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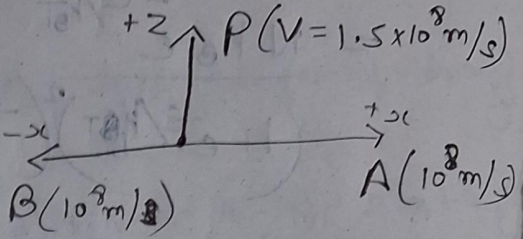
Assignment : 2

(1.) Spaceship P moves away from Earth at speed,

$V = 1.5\times10^8 m/s$

i.e along $\boxed{z\text{-axis}}$



Ships A & B are moving in opp. dir^n. wrt P at equal speeds as $\boxed{u' = 1.0\times10^8 m/s}$

⇒ When frame S' moves relative to frame S with speed V along z-axis :

⇒ obj. has velocity $u'(u'_x, u'_y, u'_z)$ in S', then in S frame :

$$u_z = \frac{u'_z + V}{1 + \frac{Vu'_z}{c^2}}, \quad u_x = \frac{u'_x}{\gamma\left(1+\frac{Vu'_z}{c^2}\right)} \quad \& \quad u_y = \frac{u'_y}{\gamma\left(1+\frac{Vu'_z}{c^2}\right)}$$

⇒ Ship A : $u'_x = +u'$, $u'_y = u'_z = 0$

⇒ Ship B : $u'_x = -u'$, $u'_y = u'_z = 0$

$\boxed{\text{P's frame is S' \& Earth's frame is S}}$

i.e Both A & B have purely transverse to V.

For both A & B :

$$u_z = \frac{u'_z + V}{1 + \frac{Vu'_z}{c^2}} = \frac{0+V}{1+0} = V$$

$\boxed{u_z = V}$

& $\boxed{u_y = 0}$

$$(u_x)_A = \frac{+u'}{\gamma(1+0)} = \frac{+u'}{\gamma}$$

$$(u_x)_B = \frac{-u'}{\gamma}$$

$$\Rightarrow \boxed{\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} = \frac{1}{\sqrt{1-(0.5)^2}} \simeq 1.15}$$

$\therefore (u_x)_A = \frac{+u'}{\gamma} = \frac{10^8 m/s}{1.15} = +8.66 \times 10^7 m/s$

$(u_x)_B = \frac{-u'}{\gamma} = \frac{-10^8 m/s}{1.15} = -8.66 \times 10^7 m/s$

& $(u_z)_A = (u_z)_B = 1.5 \times 10^8 m/s$

$u_A(u_x, u_y, u_z) = \sqrt{(u_x)^2 + (u_y)^2 + (u_z)^2}$ → wrt Earth

$= \sqrt{(8.66\times10^7)^2 + (0)^2 + (1.5\times10^8)^2}$

$= 1.732 \times 10^8 m/s$

& $\boxed{u_B = 1.732\times10^8 m/s}$ → wrt Earth

So, both ships have exactly the same speed, only their dir^n.s differ, one goes diagonally forward - north & other diagonally forward - south, & the earth observer measures both speeds to be:

$\boxed{|\vec{u}| = 1.732 \times 10^8 m/s}$ Ans

(2.) $E_T = 1.2 MeV$, $m_e = 9.1 \times 10^{-31} kg$

Rest mass energy $= m_e c^2 = 0.511 MeV$

(a) As: $K = E_T - m_e c^2 = (1.2 - 0.511) MeV = \boxed{0.689 MeV}$

Ans

(b.) Since: $E_T = \gamma m_0 c^2 = \gamma m_e c^2$

$\Rightarrow \gamma = \frac{E_T}{m_e c^2} = \frac{1.2 MeV}{0.511 MeV} = 2.35$

$\Rightarrow (1 - (\beta)^2)^{-1/2} = 2.35$

$\Rightarrow \beta = \sqrt{1 - \frac{1}{(2.35)^2}} = 0.905$

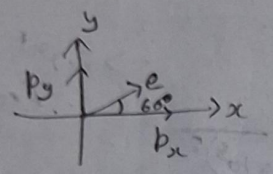
$\Rightarrow \beta = \frac{v}{c} = 0.905$ $\Rightarrow$ So, $\boxed{V_e = 0.905c}$ Ans

