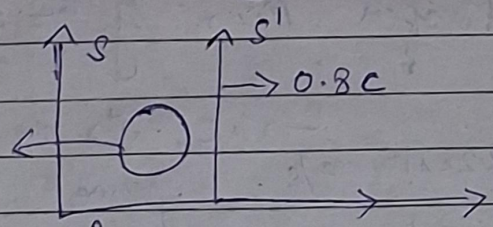
Shivam Kumar
2311166

S.T.R Assignment 1

(1.)



In S-frame:

$A = 10\ cm^2 = \pi R^2$

& $R = \sqrt{(10/\pi)}$

There is no relative motion along y-axis, so only x-coordinates of the circle will change.

So, in S'-frame:

using Length Contraction

$$R'_x = R_x \sqrt{1 - v^2/c^2}$$

$$\Rightarrow R'_x = \sqrt{10/\pi}\sqrt{1 - 0.64} \quad \left(\because \text{Rest length} = \frac{\text{moving length}}{\sqrt{1 - v^2/c^2}}\right)$$

$$\boxed{R'_x = \sqrt{\frac{10}{\pi}} \times 0.6}$$

& $$\boxed{R'_y = R_y = \sqrt{\frac{10}{\pi}}}$$

So, from S', the circle looks like an $\boxed{\text{ellipse}}$ with area:

$$A' = \pi R'_x R'_y = \pi \times \sqrt{\frac{10}{\pi}} \times 0.6 \times \sqrt{\frac{10}{\pi}}$$

$$= \cancel{\pi} \times \frac{10}{\cancel{\pi}} \times 0.6$$

$$\boxed{A' = 6\ cm^2}$$

(2.) ∴ Proper mean life (t_0) = 2.5×10^{-8} s

↳ i.e its the mean life in the particles frame

$V_{wrt\ lab} = 2.22 \times 10^{10}$ cm/s

To get mean life in lab's frame, we use time dialation:

$$\underset{\text{(in Lab's frame)}}{\text{Improper time interval}} (t) = \frac{\text{Proper Time interval}}{\sqrt{1 - v^2/c^2}}$$

$$\Downarrow$$

$$t = \frac{t_0}{\sqrt{1 - \frac{(2.22 \times 10^{10}\ cm/s)^2}{(3 \times 10^{10}\ cm/s)^2}}}$$

$$= \frac{2.5 \times 10^{-8}\ s}{\sqrt{1 - \left(\frac{2.22}{3}\right)^2}}$$

$$\boxed{t = 3.747 \times 10^{-8}\ s}$$

So,

Distance travelled in mean life $= V_{wrt\ lab} \times t$

$$= 2.22 \times 10^{10}\ cm/s \times 3.747 \times 10^{-8}\ s$$

$$= 8.25174 \times 10^{2}\ cm$$

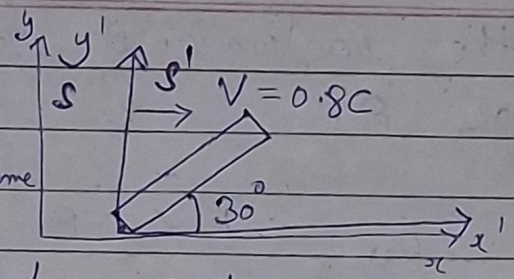
$$= 8.25174\ m$$

Ans

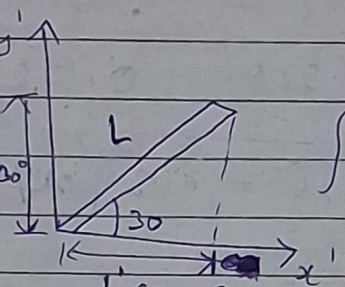
(3.)

$\boxed{L' = 1\,m}$

↳ in S'-frame



Here, we have 2 frames S-frame & rod's frame (S'-frame)



$L'_y = \frac{L'}{2} = L' \sin 30°$

$\Rightarrow L'_x = \frac{\sqrt{3}L}{2}$

There will be Length contraction only along x-axis, cause relative motion is along x-dir^n.

