Assignment 4 Report: Perceptron Learning Approaches

Name: Shivam Pawar
Course: MATH-485

Introduction

This assignment explores two approaches to Perceptron learning: the Heuristic Perceptron and the Gradient Descent Perceptron. Both are implemented and analyzed on a 2D binary classification dataset. The dataset contains 100 samples with two numerical features and a binary target.

Part 1: Heuristic Perceptron

Code Snippet

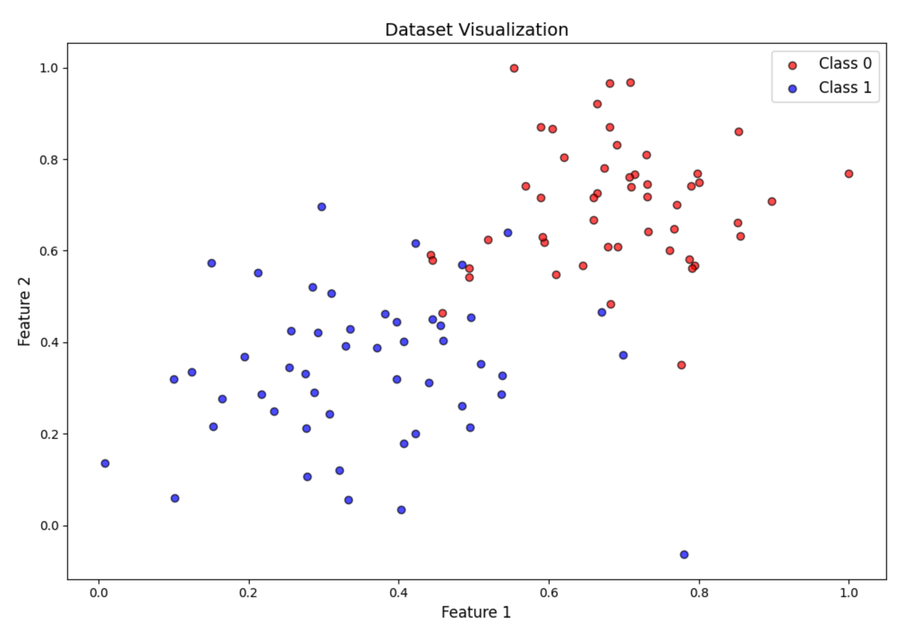
```
class HeuristicPerceptron:
    def __init__(self, eta=0.1, max_iter=1000):
        self.eta = eta
        self.max_iter = max_iter
        self.w = None
        self.b = None
        self.boundaries = []

    def _init_weights(self, X):
        n_features = X.shape[1]
        self.w = np.random.normal(loc=0.0, scale=0.01, size=n_features)
        self.b = np.random.normal(loc=0.0, scale=0.01)
        self.boundaries.append((self.w.copy(), self.b, 'initial'))

    def fit(self, X, y):
        self._init_weights(X)
        iterations = 0
        errors = True
        while errors and iterations < self.max_iter:
            errors = False
            for xi, target in zip(X, y):
                prediction = self.predict(xi.reshape(1, -1))[0]
                if prediction != target:
                    update = self.eta * (target - prediction)
                    self.w += update * xi
                    self.b += update
                    errors = True
            if errors:
                self.boundaries.append((self.w.copy(), self.b, 'intermediate'))
            iterations += 1
        self.boundaries.append((self.w.copy(), self.b, 'final'))
        return iterations
```

