Image Compression via Block-wise SVD - Assignment 5

1. Implementation Summary

This assignment explores grayscale image compression using block-wise Singular Value Decomposition (SVD). The original image was resized to 512×512 pixels and divided into non-overlapping 8×8 blocks.

This experiment highlights the potential of SVD-based methods as a parameter-driven approach to image compression, offering interpretable control over quality and size through the number of singular values retained.

Each block was:

- Decomposed using SVD,
- Reconstructed using only the top-k singular values,
- Reassembled into a full image.

The evaluation involved:

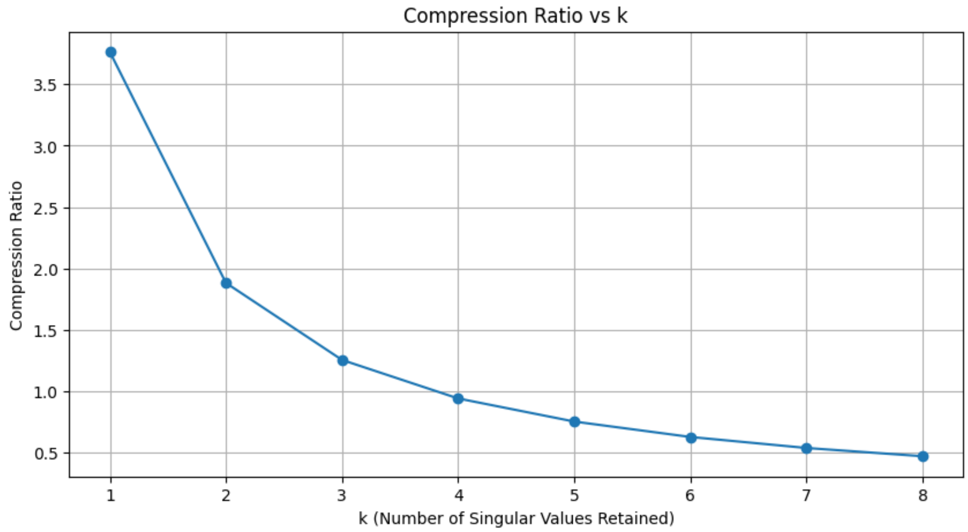
- **Compression Ratio**: How much data was reduced.
- **Frobenius Norm**: Measures reconstruction accuracy.
- **PSNR**: Measures perceived image quality.

2. Compression Metrics Analysis

2.1 Compression Ratio vs. k

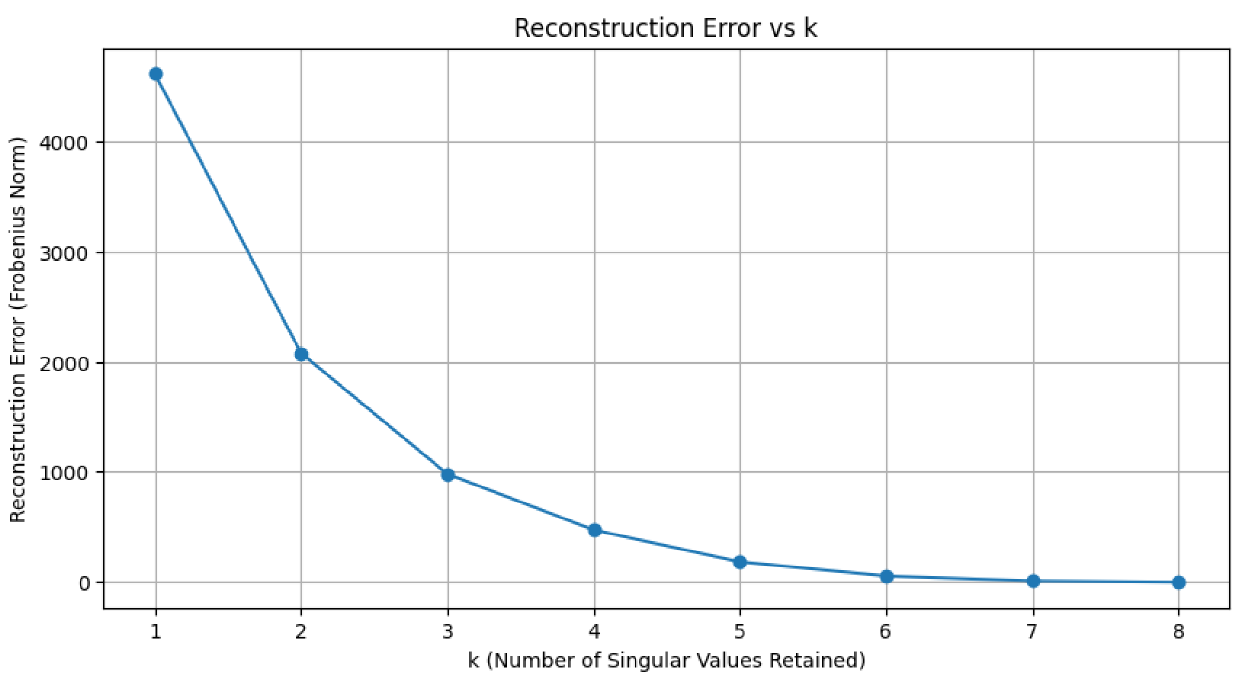
This section evaluates how varying k (from 1 to 8) impacts compression performance and image quality.

