Plan

- Basic Feasible Solution (BFS)
- Non degenerate BFS
- Degenerate BFS
- Simplex Algorithm

- Consider LPP(P)
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- A basic feasible solution of a LPP of the form (1), can have at most m strictly positive components ($rank(A) = m$).

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- The variables $x_1, \ldots, x_m$ are called **basic variables**, and $x_{m+1} = x_{m+2} = \ldots = x_n = 0$, are called **non basic variables** of the **BFS** $\mathbf{x}$.

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  where $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- Clearly $rank(A) = 2$.
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  x_2, x_3 are basic variables and x_1 is non basic.

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  where, $\mathbf{c}_B^T$ are the components of $\mathbf{c}^T$ corresponding to the basic variables.
- If LPP (P) is the diet problem, (with $\geq$ inequalities changed to equalities in the original problem),
  then $\mathbf{x}$ gives the quantities of the food products F_j,
  $j = 1, \ldots, n$ in the diet.

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- For all $k = 1, 2, \ldots, n$, let

$$\tilde{\mathbf{a}}_k = \sum_{i=1}^{m} u_{ik} \tilde{\mathbf{a}}_i = [\tilde{\mathbf{a}}_1 \ldots \tilde{\mathbf{a}}_m] \begin{bmatrix} u_{1k} \\ \vdots \\ u_{mk} \end{bmatrix} \qquad \textbf{(*)}$$

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- Then $\begin{bmatrix} u_{1k} \\ \vdots \\ u_{mk} \end{bmatrix} = B^{-1}\tilde{\mathbf{a}}_k$ for all $k = 1, 2, \ldots, n$.

- In order to obtain the same amount of nutrient as unit amount of F_k, $k = 1, \ldots, n$, one needs to consume (u_{1k} amount of F_1)+ (u_{2k} amount of F_2)+...+ (u_{mk} amount of F_m).

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- For all $k = 1, \ldots, m$, note that $z_k = c_k$.
- If $z_k > c_k$ then look for a better solution.

- the simplex table corresponding to BFS $\mathbf{x}$ is given by

	0	..	0	..	$c_s - z_s$	..	$c_k - z_k$	..	
	$B^{-1}\tilde{\mathbf{a}}_1$	..	$B^{-1}\tilde{\mathbf{a}}_m$	..	$B^{-1}\tilde{\mathbf{a}}_s$	..	$B^{-1}\tilde{\mathbf{a}}_k$	..	$B^{-1}\mathbf{b}$
$\tilde{\mathbf{a}}_1$	1	..	0	..	u_{1s}	..	u_{1k}	..	x_1
$\tilde{\mathbf{a}}_2$	0	..	0	..	u_{2s}	..	u_{2k}	..	x_2
$\vdots$	0	..	0	..	$\vdots$	..	$\vdots$	..	$\vdots$
$\tilde{\mathbf{a}}_r$	$\vdots$	..	$\vdots$	..	u_{rs}	..	u_{rk}	..	x_r
$\vdots$	$\vdots$	..	$\vdots$	..	$\vdots$	..	$\vdots$	..	$\vdots$
$\tilde{\mathbf{a}}_m$	0	..	1	..	u_{ms}	..	u_{mk}	..	x_m

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  $x_s' \geq 0$ and
  $x_i' = 0$ for $i = m+1, \ldots, n$, $i \neq s$.

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- If $u_{is} > 0$ for some $i = 1, \ldots, m$, $x'_s \geq 0$, should be such that $x'_s \leq \frac{x_i}{u_{is}}$.
- **Case 1a:** For atleast one $i = 1, \ldots, m$, $u_{is} > 0$ (where s is as defined in **Case 1**). Let $\frac{x_r}{u_{rs}} = min\{\frac{x_i}{u_{is}} : u_{is} > 0\}$.

- If $\mathbf{x}'$ is feasible for the LPP, then $x'_s \geq 0$ is such that $x'_i \geq 0$ for all $i = 1, \ldots, m$.
- If $u_{is} \leq 0$, for some $i = 1, \ldots, m$, then for all $x'_s \geq 0$, $x'_i \geq 0$ for that i.
- If $u_{is} > 0$ for some $i = 1, \ldots, m$, $x'_s \geq 0$, should be such that $x'_s \leq \frac{x_i}{u_{is}}$.
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- $\mathbf{x}' \geq \mathbf{0} \Rightarrow x'_s \leq \frac{x_r}{u_{rs}}$.

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- $\frac{x_r}{u_{rs}}$ is called the **minimum ratio.**
- $\mathbf{x}' \geq \mathbf{0} \Rightarrow x'_s \leq \frac{x_r}{u_{rs}}$.
- Also $A\mathbf{x}' = \mathbf{b}$ which implies $\mathbf{x}' \in Fea(LPP)$.

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- x_s is called the **entering variable**, and x_r is called a **leaving variable**.
- If there exists $r, t \in \{m+1, \ldots, n\}$
  $\frac{x_r}{u_{rs}} = \frac{x_t}{u_{ts}} = min\{\frac{x_i}{u_{is}} : u_{is} > 0\}$,
  then take any **one** of r, t as the **leaving variable**.