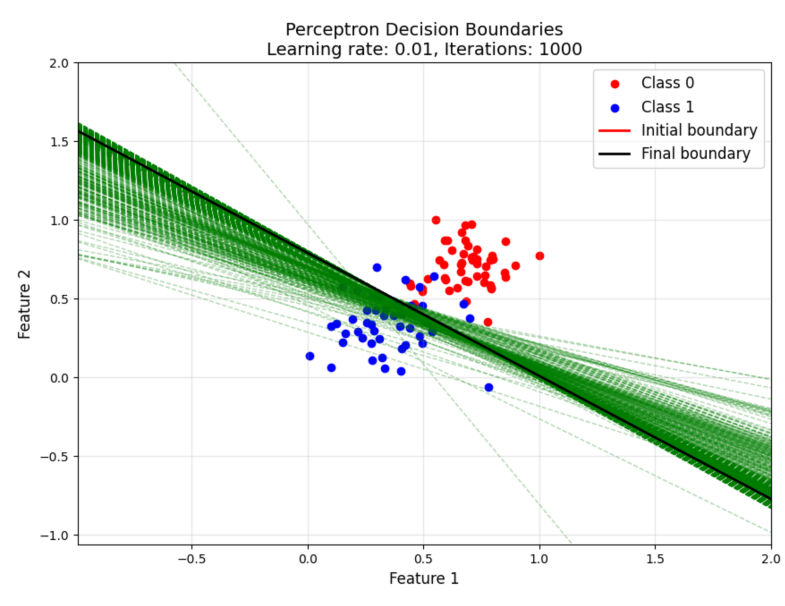
Visualizations and Analysis

Figure 1: Dataset Visualization



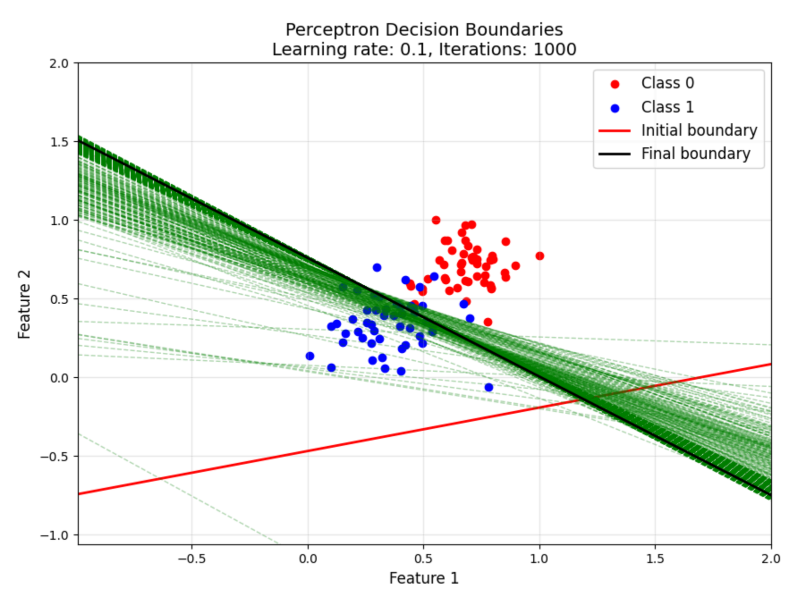
- Red points: Class 0, Blue points: Class 1
- Data is linearly separable and suitable for a perceptron.

Figure 2: Heuristic Perceptron (lr=0.01)



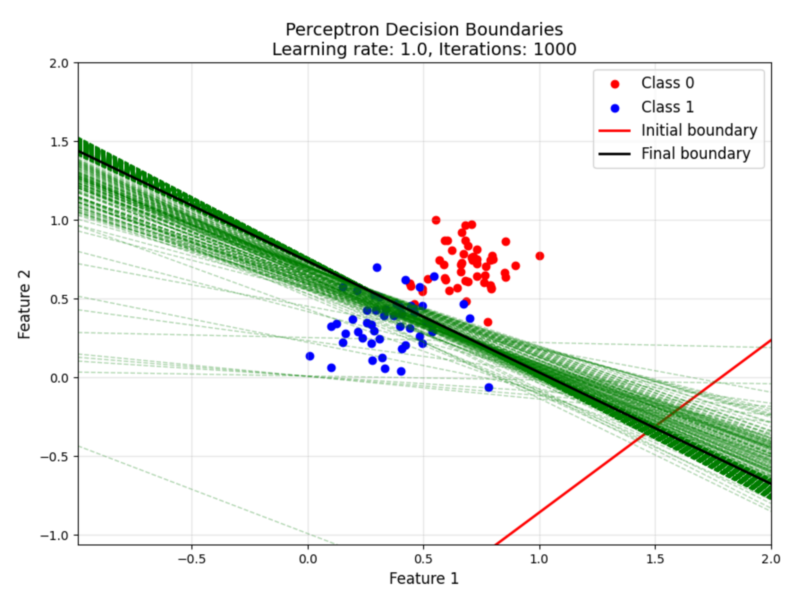
- Very slow learning. Many small updates.
- Final boundary is accurate but convergence is slow.