So;

$$\text{Rest length (S'-frame)} = \frac{\text{Moving length } (L_x)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L'_x = \frac{L_x}{\sqrt{1 - 0.64}} = \cancel{\frac{\sqrt{3}\,L}{2 \times 0.6}}$$

$\therefore L_x = L'_x (0.6)$

$\boxed{L_x = \frac{\sqrt{3}}{2}(0.6)L' = \sqrt{3} \times (0.3) L'}$ & $\boxed{L'_y = L_y}$

So;

$$L = \sqrt{(L_y)^2 + \underbrace{\left(\frac{\sqrt{3}(0.6)L}{2}\right)^2}_{(L_x)^2}} = \cancel{\sqrt{\frac{L^2}{4} + 3L^2 + 1.44}}$$

$$= \sqrt{\frac{(L')^2}{4} + (3 \times 0.09)(L')^2}$$

$$L = L'\sqrt{\frac{1}{4} + 0.27} = \sqrt{0.52} = 0.72 \times 1\,m$$

$$\boxed{L = 72\,cm}$$

Let the inclination of rod from S-frame be θ.

$$\therefore \quad L_x = L\cos\theta$$

$$\sqrt{3}\times 0.3\times 1 = 0.72\cos\theta$$

$$\Rightarrow \boxed{\cos\theta = 0.722}$$

$$\boxed{\theta = \cos^{-1}(0.722) = (43.8059)^\circ}$$

Ans

---

(4)

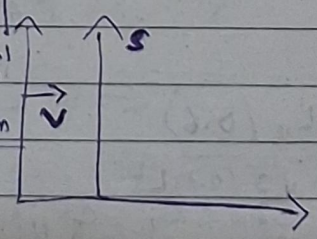
In S-frame:

$t = 2\times 10^{-4}\ s$

$x = 20\ km$

In S'-frame:

$x' = 30\ km$



$\because$ By Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$30 = \frac{20 - v(2\times 10^{-4})}{\sqrt{1 - v^2/c^2}} \Rightarrow \sqrt{1 - \frac{v^2}{c^2}} = \frac{20 - v(2\times 10^{-4})}{30}$$

$$\Rightarrow 1 - \frac{v^2}{c^2} = \frac{[400 + v^2(4\times 10^{-8}) - 80\times 10^{-4}\times v]}{900}$$

$$\Rightarrow 900 - \frac{900V^2}{c^2} = 400 + V^2(4\times10^{-8}) - 80\times10^{-4}V$$

$$\Rightarrow \boxed{900 + (4\times10^{-8})c^2}\,V^2 - (80\times10^{-4})V - 500c^2 = 0$$

On solving:

$$\boxed{V = -0.16c}$$

$$t' = \frac{t - \frac{Vx}{c^2}}{\sqrt{1 - V^2/c^2}} = \frac{(2\times10^{-4}) + \frac{(0.16c\times20)}{c\times c}}{\sqrt{1 - 0.0256}}$$