(c) $\because E_T^2 = (pc)^2 + (m_0c^2)^2$

$\Rightarrow (pc)^2 = (E_T)^2 - (m_0c^2)^2$

$$|\vec{p}| = \frac{\sqrt{(E_T)^2 - (m_0c^2)^2}}{c} = \frac{\sqrt{(1.2)^2 - (0.511)^2}}{c} \text{ MeV}$$

$$\boxed{|\vec{p}| = 1.086 \text{ MeV}/c}$$



$$|\vec{p}| = \frac{1.086 \times 10^6 \times 1.602 \times 10^{-19}}{3 \times 10^8} \text{ kg m/s}$$

$$\boxed{|\vec{p}|_e = 5.79 \times 10^{-22} \text{ kg m/s}}$$

$|\vec{p}_x| = |\vec{p}| \cos 60°$

$= (1.086) \times \frac{1}{2}$

$$\boxed{|\vec{p}_x|_e = 0.543 (\text{MeV}/c)}$$

& $|\vec{p}_y| = |\vec{p}| \sin 60° = 1.086 \times \frac{\sqrt{3}}{2}$

$$\boxed{|\vec{p}_y|_e = 0.9405 \text{ MeV}/c}$$

(d.) Velocity 4-Vector :

$\boxed{\gamma = 2.35}$

$\eta^0 = c\gamma = 2.35c$

$\eta^1 = \eta^x = \gamma V \cos 60° = (2.35 \times 0.905 \times \frac{1}{2})c$

$= \gamma V_x$

$$\boxed{\eta^1 = 1.063375 c}$$

$\eta^2 = \eta^y = \gamma V \sin 60° = (2.35 \times 0.905 \times \frac{\sqrt{3}}{2})c$

$= \gamma V_y$

$$\boxed{\eta^2 = \eta^y = 1.8418 c}$$

$$\boxed{\eta^3 = \eta^z = \gamma V_z = (2.35 \times 0) = 0}$$

4-momentum:

(e) $p^0 = m_0 c\gamma = 2.35\, c\, m_e$

$= 6.408 \times 10^{-22}$ kg m/s

$p^1 = m_0 \eta^1 = 1.063\, c\, m_e$

$= 2.899 \times 10^{-22}$ kg m/s

$p^2 = m_0 \eta^2 = 1.8418\, c\, m_e$

$= 5.0217 \times 10^{-22}$ kg m/s

$p^3 = m_0 \eta^3 = 0$

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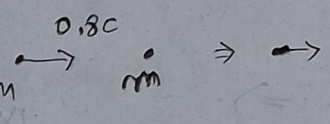
(3) In Lab's frame (where m is at rest):

Before collision:



$\Rightarrow \quad p^{\mu}_{M_{initial}} = (Mc\gamma,\ Mu_x\gamma,\ 0,\ 0)$

$\Rightarrow \quad p^{\mu}_{m_i} = (mc,\ 0,\ 0,\ 0)$

$\left(p^{\mu}_{Total}\right)_{initial} = \left((\gamma M + m)c,\ Mu_x\gamma,\ 0,\ 0\right)$

After collision:

$\left(p^{\mu}_{(M_f)}\right)_{final} = (M_T c\gamma',\ M_f (u_x)_f \gamma',\ 0,\ 0)$

$\Rightarrow \left(p^{\mu}_{T}\right)_{initial} = \left(p^{\mu}_{Total}\right)_{final}$

$\Rightarrow$ so, $\gamma M c + mc = M_f c \gamma'$

$\boxed{\gamma M + m = M_f \gamma'} \rightarrow \times c^2$

$\Rightarrow (\gamma M c^2 + mc^2)^2 = (M_f \gamma' c^2)^2$ (sq. both sides) $\rightarrow$ (i)

& $Mu_x\gamma = M_f (u_x)_f \gamma'$

sq. & $\times c^2$

$\Rightarrow (Mu_x\gamma c)^2 = (M_f (u_x)_f \gamma' c^2)^2$ $\rightarrow$ (ii)