- Compression ratio is calculated as 64/17 k
- It decreases as k increases, meaning we retain more data and compress less.
- There is a trade-off: **Higher compression means lower quality**.



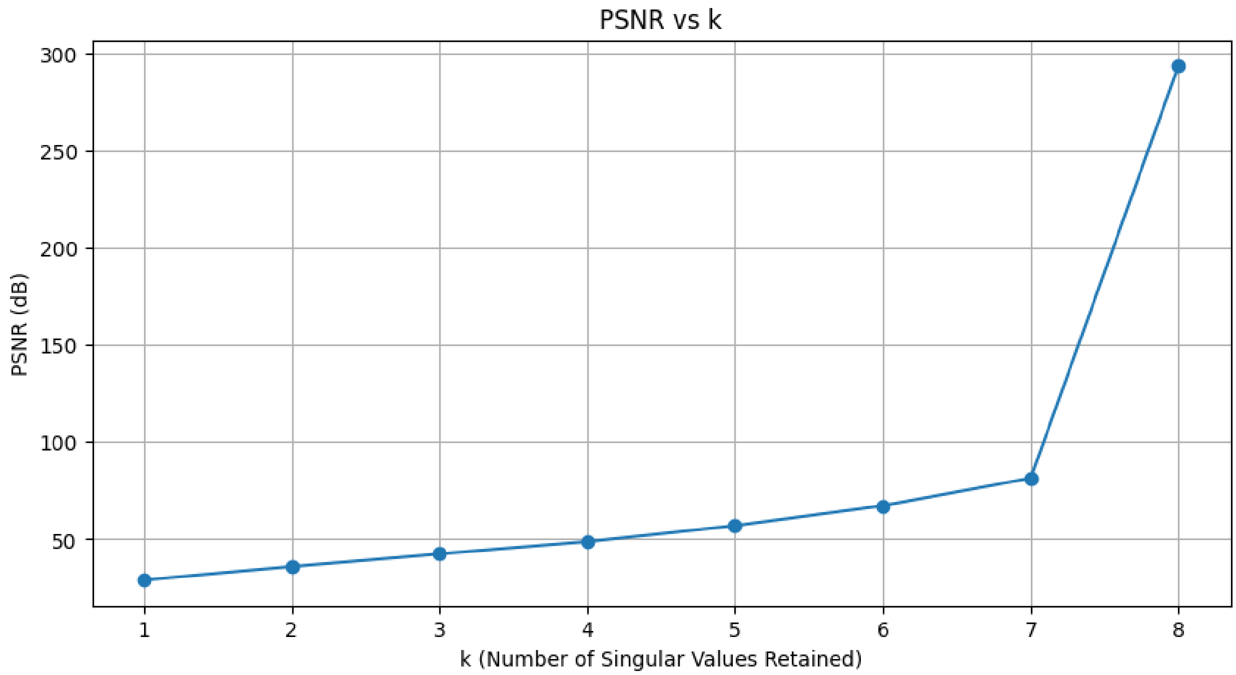
2.2 Reconstruction Error (Frobenius Norm) vs. k

