# Assignment1

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# Assignment 1

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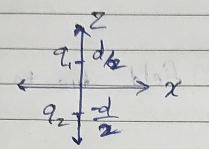


(a) $q_1 = q_2 = q$

$E_z = E_{q_1} + E_{q_2}$

$E_z = \frac{kq}{(z-\frac{d}{2})^2}\hat{k} + \frac{kq}{(z+\frac{d}{2})^2}\hat{k}$ here $k = 1/4\pi\varepsilon_0$

$$E_z = \begin{cases} \frac{2kqz}{(z^2-\frac{d^2}{4})}\hat{k} & z < d/2 \\ \frac{kq\,(z^2+d^2/4)}{(z^2-d^2/4)} & z > d/2 \end{cases}$$

(b) $E_q = E_{q_1} + E_{q_2}$

$E_x = \frac{2kqx}{(x^2+d^2/4)}\hat{i} + 0\hat{k}$

(c) $E_z = -\frac{kqd}{(z^2-d^2/4)}\hat{k}$

$E_z = 0\hat{i} - \frac{kqd}{(z^2+\frac{d^2}{4})^{1/2}}$

(2)

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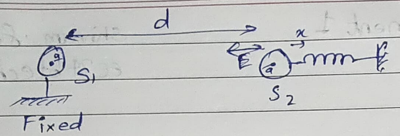



as $a << d$ we can assume field due to $S_1$ at $S_2$ be ll

$$E_{S_2S_1} \cdot q_{S_2} = -kx$$

$$\frac{qq}{4\pi\varepsilon_0 (d-x)^2} = +kx \quad \because k \text{ is constant of spring}$$

$$q^2 = k\,4\pi\varepsilon_0 (d-x)^2 \cdot x\, C^2$$

$$q = \sqrt{k4\pi\varepsilon_0}\,(d-x)\sqrt{x}\; C^2$$

$$\frac{dq}{dx} = k_0 \frac{(d-3x)}{2\sqrt{x}} \quad \because k_0 = \sqrt{4k\pi\varepsilon_0 C^2}$$

$$dq/dx = 0 \quad \text{at} \quad x = d/3$$

$$\frac{d^2q}{dx^2} = k_0\left(\frac{-3}{2\sqrt{x}} - \frac{(d-3x)}{4x^{3/2}}\right)$$

Putting $x = d/3$ we get $\frac{d^2q}{dx^2} < 0$ max$^n$

$\therefore$ q or Q is maximum at $\boxed{x = d/3}$

$$\boxed{Q_{max}(q_{max}) = \sqrt{4k\pi\varepsilon_0 C^2}\,\frac{2d^{3/2}}{3\sqrt{3}}}$$

Put why are we getting this $Q_{max}$ at $d=3$ if a higher charge is applied then $S_2$ will cross the $d/3$ mark and the electrostatic force will dominate the spring force & the sphere will meet neutrilising charge.

Here, assumptions made that the sphere 2 ($S_2$) is moved very slowly to it's equlibrium position or else oscillation ~~he~~ must happen and again the ~~spheer's~~ spheres might meet.
Now let's forset this assumption
Charges are dropped instantly and the sphere 2 ($S_2$) going kinetic enegry in middle and the stops at extremum $x = x_0$, momentailly and oscillates back.

Initial Potential Energy = Final Potential Energy

$$(V_{S_2S_1}\, q_{S_2})_i = (V_{S_2S_1}\, q_{S_2})_f + \frac{1}{2} K x_0^2$$

$$(V_i - V_f) q_{S_2} = \frac{1}{2} K x_0^2$$

$$\cancel{V = \frac{Kq_1}{4\pi}} \quad V = -\frac{q_1}{4\pi\varepsilon_0 r}$$

$$\therefore \frac{Q^2}{C^2 4\pi\varepsilon_0}\left(\frac{1}{x_0} - \frac{1}{d}\right) = \frac{1}{2} K x_0^2$$

$$Q^2 = \frac{1}{2} K \frac{x_0^3\, d\, c^2\, 4\pi\varepsilon_0}{d - x}$$

$$Q = \sqrt{\frac{1}{2} K c^2 4\pi\varepsilon_0 d}\,\sqrt{\frac{x_0^3}{d - x_0}}$$