Sub. (ii) from (i) :

$$\Rightarrow \left(M_y c^2 \gamma'\right)^2 - \left(M_y c\, u_y \gamma'\right)^2 = \left(\gamma M c^2 + m c^2\right)^2 - \left(M u_x \gamma c\right)^2$$

$$\Rightarrow \left(M_y c^2 \gamma'\right)^2 \underbrace{\left[1 - \frac{u_y^2}{c^2}\right]}_{} = \left(\gamma M c^2 + m c^2\right)^2 - \left(M u_x \gamma c\right)^2$$

$$\Rightarrow \left(M_y c^2\right)^2 \cancel{(\gamma')^2} \times \frac{1}{\cancel{(\gamma')^2}} = \left(\gamma M c^2 + m c^2\right)^2 - \left(M u_x \gamma c\right)^2$$

$$\Rightarrow M_y^2 c^4 = \gamma^2 M^2 c^4 + m^2 c^4 + 2\gamma M m c^4 - M^2 u_x^2 \gamma^2 c^2$$

$$\Rightarrow M_y^2 = \gamma^2 M^2 + m^2 + 2\gamma M m - M^2 \frac{u_x^2 \gamma^2}{c^2}$$

$$= m^2 + 2\gamma M m + M^2 \cancel{\gamma^2} \underbrace{\cancel{\left(1 - \frac{u_x^2}{c^2}\right)}}_{\frac{1}{\gamma^2}}$$

$$M_y^2 = m^2 + 2\gamma M m + M^2$$

$$M_y = \sqrt{m^2 + M^2 + 2\gamma M m}$$

As, $u_x = 0.8c \Rightarrow \boxed{\gamma = \frac{5}{3}}$

So, $\boxed{M_y = \sqrt{M^2 + m^2 + \frac{10}{3} M m}}$

By using Energy - momentum Conservation :

Initially :

$$E_i = \gamma M c^2 + m c^2 \quad , \quad p_i = \gamma M u_i$$

Finally :

$$E_f = \gamma' M_f c^2 \quad , \quad p_f = \gamma' M_f u_f$$

By using conservation laws :

$$E_i = E_f \quad \& \quad p_i = p_f$$

$$\Rightarrow (E_i)^2 - (p_i c)^2 = (E_f)^2 - (p_f c)^2 = (M_f c^2)^2$$

$$\Rightarrow M_f^2 c^4 = (\gamma M c^2 + m c^2)^2 - (\gamma M u_i c)^2$$

$$\Rightarrow M_f^2 \cancel{c^4} = \cancel{c^4}\left[(\gamma M + m)^2 - \gamma^2 M^2 \frac{u_i^2}{c^2}\right]$$

$$M_f^2 = \gamma^2 M^2 \underbrace{\left(1 - \frac{v^2}{c^2}\right)}_{\to (1)} + 2\gamma M m + m^2$$

$$\boxed{M_f = \sqrt{M^2 + 2\gamma M m + m^2}} \quad \boxed{= \sqrt{M^2 + m^2 + \frac{10}{3} M m}}$$

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(4) For a source moving directly away from the observer the relativistic wavelength shift is :

$$\lambda_{obs.} = \lambda_{emit} \sqrt{\frac{1+\beta}{1-\beta}} \quad , \quad \beta = v/c$$

$$\Rightarrow f_{obs.} = f_{emit.} \sqrt{\frac{1-\beta}{1+\beta}}$$

we have, $\beta = \frac{v}{c} = \frac{0.2\cancel{c}}{\cancel{c}} = 0.2$

& $\sqrt{\frac{1+\beta}{1-\beta}} = \sqrt{\frac{1.2}{0.8}} \simeq 1.225$

So, emitted blue wavelength, $\lambda_{emit} = 475\,nm$ becomes,

$$\lambda_{obs.} = 475 \times 1.225 \simeq 581.3\,nm$$

581.3nm lies in the yellow-orange part of the visible spectrum. So, an Earth observer sees that blue light shifted to yellow-orange light.

(5.) Here, classically, if motion is purely perpendicular to the line of sight there is no Doppler shift.
But, relativistically, time dilation makes the emitter's clock run slower by factor γ. This reduces the emitted freq. by $1/\gamma$. So, even with purely transverse motion we get a shift. ⇒ i.e Transverse Doppler's effect.

