $\vec{F} = (4 + 3\cos\phi + 3\sin\phi)\hat{s} + 3(\cos\phi - \sin\phi)\hat{\phi} - 2\hat{z}$

$$|\vec{F}| = \sqrt{(4 + 3\cos\phi + 3\sin\phi)^2 + 4}$$
$$= \sqrt{38 + 24(\sin\phi + \cos\phi)}$$

(b) $\vec{F} = \left(\dfrac{40}{s^2+1} + 3\sqrt{2}\right)\hat{s} - 2\hat{z}$

$$|\vec{F}| = \sqrt{\left(\frac{40}{s^2+1} + 3\sqrt{2}\right)^2 + 4}.$$

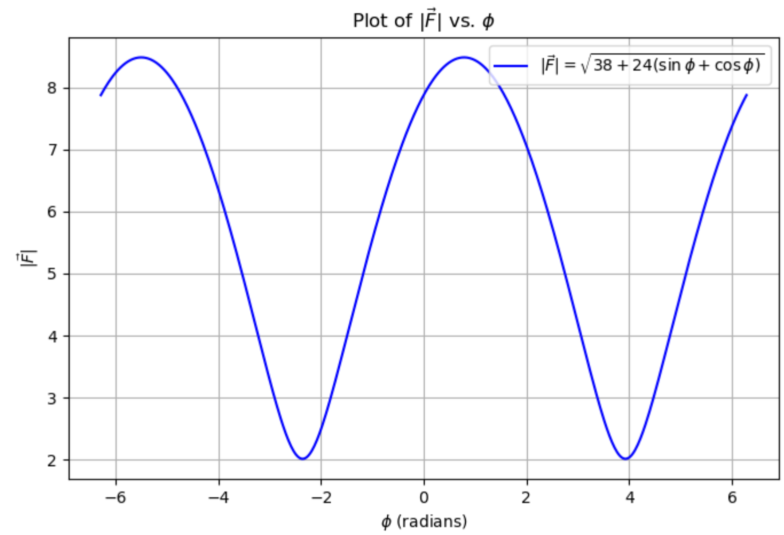
(c) $$\nabla \vec{F} = \frac{1}{s}\frac{\partial}{\partial s}(sF_s) + \frac{1}{s}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

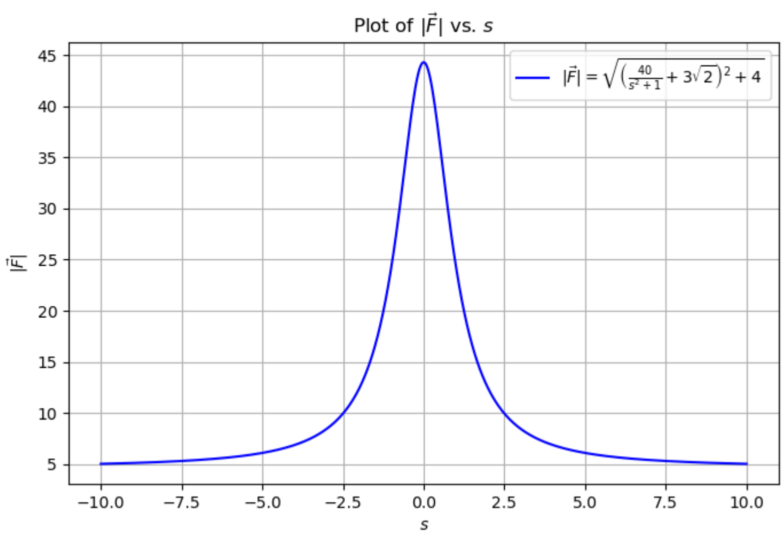
$$= \frac{1}{s}\frac{\partial}{\partial s}\left(\frac{40s}{s^2+1} + 3s(\cos\phi + \sin\phi)\right)$$
$$+ \frac{1}{s}\frac{\partial}{\partial \phi}\left(3(\cos\phi - \sin\phi)\right) - 2$$

$$= \frac{40}{(s^2+1)s} - \frac{\cancel{8}0s}{(s^2+1)^2} + \cancel{\frac{3\cos\phi + 3\sin\phi}{s}} - \cancel{\left(\frac{3\cos\phi + 3\sin\phi}{s}\right)} - 2$$

$$= \frac{40}{(s^2+1)s} - \frac{80s}{(s^2+1)^2} - 2$$

(d) $$\nabla \times \vec{F} = \begin{vmatrix} \hat{s} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{s\,\partial \phi} & \frac{\partial}{\partial z} \\ F_s & F_\phi & F_z \end{vmatrix}$$


