Regression

EE698V - Machine Learning for Signal Processing

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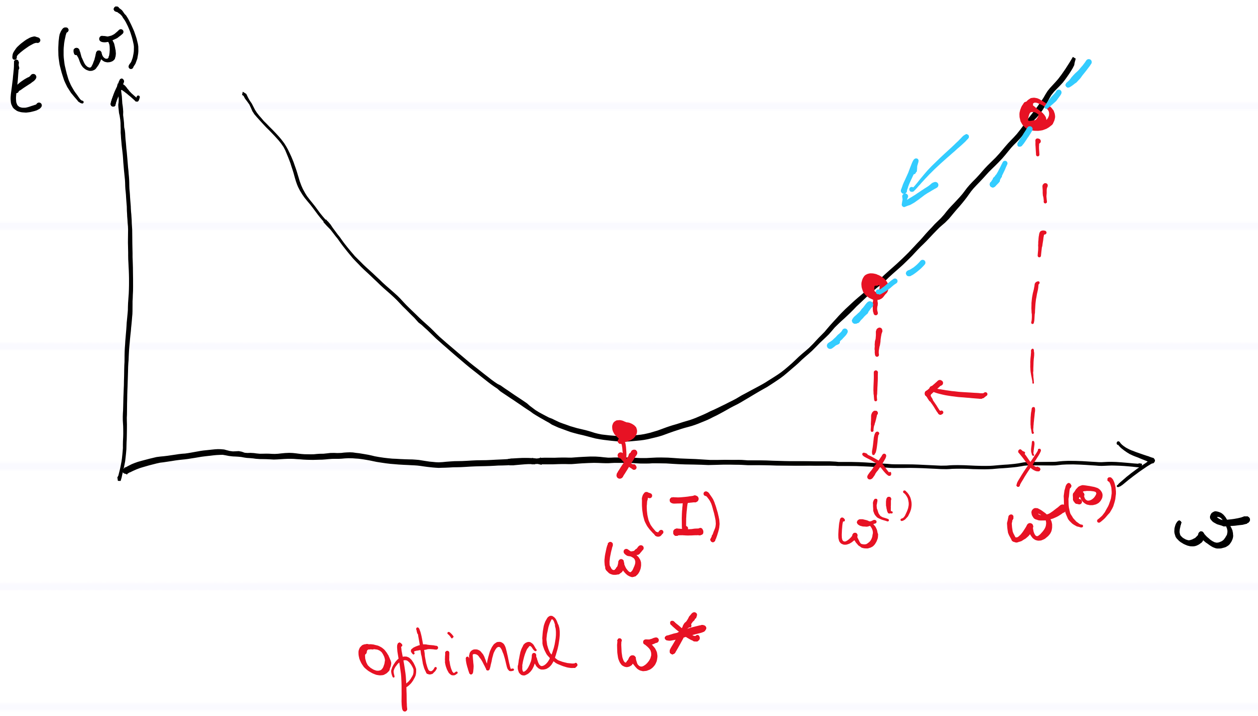
What if analytical solution not possible?

- This is true for most problems, e.g. neural networks
- Use iterative updates

Gradient Descent: scalar w



1. Start from a random $w^{(0)}$
2. find $\left.\frac{\partial E}{\partial w}\right|_{w^{(0)}}$
3. Update

$$w^{(new)} = w^{old} - \frac{\partial E}{\partial w}$$

4. goto step 2, until convergence

Gradient Descent: any dimension w

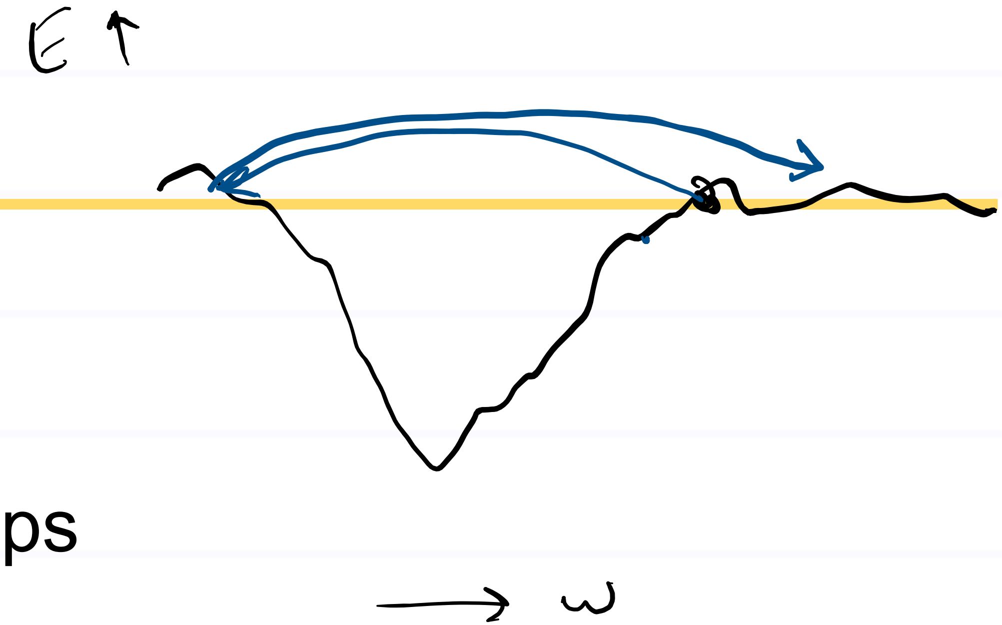
$$w^{(\tau+1)} = w^{(\tau)} - \eta \frac{\partial E}{\partial w}$$

- We have already seen how to compute $\frac{\partial \boldsymbol{E}}{\partial \boldsymbol{w}}$
- τ is iteration index
- η is learning rate or step size

Sequential Learning

- When data arriving in a stream
  - matrix inversion based solutions are difficult
  - iterative solutions are best used

Learning Rate



- Learning depends on η
  - Too small η : stuck in noisy bumps
  - Too large η : oscillates about the optimal value, or may even become unstable (explode)

Data Normalization

- If the input values vary too much, learning becomes unstable
- Good to scale to a small range (close to 0) them for better learning

$$\underbrace{\phi_i}_{\in[-100,\,100]} w_{ij} \rightarrow \underbrace{\frac{\phi_i}{c_i}}_{\in[-1,\,1]} w'_{ij}$$

if $c_i = 100$

by substituting w_{ij} by $\frac{w'_{ij}}{c_i}$ before learning

Non-linear Regression

Recap:

$$y^T = \Phi(x)\, W$$

or $Y = \Phi(X)\, W$

Φ could be a non linear function of x

but y was a linear function of w.

Consider now

$$y^T = \sigma(\Phi(x) W_1)\, W_2$$

σ is sigmoid function (element-wise operation)

This is non linear in W_1

$$y^T = \sigma(\Phi(x) W_1) W_2$$

$$y_j = \sum_{i_1} \sigma\left(\sum_{i_0} \phi_{i_0}(x) \; w_{i_0 i_1}\right) \; w_{i_1 j}$$

can we draw it :

References

- http://www.deeplearningbook.org/contents/ml.html **(highly recommended)**
- Behera, L., & Kar, I. (2010). *Intelligent Systems and control principles and applications*. Oxford University Press, Inc.. **Chapter 2**
- PRML: Chapter 5