$$Q = k_0 \frac{x^{3/2}}{\sqrt{d - x}} \qquad k_0 = \sqrt{\frac{1}{2} K c^2 4\pi\varepsilon_0 d}$$

$$\frac{dQ}{dx} = k_0\left(\frac{3}{2}\frac{x_0^{1/2}}{\sqrt{d - x_0}} + \frac{1}{2}\frac{x_0^{3/2}}{(d - x_0)^{3/2}}\right) = 0$$

$$\Downarrow$$

$$\boxed{x_0 = \frac{3d}{4}}$$

$$Q_{max} = k_0\sqrt{\frac{x^3}{d - x}} = k_0\sqrt{\frac{27 d^3 \times 3}{64 (2d)}} = \sqrt{K c^2 4\pi\varepsilon_0 d}\left(\frac{9d}{16}\right)$$

again if more than this charge is applied, then spheres will meet.

(4)

(3) $F_1 = 5a_z$

i) $x^2+y^2+z^2=a^2$, $\theta \in (0, \pi/2)$, $\phi \in (0, 2\pi)$

$$\text{Flux} = \iint_S \vec{F_1} \cdot \hat{n}\, ds$$

$$\hat{n} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{a}$$

$$\text{Flux} = \iint_S (5\hat{k}) \cdot \frac{x\hat{i}+y\hat{j}+z\hat{k}}{a}\, ds$$

$$= \iint_S \frac{5z}{a}\, ds$$

let $z = a\cos\theta$, $ds = a^2 \sin\theta\, d\theta\, d\phi$

$$\text{Flux} = 5a^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta\cos\theta\, d\theta\, d\phi$$

$$= \frac{5a^2}{2}\int_0^{2\pi} d\phi$$

$$\underline{\text{Flux} = 5\pi a^2}$$

now, trying symmetrical

$x = \rho\sin\theta\cos\phi$, $y = \rho\sin\theta\sin\phi$, $z = \rho\cos\theta$

$\hat{n}\, ds = a^2 \sin\theta\, d\theta\, d\phi$

$F_1 = 5\cos\theta\,\hat{r} - 5\sin\theta\,\hat{\theta}$

$$\text{Flux} = \iint F_1 \cdot \hat{n}\, ds$$

$$= \iint (5\cos\theta\,\hat{r} - 5\sin\theta\,\hat{\theta})(\hat{r})(a^2\sin\theta\, d\theta\, d\phi)$$

$$= \iint 5\cos\theta \sin\theta\, a^2\, d\theta\, d\phi$$

$$= 5\pi a^2$$

$\textcircled{5}$

ii) $F_2 = 5z\,\hat{k}$

$\hat{n} = (x\hat{i} + y\hat{j} + z\hat{k})\,1/a$

$\text{Flux} = \iint 5 z^2 \, 1/a \, ds$

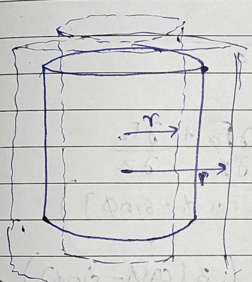
$= 5a^3 \int_0^{2\pi} \int_0^{\pi} \cos^2\theta \sin\theta \, d\theta \, d\phi$

$= \frac{10 a^3 \pi}{3}$

$\text{Flux} = \iiint \nabla F_2 \, dv$

$= \int_0^a \int_0^{2\pi} \int_0^{\pi} 5 \, d\theta \, d\phi \, dp = \frac{10}{3} a^3 \pi$

4)



$\rho_r = ar^2$

Let a guassian surface inside cylinder with same axis using guass law

$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$

$E(2\pi r L) = 1/\varepsilon_0 \int_0^r (2\pi r)(ar^2)\,dr$

$E = \frac{r^3 a}{4\varepsilon_0}$

out side cylinder

$\oint E \cdot dA = \frac{1}{\varepsilon_0} \int_0^b 2\pi r L a r^2 dr$

$E(2\pi r L) = \frac{1}{4\varepsilon_0} 2\pi \, b^4 a L$

$E = \frac{b^4 a}{4 r \varepsilon_0}$

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$$E = \begin{cases} r^3/4\varepsilon_0 & r \le b \\ \frac{b^4 a}{4r\varepsilon_0} & r > b \end{cases}$$

(5) If $\vec{F} = a\hat{s} + b\hat{\phi} + c\hat{z}$
then, $|\vec{F}| = \sqrt{a^2+c^2}$.