$$\boxed{t' = 2.1341\times10^{-4}\,s}$$

Q.5. $L_0 = 120\ m$

$t = 4.5\ \mu s$

observed length $\Rightarrow L = L_0/\gamma$

$t = L/v \Rightarrow vt = L_0/\gamma$

$= L_0\sqrt{1 - v^2/c^2}$

$\Rightarrow v^2 t^2 = L_0^2 (1 - v^2/c^2)$

$\Rightarrow v^2 \left(t^2 + \frac{L_0^2}{c^2}\right) = L_0^2$

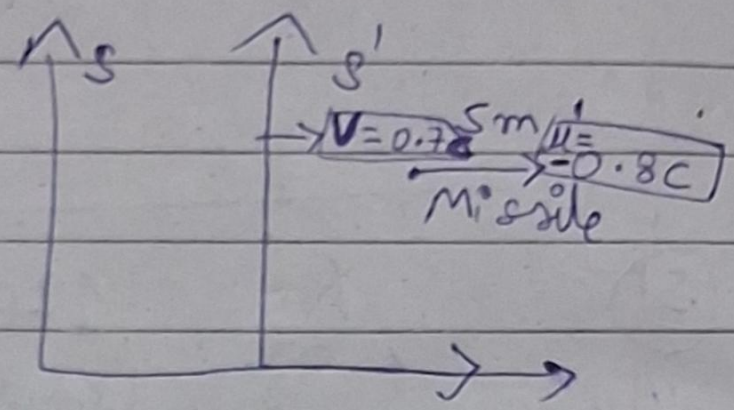
$\Rightarrow v = \frac{L_0}{\sqrt{t^2 + \frac{L_0^2}{c^2}}} = \frac{120}{\sqrt{(4.5\times10^{-6})^2 + \left(\frac{120}{c}\right)^2}}$

$= 2.656 \times 10^7\ m/s$

(6)

$V = 0.7c$

S is Earth's frame
& S' is Rocket's frame



$$\boxed{u'_x = -0.8c}$$

→ missile's velocity wrt rocket

$$u_x = \frac{u'_x + V}{1 + \frac{u'_x V}{c^2}} = \frac{-0.8c + 0.7c}{1 - \frac{0.8\cancel{c} \times 0.7\cancel{c}}{\cancel{c^2}}}$$

$$= \frac{-0.1c}{1 - 0.56} = \frac{-0.1c}{0.44}$$

$$\boxed{u_x = -0.227c}$$

→ velocity of missile wrt Earth

Now; by Length contraction:

Rest length $= \frac{M \cdot L}{\sqrt{1 - V^2/c^2}}$ (Here $V = u_x$)

⇓

$$S_m = \frac{L}{\sqrt{1 - V^2/c^2}}$$

$$\Rightarrow \quad L = 5 \times \sqrt{1-(0.227)^2}$$

$$= 5 \times 0.9738$$

$$\boxed{L = 4.8691 \text{ m}}$$

(7.) Acc. to rocket's clock missile is launched 5 seconds after the rocket launched.

$\Rightarrow$ 5 sec. in rocket in earth's frame $= 5 \times \frac{1}{\sqrt{1-0.49}} = 7.0014$ sec

In 7 sec the rocket has travelled dist. away from earth (d) acc. to earth's frame;

$$d = 0.7c \times 7 = (4.9c) \text{ m}$$

Then the missile is launched.

(acc. to earth's clock)

Now, time taken by missile to cover $(4.9c)$ m is

$$\Rightarrow \quad t = \frac{4.9c}{0.227c} = 21.5859 \text{ sec.}$$

$\therefore$ Total time to reach earth wrt earth $= 7 + 21.58$

$= 28.58$ sec

Ans

(8.) In S-frame:

E_1: $x_1 = x_0$, $t_1 = x_0/c$

E_2: $x_2 = 2x_0$, $t = x_0/2c$

So, in S' - frame :

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$\cancel{x'_1 = x_1 - Vt_1}$$

$$t'_1 = \left(t_1 - \frac{Vx_1}{c^2}\right)\gamma$$

$$t'_2 = \left(t_2 - \frac{Vx_2}{c^2}\right)\gamma$$

A/q:

$$t'_1 = t'_2 \Rightarrow \left(t_1 - \frac{Vx_1}{c^2}\right)\cancel{\gamma} = \left(t_2 - \frac{Vx_2}{c^2}\right)\cancel{\gamma}$$

$$\Rightarrow \frac{x_0}{c} - \frac{V(x_0)}{c^2} = \frac{x_0}{2c} - \frac{V(2x_0)}{c^2}$$

$$\Rightarrow \frac{V\cancel{x_0}}{\cancel{c^2}} = -\frac{\cancel{x_0}}{2\cancel{c}}$$

$$\boxed{V = -\frac{c}{2}}$$

&

So, $t'_1 = t'_2 = \left(t_1 - \frac{Vx_1}{c^2}\right)\gamma$

$$= \left(\frac{x_0}{c} + \frac{\cancel{c}x_0}{2\cancel{c^2}_{c}}\right)\left(1 - 0.25\right)^{-1/2}$$

$$t'_2 = t'_1 = \frac{\left(\frac{3x_0}{2c}\right)}{\sqrt{\frac{3}{4}}} = \frac{\sqrt{3}\,x_0}{c}$$