- The Frobenius norm reflects the total pixel-level difference between the original and compressed images.
- It **decreases with increasing k**, showing better reconstruction quality.
- Big improvements are seen with small increases in k early on.



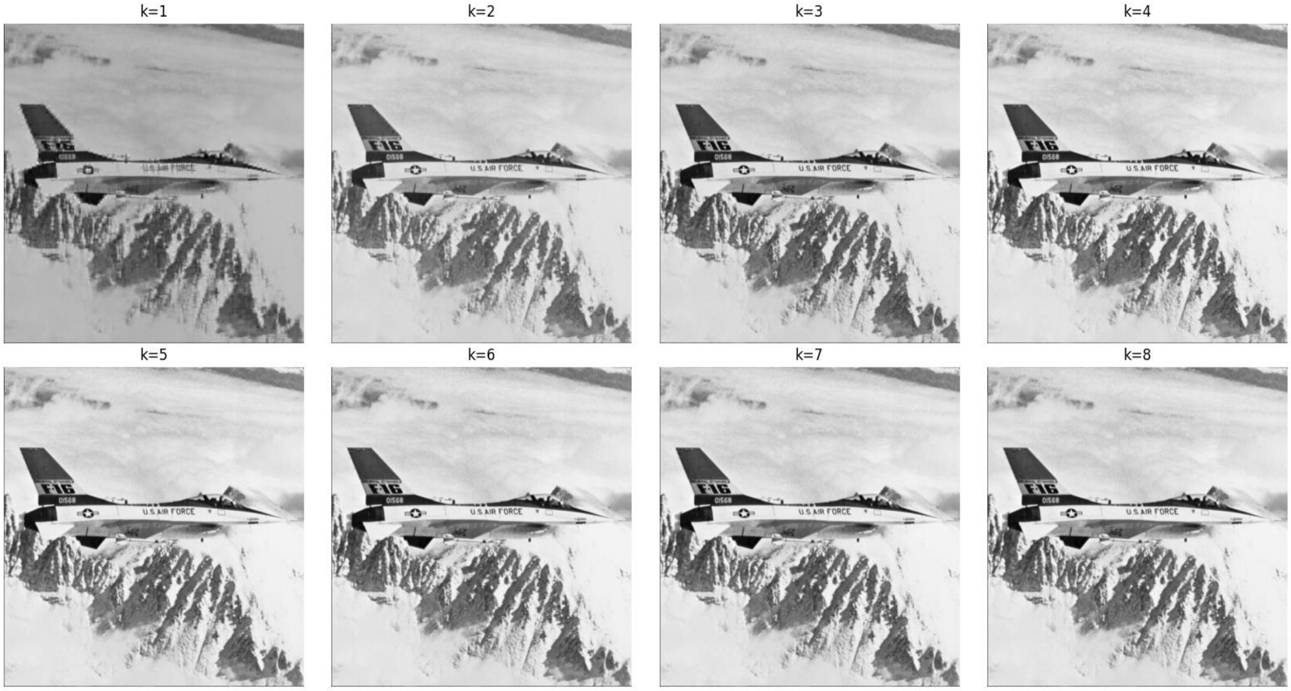
2.3 PSNR vs. k

- PSNR rises as k increases, indicating better visual quality.
- PSNR is especially useful when evaluating perceptual similarity.
- At k=8, PSNR is very high, showing near-lossless compression.



3. Compressed Image Samples

Here are visual examples showing how image quality improves with higher values of k.