(a) $\vec{F} = (4+3\cos\phi+3\sin\phi)\hat{s} + 3(\cos\phi - \sin\phi)\hat{\phi} - 2\hat{z}$

$$|\vec{F}| = \sqrt{(4+3\cos\phi+3\sin\phi)^2+4}$$
$$= \sqrt{38+24(\sin\phi+\cos\phi)}$$

(b) $\vec{F} = \left(\frac{40}{s^2+1} + 3\sqrt{2}\right)\hat{s} - 2\hat{z}$

$$|\vec{F}| = \sqrt{\left(\frac{40}{s^2+1} + 3\sqrt{2}\right)^2 + 4}.$$

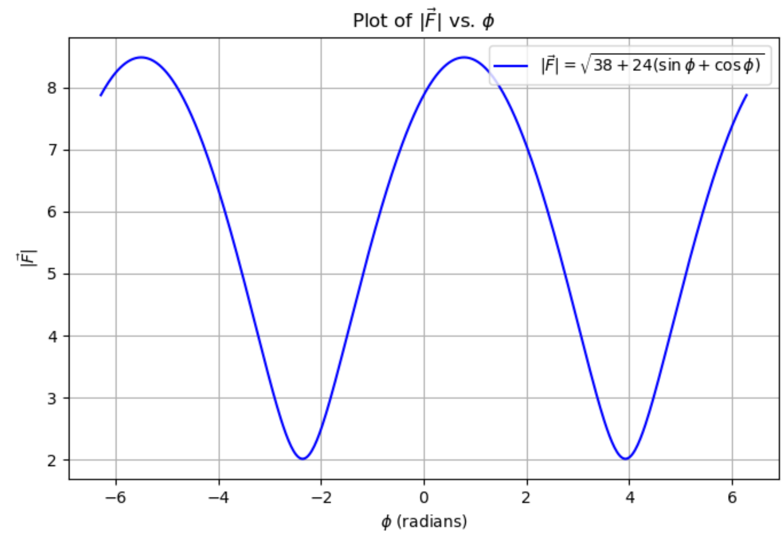
(c) $$\nabla\vec{F} = \frac{1}{s}\frac{\partial}{\partial s}(sF_s) + \frac{1}{s}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}$$

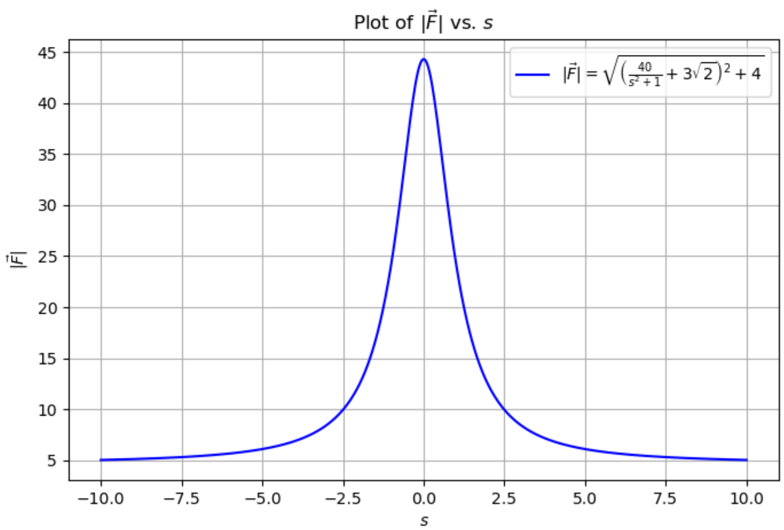
$$= \frac{1}{s}\frac{\partial}{\partial s}\left(\frac{40s}{s^2+1} + 3s(\cos\phi+\sin\phi)\right).$$
$$+\frac{1}{s}\frac{\partial}{\partial\phi}\left(3(\cos\phi-\sin\phi)\right) - 2$$

$$= \frac{40}{(s^2+1)s} - \frac{80s}{(s^2+1)^2} + \cancel{\frac{3\cos\phi+3\sin\phi}{s}} - \cancel{\left(\frac{3\cos\phi+3\sin\phi}{s}\right)} - 2$$

$$= \frac{40}{(s^2+1)s} - \frac{80s}{(s^2+1)^2} - 2$$

(d) $$\nabla\times\vec{F} = \begin{vmatrix} \hat{s} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{s\partial\phi} & \frac{\partial}{\partial z} \\ F_s & F_\phi & F_z \end{vmatrix}$$