⇒ For a photon emitted when the motion is exactly perpendicular:

$$f_{obs.} = \frac{f_0}{\gamma} \Rightarrow \boxed{\lambda_{obs.} = \gamma \lambda_0}$$

$$\Rightarrow \beta = \frac{v}{c} = 0.5, \quad \gamma = \frac{1}{\sqrt{0.75}} \simeq 1.155$$

$$\Rightarrow \lambda_{obs.} = \gamma \lambda_0 = 1.155 \times 6563\,Å \simeq 7578.2\,Å$$

<u>wavelength shift:</u>

$$\Delta\lambda = \lambda_{obs.} - \lambda_0 = (7578.2 - 6563)\,Å$$

$$= 1015.2\,Å.$$

(it is a red shift)

(6.) (a.) The hole's diameter is a $\perp^r$ length relative to the rod's motion. As Lorentz contraction acts only along the dir^n of motion, so transverse lengths ($\perp^r$) are unchanged.

$$\therefore D_{\text{seen by rod}} = D_0 = 1.0\ m$$

(b.) Contracted length in the table frame = L

$$\Rightarrow L = \frac{L_0}{\gamma} = \frac{1}{1/\sqrt{1-(0.8)^2}} = \frac{1}{1.667} = 0.6m$$

$$\therefore \boxed{L = 0.6\ m}$$

(c) In the table frame: The rod length is 0.6 m & the hole diameter is 1 m. Clearly the rod fits through the hole, so in the table's frame the rod can fall through the hole.

$\Rightarrow$ In the rod's frame: The rod has rest length = 1 m & hole still has diameter 1 m & the rod fits exactly. But here simult aneity comes, are both ends of the rod over the hole at the same time? $\Rightarrow$ This is frame-dependent.

Consider 2 events in the Table's frame:

Event A: Rear end of the rod is over the hole (at x_R) at time $t=0$

Event B: Front end of the rod is over the hole (at x_f) at the same table frame time $t=0$.

These 2 events are simultaneous in the table frame & separated by $\Delta x = x_f - x_r = L_{table} = 0.6\,m$

$\Rightarrow$ Lorentz transform for time to the rod frame (S'):

$$\Delta t' = \gamma\left(\Delta t - \frac{V\Delta x}{c^2}\right)$$

Here $\Delta t = 0$ ($\because$ simultn. in table frame)

$$\Rightarrow \boxed{\Delta t' = -\gamma \frac{V\Delta x}{c^2}}, \quad \gamma = 1.667$$

$V = 0.8c$

& $\Delta x = 0.6\,m$

so, $\boxed{\Delta t' = -\frac{0.8}{c}}$

$$\boxed{\Delta t' = -2.67\times10^{-9}\,s}$$

so, in the rod frame the front end event happens about 2.67 ns earlier than the rear-end event. So, the two "ends over the hole" events are not simultaneous in the rod frame.

7.

$\boxed{E_T = m_0c^2 + K}$ Given: $\boxed{K = m_0c^2}$

$\boxed{E_T = 2m_0c^2}$

& $\boxed{E_T = \gamma m_0c^2}$ $\rightarrow$ $\gamma \cancel{m_0c^2} = 2\cancel{m_0c^2}$

$\gamma = 2$

$$\Rightarrow \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$\beta = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ so; $\boxed{V_e = \frac{\sqrt{3}}{2}c = 0.866\,c}$

No, it does not depends upon mass of the particle.

(8)

4- momentum (in Lab frame)