Ans

(9) $t_1' = 0$, $t_2' = \Delta t'$

$(\because \Delta t' = 1 \text{ hour})$

$$\Delta t_{earth}(\text{received}) = \Delta t' \left(\sqrt{\frac{1+R}{1-R}}\right)$$

$t' = 1 \text{ hr}$

$$t = \sqrt{\frac{1+R}{1-R}} \text{ hr}$$

$$\therefore \boxed{t = \left(\sqrt{\frac{1+R}{1-R}}\right) \text{ hours}}$$

(10.) On Earth ⇒ the bacteria doubles every 24 hr.
In 8 doubling ⇒ 256 bacteria

$$t_{earth} = 8 \times 24 = 192 \text{ hr.}$$

for rocket :

$R = 0.897$

$$\gamma = \frac{1}{\sqrt{1-0.897}} = 2.26$$

⇒ doubling period in Earth frame for rocket culture
⇒ $\gamma \times 24$ hr

$t = 192$ hr is $N = \frac{192}{\gamma \times 24} = \frac{8}{\gamma} = 3.54$

population in rocket $= (2)^{8/\gamma} = 2^{(3.54)}$

≈ 11.8 Bacteria

$\Rightarrow$ If discrete doubling occur:

$2^3 = 8$ bacteria

$\Rightarrow$ 12 $\rightarrow$ according to continuous model

& 8 $\rightarrow$ according to integer doubling

(11.) $t' = \gamma\left(t - \frac{Vx}{c^2}\right)$

$t = \gamma\left(t' + \frac{Vx'}{c^2}\right)$

we set, $t' = t$

$(\gamma - 1)t = \frac{\gamma V x}{c^2} \Rightarrow x = \frac{(\gamma-1)}{\gamma}\frac{c^2}{V}t$

$\Rightarrow (\gamma - 1)t = -\frac{\gamma V x'}{c^2} \Rightarrow x' = -\frac{(\gamma-1)}{\gamma}\frac{c^2}{V}t$

$\Rightarrow \quad x' = -x \quad \& \quad \boxed{t = \frac{\gamma}{(\gamma-1)} \frac{V}{c^2} x}$

$t = t'$, $x' = -x$, besides $(x = x' = 0)$

(12.) In S-frame :

Red : $x_R = 3.00\,m$, $t_R = 1 \times 10^{-9}\,s$

Blue : $x_B = 4.80\,m$, $t_B = 8.94 \times 10^{-9}\,s$

$x'_R = x'_B$

(a) Speed :

$x' = \gamma (x - vt)$

$x_R - v t_R = x_B - V t_B$

$$V = \frac{x_B - x_R}{t_B - t_R} = \frac{1.80}{7.94 \times 10^{-9}} = 2.267 \times 10^{8}\,m/s$$

(b.) Location in S'

$$\beta = 0.756 \Rightarrow \gamma = \frac{1}{\sqrt{1-(0.756)^2}} = 1.528$$

for Red,

$$x' = \gamma (x_R - V t_R) = 1.528 \left(3 - (2.267 \times 10^{8})(1 \times 10^{-9})\right)$$

$$= 1.528 \times 2.773$$

$$= 4.24\,m$$

Same for blue

(c) Time of red flash in S' :

$$t'_R = \gamma\left(t - \frac{V x_R}{c^2}\right)$$

$$t'_R = 1.528\,(1 - 7.56) \times 10^{-9}$$

$$\boxed{t'_R = \left|-1.00 \times 10^{-8}\right| \text{ sec}}$$