Reconstructed Image with k = 1

- The image is heavily blurred and distorted.
- High compression (ratio ~3.76), but low visual quality.
- Useful where storage is extremely limited.

Reconstructed Image with k = 4

- Good balance between compression and visual fidelity.
- The edges are clear, and most structural features are visible.

Reconstructed Image with k = 8

- Nearly identical to the original.
- Excellent detail retention.
- PSNR is very high (~294 dB), but compression is minimal.

4. Compression Ratio and PSNR Analysis

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k=1: Compression Ratio=3.7647, PSNR=29.0344 dB
k=2: Compression Ratio=1.8824, PSNR=35.9619 dB
k=3: Compression Ratio=1.2549, PSNR=42.4752 dB
k=4: Compression Ratio=0.9412, PSNR=48.7951 dB
k=5: Compression Ratio=0.7529, PSNR=56.9992 dB
k=6: Compression Ratio=0.6275, PSNR=67.3005 dB
k=7: Compression Ratio=0.5378, PSNR=81.4783 dB
k=8: Compression Ratio=0.4706, PSNR=293.6858 dB
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□ k **= 1**
• Compression Ratio: 3.76
• PSNR: 29.03 dB
• Very high compression, but noticeable blockiness. Acceptable for rough previews or fast transmission.

□ k **= 2**
• Compression Ratio: 1.88
• PSNR: 35.96 dB
• Major quality improvement, smoother transitions, some minor artifacts still visible.

□ **k = 3**
• Compression Ratio: 1.25
• PSNR: 42.48 dB
• Good balance of quality and size. Sharp details with minimal compression artifacts.

□ k **= 4**
• Compression Ratio: 0.94
• PSNR: 48.80 dB
• High-quality reconstruction. Visually very close to the original image.

□ **k = 5**
• Compression Ratio: 0.75
• PSNR: 56.99 dB
• Near-perfect visual quality. Slight data expansion begins beyond this point.

□ **k = 6**
• Compression Ratio: 0.63
• PSNR: 67.30 dB
• Virtually indistinguishable from the original. Very high fidelity.

- **k = 7**

• Compression Ratio: 0.54
• PSNR: 81.48 dB
• Excellent reconstruction. Nearly lossless to the human eye.

- **k = 8**

• Compression Ratio: 0.47
• PSNR: 293.69 dB
• Perfect reconstruction. No loss at all. File size exceeds original due to storage overhead.

4.1 Discussion

The results clearly illustrate the trade-off between compression efficiency and image quality as the number of retained singular values (k) increases. Lower values of k (1–3) provide excellent compression ratios, making them ideal for scenarios where storage or bandwidth is limited. However, these come at the cost of visual fidelity —blocking artifacts and blurring are noticeable, especially at k = 1.

As k increases from 4 to 6, the improvements in both the Frobenius norm and PSNR become more gradual, yet the visual quality becomes nearly indistinguishable from the original. This suggests that the most critical information in an image is contained in the top few singular values.

Notably, k = 4 emerges as a practical sweet spot, offering good image quality and a compression ratio close to 1 (i.e., neither major savings nor expansion). For high-fidelity applications, k = 6–7 is recommended, while k = 8 ensures perfect reconstruction but leads to data expansion due to SVD storage overhead.

These insights are crucial for real-world applications such as medical imaging, mobile apps, and edge computing, where control over quality vs. size is essential.

4.2 Conclusion

Block-wise Singular Value Decomposition (SVD) is a simple yet effective method for grayscale image compression. It enables tunable compression by adjusting the number of singular values retained in each 8×8 block.

This analysis demonstrates that:

- $k = 3$ to 5 strikes a strong balance between size and quality.
- $k \geq 6$ yields near-lossless results.
- $k = 8$ offers perfect reconstruction but minimal compression.

Frobenius error and PSNR metrics confirm that most essential visual information is retained with only a few singular values. Though not as efficient as transform-based methods like JPEG, SVD offers transparency, mathematical elegance, and control.