⇒ Initial electron : $p^\mu = \left(\frac{E}{c}, p\right)$, $p = +p\hat{x}$

⇒ Initial photon : $(k^\mu) = (p_0, -p_0\hat{x})$

⇒ Scattered photon : $k'^\mu = (p', p'\hat{n}')$

By conservation Law :

$$p + k = p' + k'$$

$$\Rightarrow (p + k - k')^2 = (p')^2$$

$$\Rightarrow p^2 + k^2 + k'^2 + 2p.k - 2p.k' - 2k.k' = (p')^2$$

$$\Rightarrow 2p.(k - k') = 2k.k'$$

$(\because p^2 = p'^2 = m^2c^2$
$k^2 = k'^2 = 0)$

$$\Rightarrow \boxed{p.(k - k') = k.k'}$$

$$\Rightarrow \quad K\cdot K' = p_0 p' \cancel{(1-\cos\theta)} - (-p_0\hat{x})\cdot(p'\hat{n}')$$

$$= p_0 p' + p_0 p' \hat{x}\cdot\hat{n}' = p_0 p'(1-\cos\theta)$$

$$(\because \cos\theta = -\hat{x}\cdot\hat{n}')$$

$$\therefore K\cdot K' = p_0 p'(1-\cos\theta)$$

& $$P\cdot K = \frac{E}{c} p_0 - p\cdot(-p_0\hat{x}) = p_0\left(\frac{E}{c}+p\right)$$

$$P\cdot K = p_0\left(\frac{E+pc}{c}\right)$$

& $$p\cdot K' = \frac{E}{c} p' - p\cdot(p'\hat{n}') = p'\left(\frac{E}{c} - p\cos\beta\right)$$

$$(\because \cos\beta = -\cos\theta)$$

$$\Rightarrow \quad P\cdot K' = p'\left(\frac{E}{c} + p\cos\theta\right)$$

$$\Rightarrow \quad P\cdot K' = p'\left(\frac{E+pc\cos\theta}{c}\right)$$

So; $$P\cdot K - P\cdot K' = p_0\frac{(E+pc)}{c} - p'\frac{(E+pc\cos\theta)}{c}$$

$$\Rightarrow \quad = p_0 p'(1-\cos\theta)$$

& $$p_0(E+pc) - p'(E+pc\cos\theta) = c p_0 p'(1-\cos\theta)$$

$$\Rightarrow \quad p_0(E+pc) = p'(E+pc\cos\theta + c p_0(1-\cos\theta))$$

$$\therefore \quad p' = \frac{p_0(E+pc)}{E+pc\cos\theta + c p_0(1-\cos\theta)}$$

Now; $\lambda = h/p$

$$\therefore \Delta\lambda = \lambda' - \lambda_0 = \frac{h}{p'} - \frac{h}{p_0} = h\frac{(p_0-p')}{p'p_0}$$

$$\Rightarrow \Delta\lambda = \lambda_0 \frac{(p_0 - p')}{p'}$$

Also : $p' = \dfrac{p_0 (E + pc)}{D}$, $D = E + pc\cos\theta + cp_0(1-\cos\theta)$

$$\Rightarrow p_0 - p' = p_0 - \frac{p_0 (E+pc)}{D} = p_0 \frac{(D - (E+pc))}{D}$$

$$\Rightarrow D - (E+pc) = (E + pc\cos\theta + cp_0(1-\cos\theta)) - (E+pc)$$

$$= pc(\cos\theta - 1) + cp_0(1-\cos\theta)$$

$$\Rightarrow D - (E+pc) = c(1-\cos\theta)(p_0 - p)$$

$$\therefore p_0 - p' = p_0 \frac{c(1-\cos\theta)(p_0 - p)}{D}$$

So ; $$\frac{p_0 - p'}{p'} = \frac{c(1-\cos\theta)(p_0 - p)}{E + pc}$$

$$\therefore \Delta\lambda = \lambda_0 \cdot \frac{(p_0 - p')}{p'} = \lambda_0 \cdot \frac{c(1-\cos\theta)(p_0 - p)}{E + pc}$$

$$\Rightarrow \boxed{\Delta\lambda = \lambda_0 \frac{(p_0 - p)c}{E + pc}(1-\cos\theta)}$$

(9) (i) Electron initially at rest, $\theta = 180^\circ$ (backscatter):

As; $$\Delta\lambda = \lambda_0 \frac{(p_0 - p)c}{E + pc}(1-\cos\theta)$$

$\Rightarrow$ for e^- at rest before collision:

$$p = 0 \;\&\; E = mc^2$$

For back scattering, $\theta = 180^\circ \Rightarrow \cos\theta = -1$

& $1 - \cos\theta = 2$

$$\therefore \Delta\lambda = \lambda_0 \frac{(p_0 - 0)}{mc^2 + 0} \times (2) = \frac{\lambda_0 \, 2p_0 c}{mc^2}$$

put $\boxed{p_0 = \frac{h}{\lambda_0}}$

$$\Delta\lambda = \frac{\lambda_0 \left(2h/\lambda_0\right) \times c}{mc^2} = \frac{2h}{mc}$$

