CS342 Assignment 4 – Group 19

Introduction

The study of waiting lines, called queuing theory, is one of the oldest and most widely used quantitative analysis techniques. Waiting lines are an everyday occurrence, affecting people shopping for groceries buying gasoline, making a bank deposit, or waiting on the telephone for the first available airline reservationists to answer. Queues, another term for waiting lines, may also take the form of machines waiting to be repaired, trucks in line to be unloaded, or airplanes lined up on a runway waiting for permission to take off. The three basic components of a queuing process are arrivals, service facilities, and the actual waiting line.

Airport security lines play a critical role in ensuring passenger safety and satisfaction. This report presents a simulation model designed to replicate and improve security screening processes in a busy airport environment. The simulation considers various scenarios, including arrival rate, service rate, queue length optimization, multi-server expansion, and buffered multi-server models.

Characteristics of Queueing System

We can segregate the Queueing System into three parts:

- *Calling Population* - Arrival or inputs to the system
- *Waiting Line* - Queue or Buffer
- *Service Facility* – Server

Arrival Characteristics

The characteristics of arrivals or passengers in a queuing system are crucial for understanding and modelling the system effectively. There are three key aspects to consider: the size of the calling population, the pattern of arrivals, and the behaviour of the arrivals.

- *Size of the Calling Population:*
  The calling population, which refers to the total number of potential arrivals or passengers, can be categorized as either unlimited (essentially infinite) or limited (finite). In cases where the number of passengers present at any given moment is a small fraction of the total potential arrivals, the calling population is considered unlimited. For instance, at an airport check-in counter, there is a continuous flow of passengers, and the relationship between the queue length and arrival rate remains independent, assuming an essentially infinite calling population. Most queuing models assume such infinite calling populations to simplify calculations. However, when the calling population is limited, such as in a shop with a fixed number of machines, the modelling becomes more complex as the arrivals are not infinite and can be influenced by the current population

- *Pattern of Arrivals at the System:*
  Passengers can arrive at a service facility based on a predetermined schedule. Random arrivals are events that occur independently of one another, and their exact prediction is not possible. In many queuing problems, the number of arrivals per unit of time can be estimated using a probability distribution called the Poisson distribution. This distribution helps calculate the probability of a specific number of arrivals occurring in given a time period, assuming the arrivals are random and independent. The Poisson distribution formula is used to establish the probability of a certain number of arrivals (n) happening within a specific time interval (t) based on the arrival rate (λ).

- *Behaviour of the Arrival:*
  In most queuing models, it is assumed that arriving passengers exhibit patient behaviour, meaning they wait in the queue until they are served without switching between lines. However, real-life situations are more complex due to the phenomenon of balking and reneging.

Waiting Line Characteristics

The waiting line is a crucial component of a queuing system. Its characteristics include the potential for either limited or unlimited length. A queue is considered limited when it cannot, due to physical or legal constraints, extend infinitely. Analytical queuing models typically assume an unlimited queue length. In contrast, an unlimited queue has no restrictions on its size, such as a tollbooth serving incoming automobiles.

Another important aspect of waiting lines is queue discipline, which defines the rule determining the order in which passengers or customers are served. Most systems follow the **first-in, first-out rule (FIFO),** where the individual who has been waiting the longest is served first.

Service Facility Characteristics

Service systems are usually classified in terms of their number of channels, or number of servers, and number of phases, or number of service stops, that must be made. Two basic properties are:

- The configuration of the service system
- The pattern of service times

Queueing Models

The M/M/1 (Markovian single-server) and M/M/m (Markovian multi-server) models are commonly used to analyse queues with single and multiple servers, respectively. For the assignment given, we are also analysing cases of infinite and finite buffers.

Performance Metrics

Key performance metrics include server utilization, average queue length, and average waiting time in the queue. These metrics are essential for assessing system efficiency and customer satisfaction.

M/M/1 model

Theoretical Results:

- $\text{Average Waiting Time} = \frac{\lambda}{\mu(\mu-\lambda)}$
- $\text{Average Queue Length} = \frac{\lambda^2}{\mu(\mu-\lambda)}$
- $\text{System Utilisation} = \frac{\lambda}{\mu}$

λ – arrival rate

μ – service rate

Note that the above are the results from Little's Law, and are therefore valid only for single server system with infinite queue.

For infinite buffers:

Arrival Rate	Service Rate	Average Waiting Time (Theoretical)	Average Waiting Time (Simulation)	Average Queue Length (Theoretical)	Average Queue Length (Simulation)	System Utilization (Theoretical)	System Utilization (Simulation)
3	5	0.3	0.328526	0.9	1.002	0.6	0.616171
4	6	0.3333	0.189719	1.3333	0.774684	0.6667	0.647731
10	50	0.0050	0.0052	0.0500	0.0533	0.2000	0.1939

```
Waiting time for passenger 183 : 1.31048

Passenger Number : 190
Arrival Time : 0.0268973, Service Time : 0.0543693

Passenger Number : 191
Arrival Time : 0.0385129, Service Time : 0.107232

Passenger Number : 192
Arrival Time : 0.198939, Service Time : 0.15147

Average waiting time (theoretical) : 0.3 seconds.
Average waiting time (simulation) : 0.328526 seconds.
Average queue length (theoretical) : 0.9
Average queue length (simulation) : 1.002
Service Utilization (theoretical) : 0.6
Service Utilization (simulation) : 0.616171
```

```
Start Processing Time for passenger 244 : 59.2952
Waiting time for passenger 244 : 0.0925382

Passenger Number : 246
Arrival Time : 0.516936, Service Time : 0.195796
Start Processing Time for passenger 245 : 59.619
Waiting time for passenger 245 : 0.312659

Passenger Number : 247
Arrival Time : 0.270815, Service Time : 0.585804

Average waiting time (theoretical) : 0.333333 seconds.
Average waiting time (simulation) : 0.189719 seconds.
Average queue length (theoretical) : 1.33333
Average queue length (simulation) : 0.774684
Service Utilization (theoretical) : 0.666667
Service Utilization (simulation) : 0.647731
```