Plot of $|\vec{F}|$ vs. ϕ
$|\vec{F}| = \sqrt{38 + 24(\sin\phi + \cos\phi)}$
$|\vec{F}|$
ϕ (radians)
−6
−4
−2
0
2
4
6
2
3
4
5
6
7
8


Plot of $|\vec{F}|$ vs. s
$|\vec{F}| = \sqrt{\left(\frac{40}{s^2+1} + 3\sqrt{2}\right)^2 + 4}$
$|\vec{F}|$
s
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
5
10
15
20
25
30
35
40
45

(7)

$= 0\hat{s} + 0\hat{\phi} - 0\hat{z}$

$= 0 - \frac{3(-\sin\phi + \cos\phi)}{s}$

$\therefore$ Not conservative

(6)

(c) Considering all interactions

let charges be 1 C at (0,1) q_1

-1 C (1, 0) q_2

1 C (0, -1) q_3

-1 C (-1, 0) q_4

$$\text{Energy} = \Sigma \frac{q q}{4\pi\varepsilon_0 r}$$

$$= \underbrace{4\left(\frac{-1}{4\pi\varepsilon_0\sqrt{2}}\right)}_{(q_i, q_{i+1})} + \underbrace{2\left(\frac{1}{4\pi\varepsilon_0 2}\right)}_{(q_i, q_{i+2})}$$

$$= \frac{1 - 2\sqrt{2}}{4\pi\varepsilon_0} \ \text{J}$$

(7)