Figure 3: Heuristic Perceptron (lr=0.1)



- Faster convergence. Fewer intermediate updates.
- Final boundary is clear and balanced. Optimal result.

Figure 4: Heuristic Perceptron (lr=1.0)



- Aggressive jumps in boundaries.
- Converged quickly, but less stable during training.

Results Table

Learning Rate	Iterations	Final Weights	Final Bias
0.01	1000	[-0.0885, -0.1132]	0.0891
0.1	1000	[-0.8096, -1.0734]	0.8111
1.0	1000	[-7.7065, -10.8941]	8.0098

Part 2: Gradient Descent Perceptron

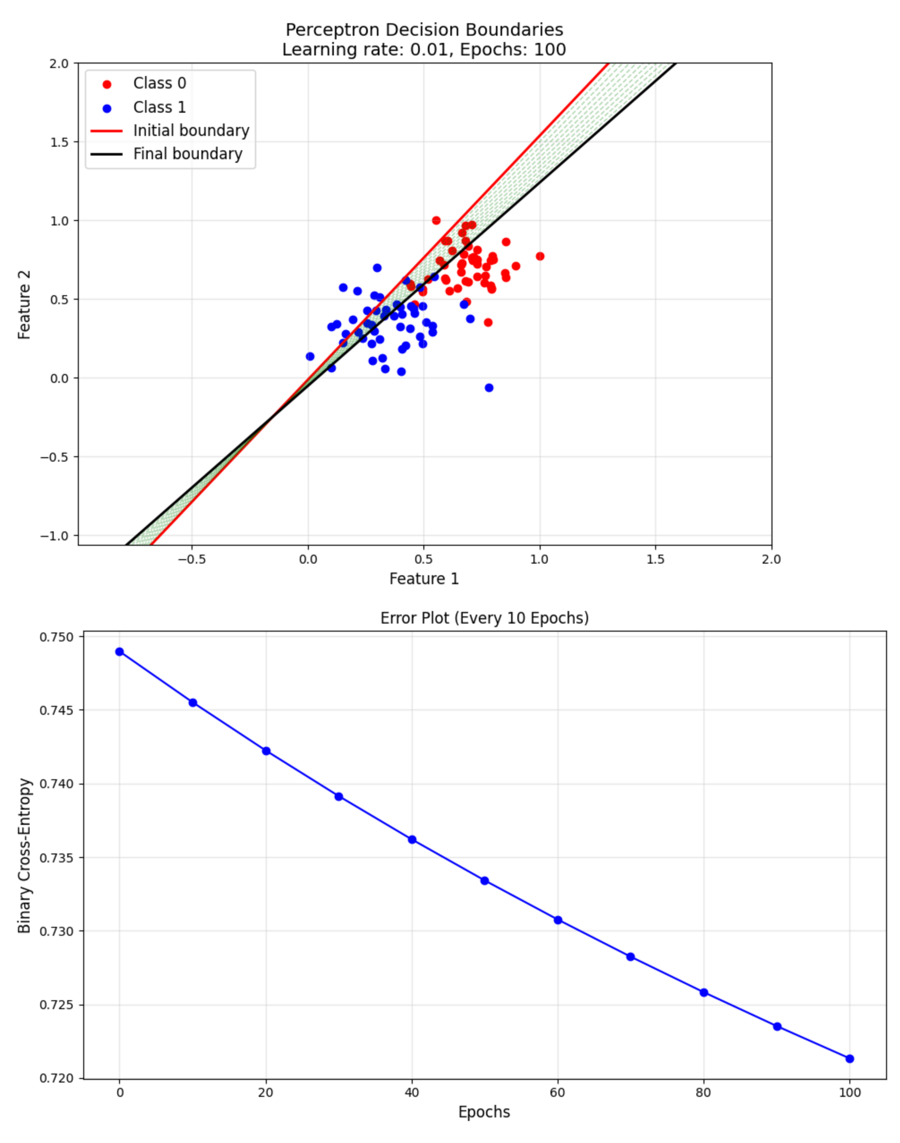
Code Snippet

```
class GradientDescentPerceptron:
    def __init__(self, eta=0.1, n_epochs=100):
        self.eta = eta
        self.n_epochs = n_epochs
        self.w = None
        self.b = None
        self.boundaries = []
        self.cost_history = []
        self.epoch_records = []

    def fit(self, X, y):
        self._init_weights(X)
        n_samples = X.shape[0]
        self.cost_history.append(self.compute_cost(X, y))
        self.epoch_records.append(0)
        for epoch in range(1, self.n_epochs + 1):
            output = self.activation(X)
            error = output - y
            grad_w = np.dot(X.T, error) / n_samples
            grad_b = np.sum(error) / n_samples
            self.w -= self.eta * grad_w
            self.b -= self.eta * grad_b
            if epoch % 10 == 0:
                self.cost_history.append(self.compute_cost(X, y))
                self.epoch_records.append(epoch)
        self.boundaries.append((self.w.copy(), self.b, 'final'))
```