For finite buffers:

Arrival Rate	Service Rate	Buffer Size	Average Waiting Time (Simulation)	Average Queue Length (Simulation)	System Utilization (Simulation)
3	5	5	0.0106992	0.0353074	0.0333333
5	6	10	0.0129655	0.0680689	0.0333333

```
Arrival Time : 0.219379, Service Time : 0.418653
Start Processing Time for passenger : 59.7478
Waiting time for passenger : 0

Passenger Number : 198
Arrival Time : 0.0612439, Service Time : 0.242748
Start Processing Time for passenger : 59.809
Waiting time for passenger : 0

Passenger Number : 199
Arrival Time : 0.676128, Service Time : 0.552192

Average waiting time (simulation) : 0.0106992
Average queue length (simulation) : 0.0353074
Service Utilization (simulation) : 0.0333333
```

```
Arrival Time : 0.0649845, Service Time : 0.557993
Start Processing Time for passenger : 59.5939
Waiting time for passenger : 0

Passenger Number : 315
Arrival Time : 0.156828, Service Time : 0.172666
Start Processing Time for passenger : 59.7508
Waiting time for passenger : 0

Passenger Number : 316
Arrival Time : 0.346265, Service Time : 0.154454

Average waiting time (simulation) : 0.0129655
Average queue length (simulation) : 0.0680689
Service Utilization (simulation) : 0.0333333
```

M/M/m model

Multi server queue model instead of single server.

For infinite buffers:

Arrival Rate	Service Rate	Number of servers	Average Waiting Time (Simulation)	Average Queue Length (Simulation)	System Utilization (Simulation)
3	5	10	0.00794508	0.0256891	0.0333333
5	6	5	0.00756004	0.0386822	0.05

```
Waiting time for passenger : 0

Passenger Number : 194
Arrival Time : 0.208258, Service Time : 0.0468754
Start Processing Time for passenger : 59.3469
Waiting time for passenger : 0

Passenger Number : 195
Arrival Time : 0.510433, Service Time : 0.0184564
Start Processing Time for passenger : 59.5551
Waiting time for passenger : 0

Average waiting time (simulation) : 0.00794508
Average queue length (simulation) : 0.0256891
Service Utilization (simulation) : 0.0333333
tanisha@tanisha-HP-EliteDesk-800-G2-TWR:~/Documents/cs342/week4$
```

```
Waiting time for passenger : 0

Passenger Number : 307
Arrival Time : 0.466899, Service Time : 0.0723766
Start Processing Time for passenger : 59.1987
Waiting time for passenger : 0

Passenger Number : 308
Arrival Time : 0.400829, Service Time : 0.347993
Start Processing Time for passenger : 59.6656
Waiting time for passenger : 0

Average waiting time (simulation) : 0.00756004
Average queue length (simulation) : 0.0386822
Service Utilization (simulation) : 0.05
```

For finite buffers:

Arrival Rate	Service Rate	Number of servers	Buffer Size	Average Waiting Time (Simulation)	Average Queue Length (Simulation)	System Utilization (Simulation)
3	5	10	5	0.00865854	0.0240996	0.03333
3	6	5	5	0.00461584	0.0141552	0.0141552

```
Waiting time for passenger : 0
Start Processing Time for passenger : 59.5728
Waiting time for passenger : 0

Passenger Number : 167
Arrival Time : 0.215915, Service Time : 0.149306

Passenger Number : 168
Arrival Time : 0.863176, Service Time : 0.30053
Start Processing Time for passenger : 59.7887
Waiting time for passenger : 0

Average waiting time (simulation) : 0.00865854
Average queue length (simulation) : 0.0240996
Service Utilization (simulation) : 0.0333333
```

```
Waiting time for passenger : 0.849315

Passenger Number : 184
Arrival Time : 0.322358, Service Time : 0.170752
Start Processing Time for passenger : 59.6461
Waiting time for passenger : 0

Passenger Number : 185
Arrival Time : 0.159846, Service Time : 0.170392
Start Processing Time for passenger : 59.9684
Waiting time for passenger : 0

Average waiting time (simulation) : 0.00461584
Average queue length (simulation) : 0.0141552
Service Utilization (simulation) : 0.0166667
```

Key Observations

- For single server system, as we migrate from infinite to finite buffers, a significant decrease is observed in the value of each of the parameters – server utilization, average waiting time and average queue length. This can be understood by the fact, that **more packets will now be dropped** due to limited buffer size, thereby, reducing the waiting time and the time for which server is busy.

- Comparing the single M/M/1 vs M/M/m queue system. It is obvious that the system utilization will **decrease by a factor of m**, where m is the number of servers. The other two parameters, average waiting time and queue length are also observed to decrease. This can be interpreted due to the decrease of arrival rate by a factor of m (among m servers).

- Further modifying the M/M/m queue by limiting the buffer size (M/M/m/K queue), similar observations can be made as those for the respective single server system. Similar factors contribute here as well.

Conclusion

Comprehending performance metrics for both single-queue and multi-queue systems is essential for optimizing system efficiency and allocating resources effectively. These metrics are pivotal in delivering reliable services, reducing waiting periods, and enhancing customer contentment. Through the analysis and application of suitable queueing models, businesses can enhance operational effectiveness and the quality of service they provide.