$\therefore$ For back scattering:

$$\boxed{\Delta\lambda = \lambda' - \lambda_0 = \frac{2h}{mc}}$$

(ii) For ultra-relativistic electron ($E >> mc^2$) :

$$\lambda' = \lambda_0 \frac{(E - pc + 2cp_0)}{E + pc} \rightarrow (a)$$

$E >> mc^2$ (ultra relativistic limit)

$\boxed{\frac{m^2c^4}{E^2} <<< 1}$

$$pc = \sqrt{E^2 - m^2c^4}$$

(expanding)

$$\Rightarrow pc = E\sqrt{1 - \frac{m^2c^4}{E^2}} \simeq E\left(1 - \frac{m^2c^4}{2E^2}\right)$$

$$= E - \frac{m^2c^4}{2E} + O\left(\frac{m^4c^8}{E^3}\right) \rightarrow (i)$$

Put (i) in (a) ;

$N = E - pc + 2cp_0$ (Numerator)

$$\Rightarrow E - pc \simeq \frac{m^2c^4}{2E}$$

$$\Rightarrow N \simeq \frac{m^2c^4}{2E} + 2cp_0$$

Similarly, Denominator $= D = \cancel{2E\left(1 + m^2\right)}\ 2E\left(1 + \frac{m^2c^4}{4E^2}\right)$

$$\text{so; } \lambda' = \lambda_0 \frac{N}{D} \simeq \frac{\lambda\left(\frac{m^2c^4}{2E} + 2cp_0\right)}{2E}\left(1 + \frac{m^2c^4}{4E^2}\right)$$

$$\lambda' = \lambda_0\left[\frac{cp_0}{E} + \frac{m^2c^4}{4E^2}\right] + (\text{Higher orders})$$

we use $cp_0 = hc/\lambda_0$

$$\Rightarrow \lambda_0 \cdot \frac{cp_0}{E} = \lambda_0 \cdot \frac{hc/\lambda_0}{E} = \frac{hc}{E}$$

$\therefore$ 1 order app. for the scattered wavelength is:

$$\lambda' \simeq \frac{hc}{E} + \lambda_0 \frac{m^2c^4}{4E^2} \quad (E \gg mc^2, \theta = \pi)$$

$$\Rightarrow \lambda' \simeq \frac{hc}{E}\left(1 + \frac{m^2c^4\lambda_0}{4hcE}\right)$$

(10.) The time diff. b/n the same two events measured in the train frame (S') is ,,

$$\Delta t' = \gamma\left(\Delta t - \frac{V\Delta x}{c^2}\right)$$

Here $\boxed{\Delta t = 0}$ (simultaneity) ;

$$\boxed{\Delta t' = -\gamma \frac{V\Delta x}{c^2}}$$

Let $\boxed{K = \gamma\beta}$, $\boxed{\beta = V/c}$

$$\Rightarrow \boxed{\Delta t' = -K\frac{\Delta x}{c}}$$

we have , $\boxed{|\Delta t'| = 0.5\,s}$

& $\boxed{\frac{\Delta x}{c} = \frac{1000}{3\times10^8} = 3.33\times10^{-6}\,s}$

So, $K\frac{\Delta x}{c} = 0.5 \Rightarrow K = \frac{0.5}{3.33\times10^{-6}} = 1.5\times10^5$

$\therefore \boxed{\gamma\beta = 1.5\times10^5} = K$

As ; $\gamma\beta = \frac{\beta}{\sqrt{1-\beta^2}} = K \Rightarrow \cancel{\beta^2 = K^2(1-\beta^2)}\ \boxed{\frac{\beta^2}{1-\beta^2} = K^2}$