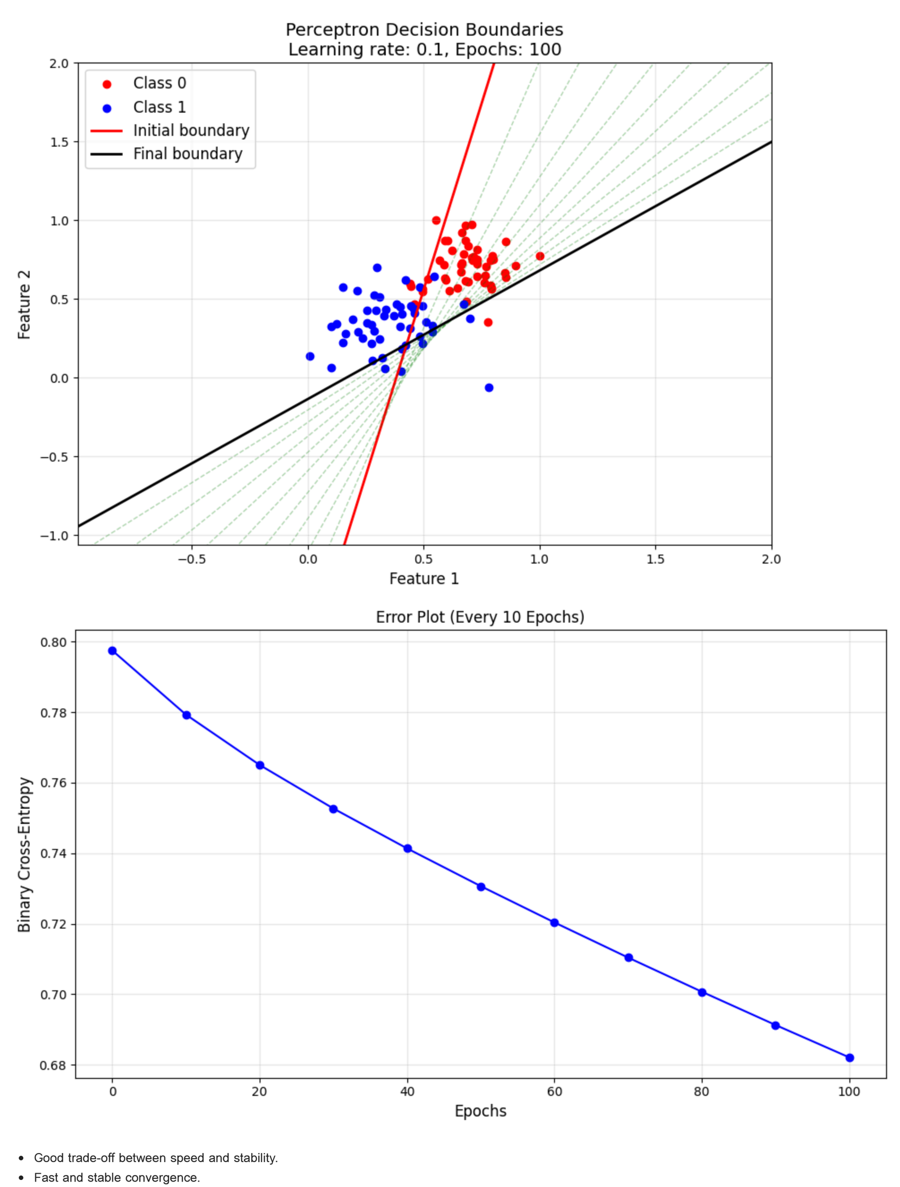
Visualizations and Analysis

Figure 5–6: GD Perceptron (lr=0.01)



