1. Physical Quantities and Units

Definition:
Physical quantities are quantities that can be measured and expressed in terms of numbers and units.

Examples:

- Length (meter, m)
- Mass (kilogram, kg)
- Time (second, s)
- Temperature (kelvin, K)

Numerical:

- **Q:** Convert 5 kilometers to meters.
- **A:** $5\,\mathrm{km} = 5 \times 1000\,\mathrm{m} = 5000\,\mathrm{m}$

2. Fundamental and Derived Units

Definition:

- **Fundamental Units:** Basic units from which other units are derived (e.g., meter, kilogram, second).
- **Derived Units:** Units obtained from fundamental units (e.g., speed in meters per second, m/s).

Examples:

- Fundamental Units: Meter (m), Kilogram (kg), Second (s)
- Derived Units: Velocity (m/s), Force (Newton, N)

Numerical:

- **Q:** Express force in terms of fundamental units.
- **A:** Force (Newton) = $\mathrm{kg} \cdot \mathrm{m/s^2}$

3. System of Units

Definition:
A set of units used to measure various physical quantities. The most common systems are the CGS (centimeter-gram-second), MKS (meter-kilogram-second), and SI (International System of Units).

Examples:

- CGS: Length (cm), Mass (g), Time (s)
- MKS: Length (m), Mass (kg), Time (s)
- SI: Incorporates MKS and additional units for electric current (ampere), temperature (kelvin), etc.

Numerical:

- **Q:** Convert 100 cm to meters.
- **A:** $100\,\mathrm{cm} = 100 \times 0.01\,\mathrm{m} = 1\,\mathrm{m}$

4. Dimensional Analysis

Definition:
A method to check the correctness of equations and derive relations between physical quantities by using their dimensions.

Examples:

- The dimension of velocity: $[LT^{-1}]$
- The dimension of force: $[MLT^{-2}]$

Numerical:

- **Q:** Check the dimensional consistency of the equation $v = u + at$ where v is final velocity, u is initial velocity, a is acceleration, and t is time.
- **A:** Dimensions: $[LT^{-1}] = [LT^{-1}] + [LT^{-2}] \cdot [T]$
  $[LT^{-1}] = [LT^{-1}] + [LT^{-1}]$
  The equation is dimensionally consistent.

5. Accuracy, Precision, and Errors

Definition:

- **Accuracy:** The closeness of a measurement to the true value.
- **Precision:** The repeatability of measurements.
- **Errors:** The deviation of the measured value from the true value. Types include systematic and random errors.

Examples:

- If a balance measures the mass of a 100g weight as 99.8g, 99.9g, and 100.1g in different trials:
  - The measurements are precise but not accurate if the true mass is 100g.

Numerical:

- **Q:** Calculate the absolute error if the measured values of a length are 2.5m, 2.6m, and 2.4m, while the true value is 2.5m.
- **A:** Absolute error for each measurement:
  - $|2.5 - 2.5| = 0$
  - $|2.6 - 2.5| = 0.1$
  - $|2.4 - 2.5| = 0.1$
  - Mean absolute error: $(0 + 0.1 + 0.1)/3 = 0.0667\,\mathrm{m}$

6. Significant Figures

Definition:
Digits in a number that are reliable and necessary to indicate the precision of the measurement.

Examples:

- 123.45 has 5 significant figures.
- 0.00456 has 3 significant figures.

Numerical:

- **Q:** How many significant figures are there in 0.03040?
- **A:** 4 significant figures (3, 0, 4, 0).

7. Measurement of Length, Mass, and Time

Definition:
Techniques and instruments used to measure physical quantities accurately.

Examples:

- Vernier caliper for length
- Beam balance for mass
- Stopwatch for time

Numerical:

- **Q:** If a stopwatch measures 100.0 s with a least count of 0.1 s, calculate the percentage uncertainty.
- **A:** Uncertainty = $0.1/100.0 \times 100\% = 0.1\%$

1. **Conversion of Units**

   Q: Convert 72 kilometers per hour to meters per second.

   A:

$$1\,\text{km/h} = \frac{1000\,\text{m}}{3600\,\text{s}} = \frac{5}{18}\,\text{m/s}$$

$$72\,\text{km/h} = 72 \times \frac{5}{18}\,\text{m/s} = 20\,\text{m/s}$$

2. **Dimensional Analysis**

   Q: Check the dimensional consistency of the formula for kinetic energy, $E = \frac{1}{2}mv^2$.

