Linear Algebra Review

Matrix :-

2-D arrays. Set of numbers arranged in rows & columns and enclosed in square brackets.

Eg:-

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix}$$

Vector :-

A vector is a matrix with one column and many rows. Vectors are subset of matrices.

Notations :-

- A_{ij} refers to the element in the i^{th} row and j^{th} column of matrix A.

- A vector with 'n' rows is referred as an n-dimensional vector.
- v_i refers to the element in the i^{th} row of the vector.
- In general, all our vectors & matrices will be 1-indexed.
- Matrices are usually denoted by uppercase names while vectors are lowercase.
- Scalar means that an object is a single value.
- $\mathbb{R}$ refers to the set of scalar real no.s.
- $\mathbb{R}^n$ refers to the set of n-dimensional vectors of real numbers.

Addition and Subtraction of Matrices :-

Addition & subtraction are done element-wise, so we simply add / subtract each corresponding element :

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} + \begin{bmatrix} e & g \\ f & h \end{bmatrix} = \begin{bmatrix} a+e & c+g \\ b+f & d+h \end{bmatrix}$$

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} - \begin{bmatrix} e & g \\ f & h \end{bmatrix} = \begin{bmatrix} a-e & c-g \\ b-f & d-h \end{bmatrix}$$

- To add or subtract two matrices, the dimensions must be same.

Premium

* In scalar multiplication, we simply multiply every element by the scalar value.

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} * x = \begin{bmatrix} a*x & c*x \\ b*x & d*x \end{bmatrix}$$

* In scalar division, we simply divide every element by the scalar value.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} / x = \begin{bmatrix} a/x & b/x \\ c/x & d/x \end{bmatrix}$$

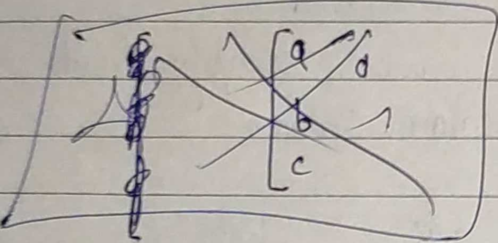
* **Matrix - Vector Multiplication :-**

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a*x + b*y \\ c*x + d*y \\ e*x + f*y \end{bmatrix}$$

- Result is a vector.
- No. of columns of matrix must be equal to no. of rows of the vector.
- An $m \times n$ matrix multiplied by an $n \times 1$ vector results in an $m \times 1$ vector.

★ Matrix - Matrix Multiplication :-

We multiply two matrices by breaking it into several vector multiplications and concatenating the result.



$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} * \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} a*w+b*y & a*x+b*z \\ c*w+d*y & c*x+d*z \\ e*w+f*y & e*x+f*z \end{bmatrix}$$

- No. of columns of first matrix must be equal to no. of rows of second matrix.

- An $m \times n$ matrix multiplied by an $n \times o$ matrix results in an $m \times o$ matrix.

★ Properties of Matrix Multiplication :-

- It is not commutative.

If A & B are two matrices, then in general,

$$\boxed{A \times B \neq B \times A}$$

- Matrix multiplication is associative.

$$\boxed{(A \times B) \times C = A \times (B \times C)}$$

★ Identity Matrix :-

- When multiplied by any matrix of same dimension, results in original matrix.
- Denoted by I (or $I_{n \times n}$).
- Examples :-

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2×2 $\quad$ 3×3 $\quad$ 4×4

- Informally written as :-

$$\begin{bmatrix} 1 & & & O \\ & 1 & & \\ & & 1 & \\ O & & & 1 \end{bmatrix}$$

- For any matrix A :-

$$\boxed{A \cdot I = I \cdot A = A}$$

★ Inverse of Matrix / Matrix Inverse :-

- If A is a $m \times m$ matrix, and if it has an inverse,

$$\boxed{A A^{-1} = A^{-1} A = I}$$

- Matrices that don't have an inverse are called "singular" or "degenerate".

- A non square matrix does not have an inverse matrix.

★ Transpose of Matrix / Matrix Transpose :-

- Let A be an $m \times n$ matrix, and let $B = A^T$. Then B is an $n \times m$ matrix, and :-

$$\boxed{B_{ij} = A_{ji}}$$

- Transpose or Transposition of a matrix is like rotating the matrix 90° in clockwise direction and then reversing it.