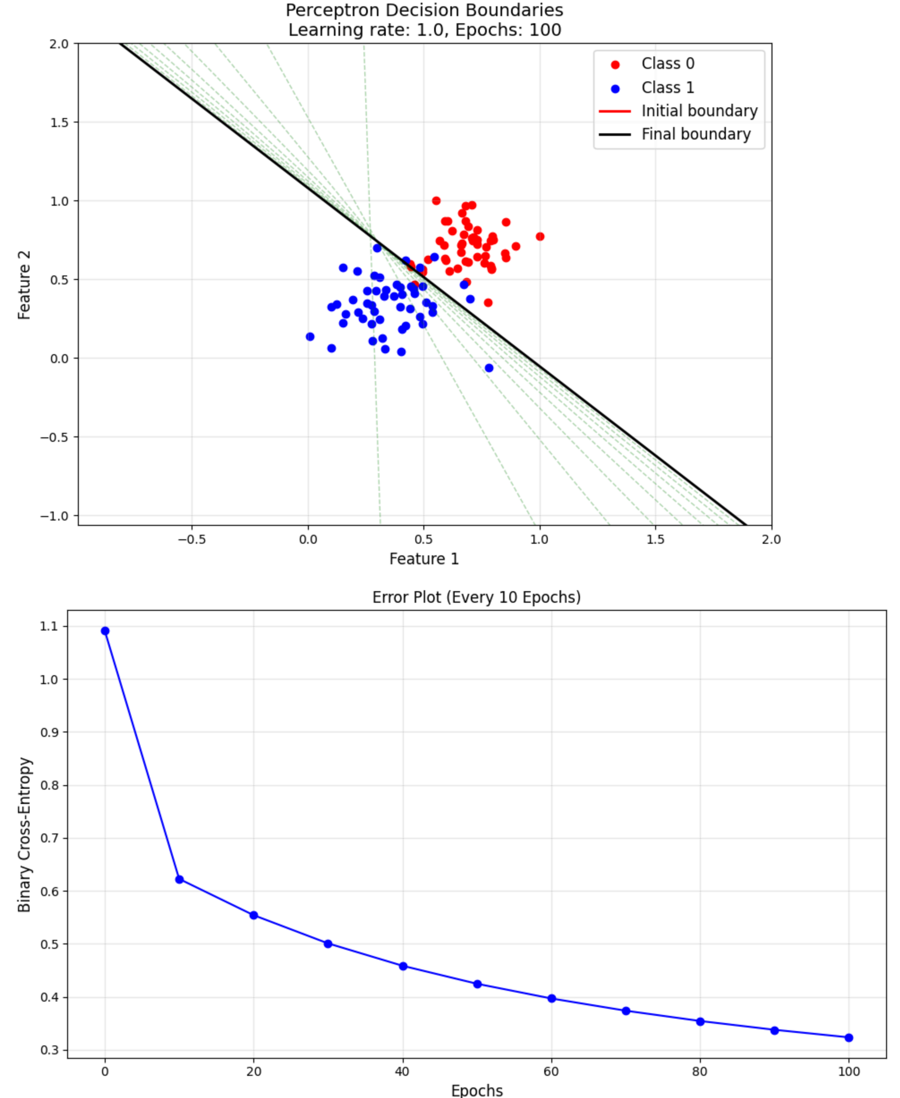
- Boundary updates are slow and subtle.
- Loss decreases slowly but steadily.