   A:

$$\text{Dimensions of } E = \frac{1}{2}mv^2 = [M][L^2T^{-2}]$$

$$\text{Dimensions of kinetic energy} = [ML^2T^{-2}]$$

   The equation is dimensionally consistent.

3. **Density Calculation**

Q: A metal cube has a side length of 4 cm and a mass of 256 grams. Calculate its density in $\mathrm{g/cm^3}$.

A:

$$\text{Volume of the cube} = \text{side}^3 = 4^3 = 64\,\mathrm{cm}^3$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{256\,\mathrm{g}}{64\,\mathrm{cm}^3} = 4\,\mathrm{g/cm}^3$$

4. **Error Calculation**

Q: If the measured length of a rod is 2.50 m, 2.52 m, and 2.48 m, calculate the mean absolute error.

A:

$$\text{Mean length} = \frac{2.50 + 2.52 + 2.48}{3} = 2.50\,\mathrm{m}$$

$$\text{Absolute errors} = |2.50 - 2.50|, |2.52 - 2.50|, |2.48 - 2.50|$$

$$= 0, 0.02, 0.02$$

$$\text{Mean absolute error} = \frac{0 + 0.02 + 0.02}{3} = 0.013\,\mathrm{m}$$

5. **Significant Figures**

   Q: Round off the number 0.006789 to three significant figures.

   A:

$$0.006789 \text{ rounded to three significant figures is } 0.00679$$

6. **Speed Calculation**

   Q: A car travels a distance of 150 km in 2.5 hours. Calculate its average speed in $\mathrm{m/s}$.

   A:

$$\text{Speed in km/h} = \frac{150\,\mathrm{km}}{2.5\,\mathrm{h}} = 60\,\mathrm{km/h}$$

$$\text{Speed in m/s} = 60 \times \frac{5}{18} = 16.67\,\mathrm{m/s}$$

7. **Force Calculation**

   Q: Calculate the force exerted by a 5 kg object accelerating at $2\,\mathrm{m/s^2}$.

   A:

$$\text{Force} = \text{mass} \times \text{acceleration} = 5\,\mathrm{kg} \times 2\,\mathrm{m/s^2} = 10\,\mathrm{N}$$

8. **Volume Conversion**

   Q: Convert 2 liters to cubic meters.

   A:

$$1\,\text{liter} = 0.001\,\text{m}^3$$

$$2\,\text{liters} = 2 \times 0.001\,\text{m}^3 = 0.002\,\text{m}^3$$

9. **Uncertainty Calculation**

   Q: A measurement is recorded as 5.45 ± 0.05 m. Calculate the relative uncertainty.

   A:

$$\text{Relative uncertainty} = \frac{\text{absolute uncertainty}}{\text{measured value}} \times 100\%$$

$$= \frac{0.05}{5.45} \times 100\% = 0.917\%$$

10. **Pressure Calculation**

    Q: Calculate the pressure exerted by a force of 500 N on an area of $0.5\,\text{m}^2$.

    A:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{500\,\text{N}}{0.5\,\text{m}^2} = 1000\,\text{Pa}$$

1. Systematic Error

Definition: Systematic errors are consistent, repeatable errors associated with faulty equipment or biased techniques. They lead to a measurement being consistently off in the same direction.

Example Numerical:

Q: A thermometer always reads 0.5°C higher than the actual temperature. If it shows a reading of 37.5°C, what is the actual temperature?

A:

$$\text{Actual temperature} = \text{Measured temperature} - \text{Systematic error}$$

$$= 37.5^{\circ}C - 0.5^{\circ}C = 37.0^{\circ}C$$

2. Random Error

Definition: Random errors are unpredictable variations that arise due to unpredictable factors affecting the measurement process. They can be reduced by taking multiple measurements and averaging them.

Example Numerical:

Q: A student measures the length of a rod five times as 20.1 cm, 20.2 cm, 20.0 cm, 20.1 cm, and 20.3 cm. Calculate the mean length and the standard deviation.