$\Rightarrow \frac{\beta^2}{1-\beta^2} + 1 = (K^2+1)$ Put

$\Rightarrow \boxed{\frac{1}{1-\beta^2} = (K^2+1)}$

$\Rightarrow \beta^2(K^2+1) = K^2$

$\Rightarrow \beta = \frac{K}{\sqrt{K^2+1}} \simeq 1 - \frac{1}{2K^2}$ (using approximation)

$\beta = 1 - 2.22\times10^{-11}$

$\therefore V = \beta c = (1 - 2.2\times10^{-11})\,c = (3\times10^8 - 6.667\times10^{-3})$ m/s

$= 2.999\times10^8$ m/s

Thus if $\boxed{V = c - u}$

$$\Rightarrow \boxed{u \simeq 6.667 \times 10^{-3}\ m/s}$$

$\Rightarrow$ Also; from the sign of $\Delta t' = -u\left(\Delta x / c\right)$:

$\Delta t' < 0$ i.e the event at larger x (the lamp ahead in the dir^n. of motion) occurs earlier in the train frame.

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(11)

In the electron rest frame the photon has wavelength λ_0 & momentum mag. $p' = h/\lambda_0$

we need p_{lab}

=> The photon's dir^n observed in the lab relates to the emission dir^n in the e^- frame. For the lab detection angle $\theta_{lab} = 90°$, the corresponding emission angle in the e^- frame satisfies $\cos\theta' = -\beta$, $\beta = \frac{v}{c}$

=> Energy Lorentz transform for photons:

$$E_{lab} = \gamma\left(E' + \beta p'c \cos\theta'\right)$$

For photon, $E' = p'c$ & $\cos\theta' = -\beta$

$$\therefore E_{lab} = \gamma p'c\left(1 - \beta^2\right) = \frac{p'c}{\gamma}$$

$$\text{so: } p_{lab} = \frac{p'}{\gamma} = \frac{h}{\gamma\lambda_0} \quad \left(\because p_{lab} = \frac{E_{lab}}{c}\right)$$

So: $\lambda_0 = 1 \times 10^{-11}$ m

$v = 0.5c$ & $\gamma = \frac{1}{\sqrt{0.75}} \simeq 1.155$

$$\boxed{\therefore p_{detector} = p_{lab} = \frac{6.626 \times 10^{-34}}{1.155 \times 10^{-11}} = 5.738 \times 10^{-23} \text{ kg m/s}}$$

(9) (i) (12.) Total energy before decay in the Lab frame:

$$E_{tot.} = m_x c^2$$

After decay, energies of a & b are

$$E_a = \sqrt{p^2c^2 + m_a^2c^4} \quad , \quad E_b = \sqrt{p^2c^2 + m_b^2c^4}$$

By energy conservation:

$$m_x c^2 = E_a + E_b$$

$$\left(m_x c^2 - \sqrt{p^2c^2 + m_a^2c^4}\right)^2 = \left(\sqrt{p^2c^2 + m_b^2c^4}\right)^2$$

$$\Rightarrow \quad (m_x c^2)^2 + \cancel{p^2c^2} + m_a^2c^4 - 2m_x c^2 \underbrace{\sqrt{p^2c^2 + m_a^2c^4}}_{E_a} = \cancel{p^2c^2} + m_b^2c^4$$

$$\Rightarrow \quad m_x^2c^4 - 2m_x c^2(E_a) + m_a^2c^4 = m_b^2c^4$$

$$\Rightarrow \quad +2m_x c^2(E_a) = -m_b^2c^4 + m_a^2c^4 + m_x^2c^4$$

$$\Rightarrow \quad E_a = \frac{c^2}{2m_x}\left(m_x^2 + m_a^2 - m_b^2\right)$$

$$\Rightarrow \quad p^2c^2 + m_a^2c^4 = \frac{c^4}{4m_x^2}\left(m_x^2 + m_a^2 - m_b^2\right)^2$$

$$\Rightarrow \quad p^2 = \frac{c^2}{4m_x^2}\left[\left(m_x^2 + m_a^2 - m_b^2\right)^2 - 4m_x^2m_a^2\right]$$

So;

$$\boxed{p = \frac{c}{2m_x}\left[\left(m_x^2 + m_a^2 - m_b^2\right)^2 - 4m_x^2m_a^2\right]^{1/2}}$$