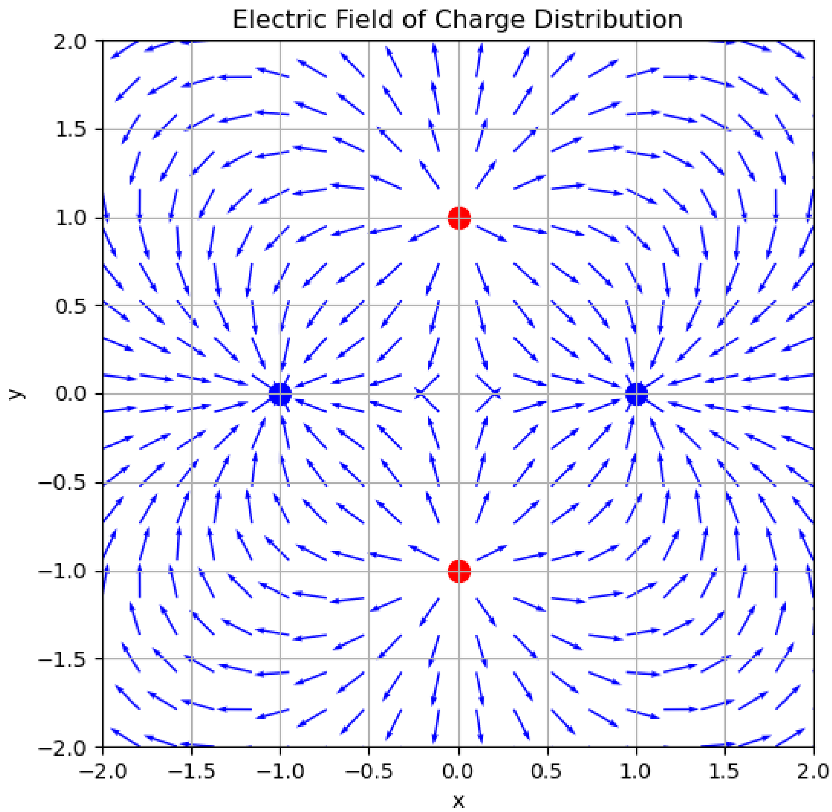
(a) $E = \nabla V$

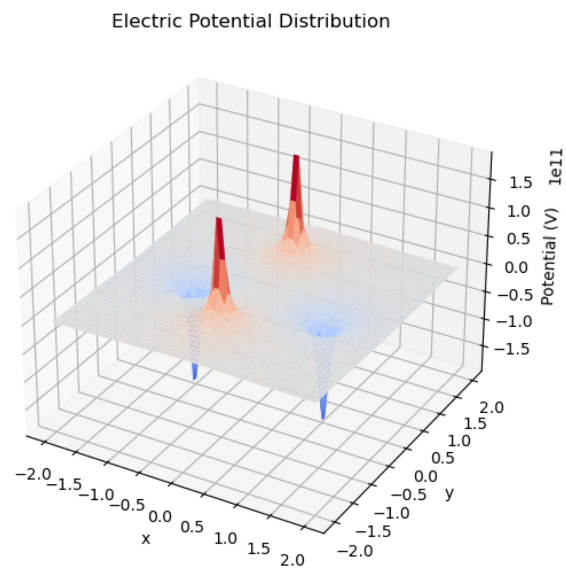
$$\vec{E} = \frac{V_0}{a} e^{-r/a}\hat{r}, \quad \nabla\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{V_0 r^2}{a} e^{-r/a}\right)$$

$$\nabla\cdot\vec{E} = \frac{V_0}{r^2 a}\left(2re^{-r/a} - \frac{r^2 e^{-r/a}}{a}\right)$$

$$-\nabla^2 V = \frac{\rho}{\varepsilon_0}$$

$$\rho = \frac{V_0\,\varepsilon}{r^2 a}\left(2re^{-r/a} - \frac{r^2}{a}e^{-r/a}\right)$$


Electric Field of Charge Distribution
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x


Electric Potential Distribution
1e11
Potential (V)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
x
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
y
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0

Put $r = a$

$$\boxed{\rho = \frac{V_0 \varepsilon_0 e^{-1}}{a}}$$

(b) $\vec{E} = \frac{V_0}{a} e^{-r/a} \hat{r}$

$\vec{E} = \frac{V_0 e^{-1}}{d}$

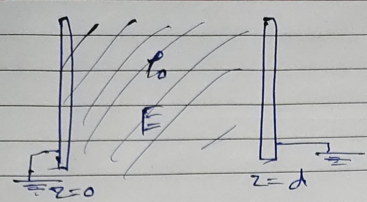
(c) $\iiint \rho \, dv$

$= 2\pi^2 \int \frac{V_0 \varepsilon_0 A}{r^2} \left(2 r e^{-r/a} - \frac{r^2 e^{-r/a}}{a} \right)$

$= 2\pi^2 \int_0^\infty e^{-r/a} \left(\frac{2}{r} - \frac{1}{a} \right) dr$

9

8



(a) $-\nabla^2 V = \rho_0/\varepsilon_0$

$$V = \frac{-\rho_0 z^2}{2\varepsilon} + c_1 z + c_2$$ as $x_2\ z$ infigu

$V(0) = 0 \Rightarrow c_2 = 0$

$V(d) = 0 \Rightarrow c_1 = 0$

$$V(z) = \frac{-\rho_0 z^2}{2\varepsilon}$$

(b) $\vec{E} = -\nabla V$

$$\vec{E} = \frac{\rho_0 z}{\varepsilon}\hat{k}$$

(c) $V = -\rho z^2 + C_1 z + C_2$

$V(0) = 0 \rightarrow C_2 = 0$, $V(d) = V_0$

$$V_0 = \frac{-\rho_0 d^2}{2\varepsilon} + C_1 d \Rightarrow C_1 = \left(\frac{1}{d}\right)\left(V_0 + \frac{\rho d^2}{2\varepsilon}\right)$$

$$V = \frac{-\rho_0 z^2}{2\varepsilon} + \left(V_0 + \frac{\rho d^2}{2\varepsilon}\right)\frac{z}{d}$$

$$\Rightarrow E = \frac{\rho_0 z}{\varepsilon} - \left(V_0 + \frac{\rho d^2}{2\varepsilon}\right)\frac{1}{d}\ \hat{k}$$

For codes refer to
https://github.com/ArnavYadnopavit/Electromagnetics/tree/main/Assignment1