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- $\mathbf{b} = \sum_{i=1, i\neq r}^{m} (x_i - \frac{u_{is}}{u_{rs}} x_r) \tilde{\mathbf{a}}_i + \tilde{\mathbf{a}}_s (\frac{x_r}{u_{rs}})$

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- If z'_k denotes the new values of z_k then $z'_k = z_k + \frac{(c_s - z_s)}{u_{rs}} u_{rk}$ and $c_k - z'_k = (c_k - z_k) - \frac{(c_s - z_s)}{u_{rs}} u_{rk}$.

- The simplex table corresponding to the new BFS $\mathbf{x}'$ is given by

	$c_s - z'_s$ $c_s - z_s - \frac{(c_s - z_s)}{u_{rs}} u_{rs} = 0$	$c_k - z'_k$ $(c_k - z_k) - \frac{(c_s - z_s)}{u_{rs}} u_{rk}$	
	$B'^{-1}\tilde{\mathbf{a}}_s$	$B'^{-1}\tilde{\mathbf{a}}_k$	$B'^{-1}\mathbf{b}$
$\tilde{\mathbf{a}}_1$	$u_{1s} - \frac{u_{1s}}{u_{rs}} u_{rs} = 0$	$u_{1k} - \frac{u_{1s}}{u_{rs}} u_{rk}$	$x_1 - \frac{u_{1s}}{u_{rs}} x_r$
$\tilde{\mathbf{a}}_2$	$u_{2s} - \frac{u_{2s}}{u_{rs}} u_{rs} = 0$	$u_{2k} - \frac{u_{2s}}{u_{rs}} u_{rk}$	$x_2 - \frac{u_{2s}}{u_{rs}} x_r$
$\vdots$	0	$\vdots$	$\vdots$
$\tilde{\mathbf{a}}_r$	$\frac{u_{rs}}{u_{rs}} = 1$	$\frac{u_{rk}}{u_{rs}}$	$\frac{x_r}{u_{rs}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\tilde{\mathbf{a}}_m$	$u_{ms} - \frac{u_{ms}}{u_{rs}} u_{rs} = 0$	$u_{mk} - \frac{u_{ms}}{u_{rs}} u_{rk}$	$x_m - \frac{u_{ms}}{u_{rs}} x_r$

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$$
\begin{array}{c|cc|c}
 & c_s - z'_s & c_k - z'_k & \\
 & c_s - z_s - \frac{(c_s - z_s)}{u_{rs}} u_{rs} = 0 & (c_k - z_k) - \frac{(c_s - z_s)}{u_{rs}} u_{rk} & \\
\hline
 & B'^{-1}\tilde{\mathbf{a}}_s & B'^{-1}\tilde{\mathbf{a}}_k & B'^{-1}\mathbf{b} \\
\hline
\tilde{\mathbf{a}}_1 & u_{1s} - \frac{u_{1s}}{u_{rs}} u_{rs} = 0 & u_{1k} - \frac{u_{1s}}{u_{rs}} u_{rk} & x_1 - \frac{u_{1s}}{u_{rs}} x_r \\
\tilde{\mathbf{a}}_2 & u_{2s} - \frac{u_{2s}}{u_{rs}} u_{rs} = 0 & u_{2k} - \frac{u_{2s}}{u_{rs}} u_{rk} & x_2 - \frac{u_{2s}}{u_{rs}} x_r \\
\vdots & 0 & \vdots & \vdots \\
\tilde{\mathbf{a}}_r & \frac{u_{rs}}{u_{rs}} = 1 & \frac{u_{rk}}{u_{rs}} & \frac{x_r}{u_{rs}} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{\mathbf{a}}_m & u_{ms} - \frac{u_{ms}}{u_{rs}} u_{rs} = 0 & u_{mk} - \frac{u_{ms}}{u_{rs}} u_{rk} & x_m - \frac{u_{ms}}{u_{rs}} x_r
\end{array}
$$

- The entry u_{rs} of the previous table which is made 1 (by dividing) in this table is called the **pivot element**.

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  So given any $M \in \mathbb{R}$, by taking x'_s sufficiently large we can make $\mathbf{c}^T\mathbf{x}'$ smaller than M.
  So the LPP does **not** have an optimal solution.

- $D = \{\mathbf{d} \in \mathbb{R}^n : \mathbf{d} \neq 0, A_{m\times n}\mathbf{d} = \mathbf{0}, \mathbf{d} \geq \mathbf{0}\}$ is the set of all directions of $S = Fea(LPP) = \{\mathbf{x} \in \mathbb{R}^n : A_{m\times n}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq \mathbf{0}, rank(A) = m\}$

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- The above optimality conditions (for max and min problems) are sufficient but not necessary for an optimal solution.

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  The simplex table corresponding to the BFS with basic variables x_2 and s_1 is given by:
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$c_j - z_j$	2	0	0	-1	
	x_1	x_2	s_1	s_2	$B^{-1}\mathbf{b}$
x_2	-1	1	0	1	1
s_1	1	0	1	-2	1

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$c_j - z_j$	1	1	0	0	
	x_1	x_2	s_1	s_2	$B^{-1}\mathbf{b}$
s_2	-1	1	0	1	1
s_1	-1	2	1	0	4

  Note that the above table is optimal.

- Suppose if we change the objective function to Min $x_1 - 2x_2$
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$c_j - z_j$	-1	0	0	2	
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$c_j - z_j$	0	0	1	0	
	x_1	x_2	s_1	s_2	$B^{-1}\mathbf{b}$
x_2	0	1	1	-1	3
x_1	1	0	1	-2	2

The above table is optimal.