A:

$$\text{Mean length} = \frac{20.1 + 20.2 + 20.0 + 20.1 + 20.3}{5} = 20.14\,\text{cm}$$

$$\text{Deviation} = \sqrt{\frac{(20.1 - 20.14)^2 + (20.2 - 20.14)^2 + (20.0 - 20.14)^2 + (20.1 - 20.14)^2 +}{5}}$$

$$= \sqrt{\frac{0.0016 + 0.0036 + 0.0196 + 0.0016 + 0.0256}{5}}$$

$$= \sqrt{0.0104} = 0.102\,\text{cm}$$

3. Absolute Error

Definition: Absolute error is the difference between the measured value and the true value. It is a measure of the magnitude of the error without regard to direction.

Example Numerical:

Q: The true length of a rod is 50.0 cm. If the measured length is 49.5 cm, calculate the absolute error.

A:

$$\text{Absolute error} = |\text{Measured value} - \text{True value}|$$

$$= |49.5\,\text{cm} - 50.0\,\text{cm}| = 0.5\,\text{cm}$$

4. Relative Error

Definition: Relative error is the ratio of the absolute error to the true value, often expressed as a percentage.

Example Numerical:

Q: The true length of a rod is 50.0 cm, and the measured length is 49.5 cm. Calculate the relative error.

A:

$$\text{Relative error} = \frac{\text{Absolute error}}{\text{True value}} \times 100\%$$

$$= \frac{0.5\,\text{cm}}{50.0\,\text{cm}} \times 100\% = 1.0\%$$

5. Percentage Error

Definition: Percentage error is the relative error expressed as a percentage.

Example Numerical:

Q: If the absolute error in measuring the length of a rod is 0.5 cm and the true length is 50.0 cm, calculate the percentage error.

A:

$$\text{Percentage error} = \frac{\text{Absolute error}}{\text{True value}} \times 100\%$$

$$= \frac{0.5\,\text{cm}}{50.0\,\text{cm}} \times 100\% = 1.0\%$$

6. Mean Absolute Error

Definition: Mean absolute error (MAE) is the average of the absolute errors of a set of measurements.

Example Numerical:

Q: The measured lengths of a rod are 50.1 cm, 49.9 cm, and 50.2 cm. Calculate the mean absolute error if the true length is 50.0 cm.

A:

$$\text{Absolute errors} = |50.1 - 50.0|, |49.9 - 50.0|, |50.2 - 50.0|$$

$$= 0.1\,\text{cm}, 0.1\,\text{cm}, 0.2\,\text{cm}$$

$$\text{Mean absolute error} = \frac{0.1 + 0.1 + 0.2}{3} = \frac{0.4}{3} = 0.133\,\text{cm}$$

7. Standard Deviation

Definition: Standard deviation is a measure of the amount of variation or dispersion in a set of values.

Example Numerical:

Q: The measurements of a rod are 50.1 cm, 50.3 cm, 50.2 cm, 50.4 cm, and 50.0 cm. Calculate the standard deviation.

A:

$$\text{Mean} = \frac{50.1 + 50.3 + 50.2 + 50.4 + 50.0}{5} = 50.2\,\text{cm}$$

$$\text{Variance} = \frac{(50.1 - 50.2)^2 + (50.3 - 50.2)^2 + (50.2 - 50.2)^2 + (50.4 - 50.2)^2 + (50.0 -}{5}$$

$$= \frac{0.01 + 0.01 + 0 + 0.04 + 0.04}{5} = \frac{0.1}{5} = 0.02$$

$$\text{Standard deviation} = \sqrt{0.02} \approx 0.141\,\text{cm}$$

8. Instrumental Error

Definition: Instrumental errors are caused by imperfections in the measuring instrument or its misuse by the operator.

Example Numerical:

Q: A balance is known to have a zero error of -0.2 g. If it measures the mass of an object as 100.0 g, what is the corrected mass?

A:

$$\text{Corrected mass} = \text{Measured mass} - \text{Instrumental error}$$

$$= 100.0\,\text{g} - (-0.2\,\text{g}) = 100.2\,\text{g}$$

9. Parallax Error

Definition: Parallax errors occur when the measurement is not taken from the correct angle, leading to inaccurate readings.

Example Numerical:

Q: If a student reads the level of mercury in a thermometer from an angle and records 25.3°C instead of the true value of 25.0°C, what is the parallax error?

A:

$$\text{Parallax error} = \text{Measured value} - \text{True value}$$

$$= 25.3^{\circ}C - 25.0^{\circ}C = 0.3^{\circ}C$$

10. Zero Error

Definition: Zero error is a type of systematic error where the measuring instrument does not start from exactly zero.

Example Numerical:

Q: A vernier caliper