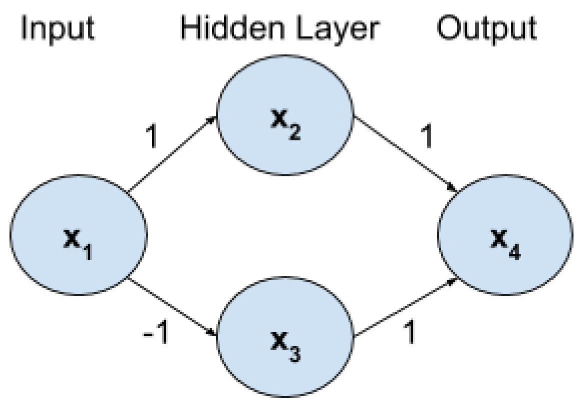
Simplex Example



Consider the above network with one hidden layer. For simplicity, I have omitted the bias variable. Suppose we want to check the property $x_1 \in [0,1]$ and $x_4 \notin [0.5,1]$. To verify this property we simply negate it and try to find a solution with the given constraints.

Equations for this network are:

- $x_2 - x_1 = x_5$ …… Equation 1
- $x_3 + x_1 = x_6$ …… Equation 2
- $x_4 - x_3 - x_2 = x_7$ …… Equation 3

Bounds are:

- $x_1 \in [0, 1]$
- $x_4 \in [0.5, 1]$
- x_2 , x 3 unbounded
- x_5 , x_6 , $x_7 \in [0, 0]$

Variable	Lower Bound	Value	Upper Bound
x_1	0	0	1
x_2	-INF	0	+INF
x_3	-INF	0	+INF
x_4	0.5	0	1
x_5	0	0	0
x_6	0	0	0
x_7	0	0	0

Update $x_4 = x_4 + 0.5$. This updates x_7 in equation 3, which is now out of bounds. We pivot x_7 and x_2 giving equation:
$x_2 = x_4 - x_3 - x_7$
Now replace the value of x_2 in equation 1 and get the following set of equations:

- $x_5 = x_4 - x_3 - x_7 - x_1$ …… Equation 1
- $x_6 = x_3 + x_1$ …… Equation 2
- $x_2 = x_4 - x_3 - x_7$ …… Equation 3

This would given the following table:

Variable	**Lower Bound**	**Value**	**Upper Bound**
$\mathbf{x_1}$	0	0	1
$\mathbf{x_2}$	-INF	0.5	+INF
$\mathbf{x_3}$	-INF	0	+INF
$\mathbf{x_4}$	0.5	0.5	1
$\mathbf{x_5}$	0	0.5	0
$\mathbf{x_6}$	0	0	0
$\mathbf{x_7}$	0	0	0

Now, x_5 is out of bounds. So, we pivot x_5 and x_1,

- $x_1 = x_4 - x_3 - x_7 - x_5$ [Pivot operation] …… Equation 1
- $x_6 = x_4 - x_7 - x_5$ [change value of x_1] …… Equation 2
- $x_2 = x_4 - x_3 - x_7$ …… Equation 3

Also, do $x_5 = x_5 - 0.5$

Variable	Lower Bound	Value	Upper Bound
x_1	0	0.5	1
x_2	-INF	0.5	+INF
x_3	-INF	0	+INF
x_4	0.5	0.5	1
x_5	0	0	0
x_6	0	0.5	0
x_7	0	0	0

Clearly, we cannot pivot x_6 with any other variables in equation 2. Hence, this is an UNSAT.