Figure 7–8: GD Perceptron (lr=0.1)



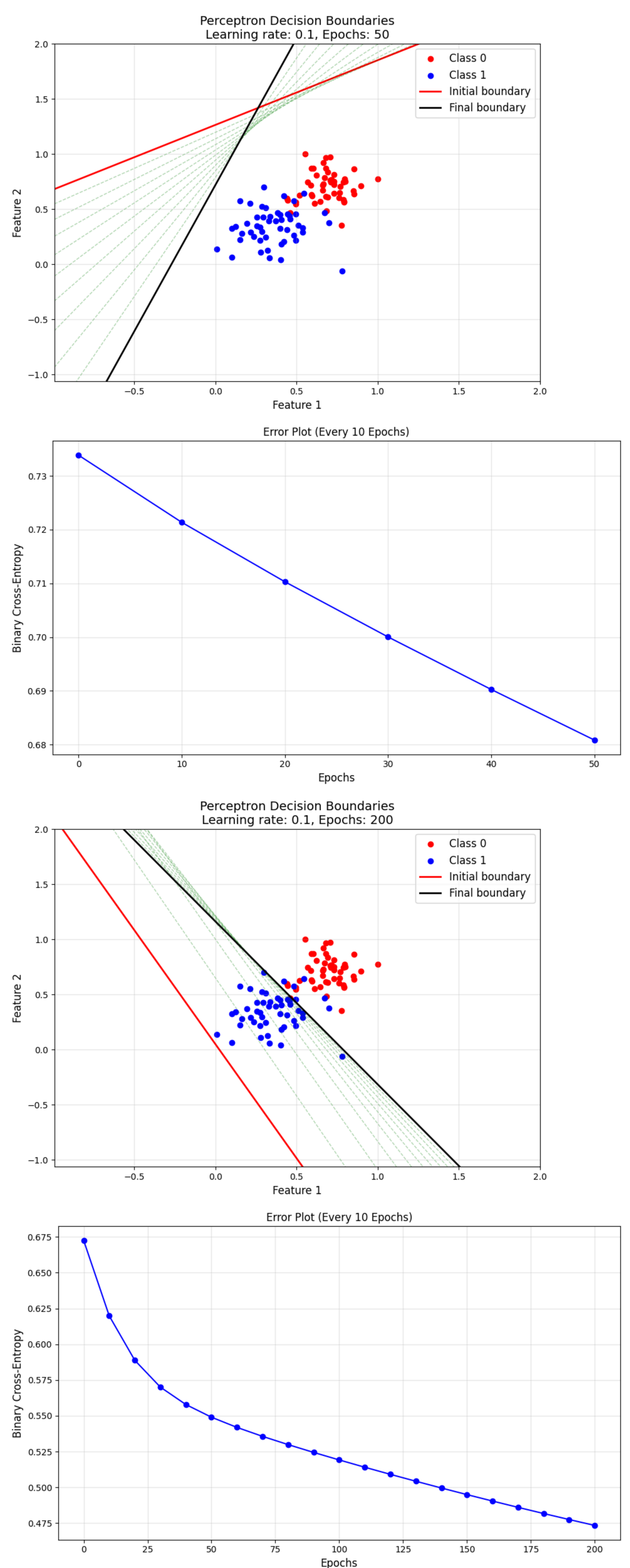
- Good trade-off between speed and stability.
- Fast and stable convergence.

Figure 9–10: GD Perceptron (lr=1.0)



- Fast learning, but less smooth.
- Risk of instability if data were noisy.

Figure 11–14: Epoch Comparison (lr=0.1)



- Epoch 50: Underfitting, high loss.
- Epoch 100: Balanced learning.
- Epoch 200: Slight improvement, diminishing returns.

Results Table

Learning Rate	Epochs	Final Weights	Final Bias	Final Loss
0.01	100	[1.4314, -1.1076]	-0.0625	0.7213
0.1	100	[0.8021, -0.9818]	-0.1373	0.6821
1.0	100	[-3.5249, -3.1071]	3.3478	0.3234
0.1	50	[-0.4394, 0.1654]	-0.1184	0.6808
0.1	200	[-1.9050, -1.2881]	1.4938	0.4734

Conclusion

- **Heuristic Perceptron** is simple and intuitive, suitable for small datasets.
- **Gradient Descent Perceptron** offers smoother convergence and loss tracking.
- Learning rate and number of epochs significantly affect convergence quality.

- Gradient descent outperforms heuristic in terms of training stability and loss reduction.