Plot of $|\vec{F}|$ vs. $\phi$
$|\vec{F}| = \sqrt{38 + 24(\sin\phi + \cos\phi)}$
$|\vec{F}|$
$\phi$ (radians)


Plot of $|\vec{F}|$ vs. $s$
$|\vec{F}| = \sqrt{\left(\frac{40}{s^2+1} + 3\sqrt{2}\right)^2 + 4}$
$|\vec{F}|$
$s$

$= 0\hat{s} + 0\hat{\phi} - 0\hat{z}$

$= 0 - \frac{3(-\sin\phi + \cos\phi)}{s}$

$\therefore$ Not conservative

(6)

(c) Considering all interactions

let charges be 1 C at (0,1) $q_1$

-1 C (1, 0) $q_2$

1 C (0, -1) $q_3$

-1 C (-1, 0) $q_4$

$$\text{Energy} = \Sigma \frac{q q}{4\pi\varepsilon_0 r}$$

$$= 4\left(\frac{-1}{4\pi\varepsilon_0\sqrt{2}}\right) + 2\left(\frac{1}{4\pi\varepsilon_0 2}\right)$$

$$(q_i, q_{i+1}) \qquad (q_i, q_{i+2})$$

$$= \frac{1-2\sqrt{2}}{4\pi\varepsilon_0} \text{ J}$$

(7)

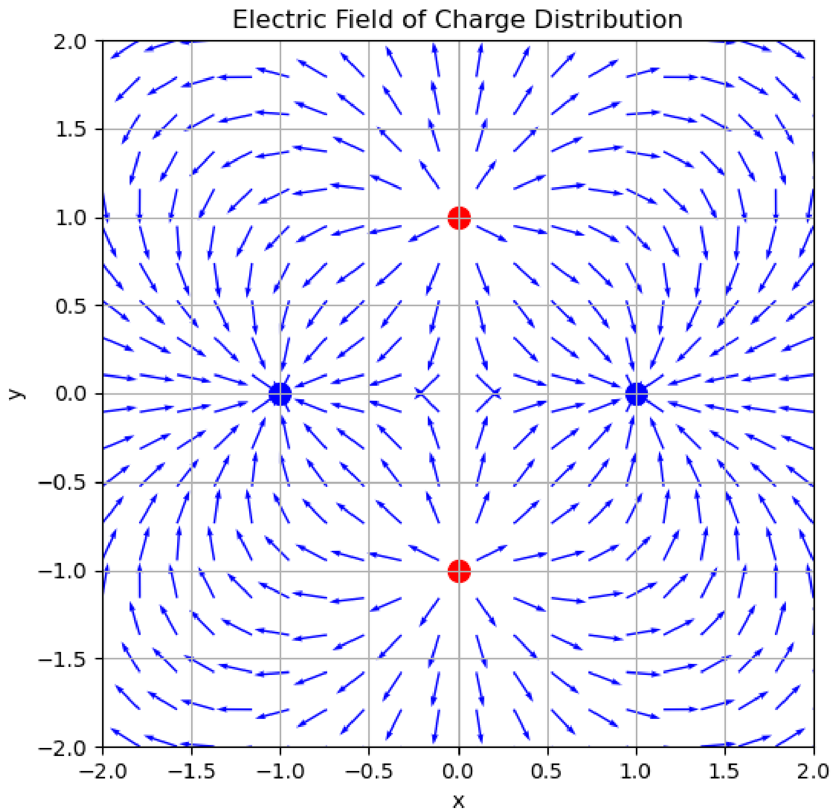
(a) $E = \nabla V$

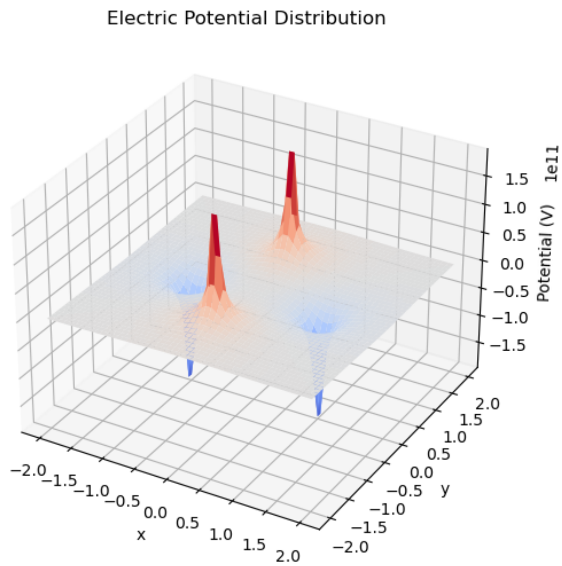
$$\vec{E} = \frac{V_0}{a} e^{-r/a}\hat{r}, \quad \nabla\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{V_0 r^2}{a} e^{-r/a}\right)$$

$$\nabla\cdot\vec{E} = \frac{V_0}{r^2 a}\left(2re^{-r/a} - \frac{r^2 e^{-r/a}}{a}\right)$$

$$-\nabla^2 V = \frac{\rho}{\varepsilon_0}$$

$$\rho = \frac{V_0\,\epsilon}{r^2 a}\left(2re^{-r/a} - \frac{r^2}{a}e^{-r/a}\right)$$


Electric Field of Charge Distribution
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x


Electric Potential Distribution
1e11
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Potential (V)
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y

(8)

Put $r = a$

$$\boxed{\rho = \frac{V_0 \varepsilon_0 e^{-1}}{a}}$$

(b) $\vec{E} = \frac{V_0}{a} e^{-r/a} \hat{r}$

$\vec{E} = \frac{V_0 e^{-1}}{d}$

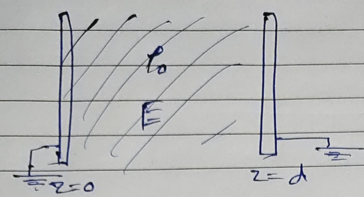
(c) $\iiint \rho \, dv$

$= 2\pi^2 \int \frac{V_0 \varepsilon_0 A}{r^2} \left( 2 r e^{-r/a} - \frac{r^2 e^{-r/a}}{a} \right)$

$= 2\pi^2 \int_0^{\infty} e^{-r/a} \left( \frac{2}{r} - \frac{1}{a} \right) dr$

(9)

(8)




(a) $-\nabla^2 V_0 = \rho_0/\epsilon_0$

$$V = \frac{-\rho_0 z^2}{2\epsilon} + c_1 z + c_2$$ as $X_2\, z$ infigu

$V(0) = 0 \Rightarrow C_2 = 0$

$V(d) = 0 \Rightarrow C_1 = 0$

$$V(z) = \frac{-\rho_0 z^2}{2\epsilon}$$

(b) $\vec{E} = -\nabla V$

$$\vec{E} = \frac{\rho_0 z}{\epsilon}\,\hat{k}$$

(c) $V = -\rho z^2 + C_1 z + C_2$

$V(0) = 0 \rightarrow C_2 = 0$, $V(d) = V_0$

$$V_0 = \frac{-\rho_0 d^2}{2\epsilon} + C_1 d \Rightarrow C_1 = \left(\frac{1}{d}\right)\left(V_0 + \frac{\rho d^2}{2\epsilon}\right)$$

$$V = \frac{-\rho_0 z^2}{2\epsilon} + \left(V_0 + \frac{\rho d^2}{2\epsilon}\right)\frac{z}{d}$$

$$\Rightarrow E = \frac{\rho_0 z}{\epsilon} - \left(V_0 + \frac{\rho d^2}{2\epsilon}\right)\frac{1}{d}\;\hat{k}$$

For codes refer to
https://github.com/Shivam25-stack/Electromagnetics/tree/42b591c5d3f493c295685900f3687d902d02
assignment1