Plan

- Examples of optimization problems
- Examples of Linear Programming Problems (LPP)
- Solution of an LPP by the Graphical Method
- Extreme points and Corner points
- Exercises and Questions

Optimization Problems

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- Laws of refraction of light.
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Linear Programming Problems

Diet Problem: For U.S soldiers, World War II

- Let there be m nutrients $N_1, N_2, ..., N_m$ and n food products, $F_1, F_2, ..., F_n$, available in the market which can supply these nutrients.

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  So the problem is to decide on a diet of minimum cost consisting of the n food products (in various quantities) so that one gets the required amount of each of the nutrients.

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$\mathbf{b} = [b_1, b_2, \ldots, b_m]^T$ and $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$, $\mathbf{O}$ is the zero vector with n components.

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and $\mathbf{b} = [s_1, \ldots, s_m, -d_1, \ldots, -d_n]^T$.

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A linear programming problem(LPP) is given by :
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Given $\mathbf{c} \in \mathbb{R}^n$, a column vector with n components, $\mathbf{b} \in \mathbb{R}^m$, a column vector with m components, and an $A \in \mathbb{R}^{m \times n}$, a matrix with m rows and n columns

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The function $f(\mathbf{x}) = \mathbf{c}^T\mathbf{x}$ is called the objective function, the constraints $\mathbf{x} \geq \mathbf{0}$ are called the non negativity constraints.

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and $\mathbf{c}^T\mathbf{x}$ are all linear functions from $\mathbb{R}^n \to \mathbb{R}$, hence the name **linear programming problem**.

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- If the LPP has an optimal solution, then the value of the objective function $\mathbf{c}^T\mathbf{x}$ where $\mathbf{x}$ is an optimal solution of the LPP is called the optimal value of the LPP.

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Hyperplanes, Normals, Closed Half Spaces, Polyhedral set

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- The vector $\mathbf{a}$ is called a normal to the hyperplane H, since it is orthogonal (or perpendicular) to each of the vectors $\mathbf{x} - \mathbf{x}_0$ on the hyperplane with tail at $\mathbf{x}_0$.

- A collection of hyperplanes $H_1, \ldots, H_k$ in $\mathbb{R}^n$ is said to be **Linearly Independent (LI)** if the corresponding normal vectors $\mathbf{a}_1, \ldots, \mathbf{a}_k$ are linearly independent as vectors in $\mathbb{R}^n$.

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- If they are not **LD** then the vectors are called **LI** ,that is, the only solution to (**) is $c_1 = \ldots = c_k = 0$.

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- The feasible region of a LPP is a polyhedral set.
- Since the intersection of any collection of **closed subsets** of $\mathbb{R}^n$ is again a **closed subset** of $\mathbb{R}^n$, hence Fea(LPP) is a **closed** subset of $\mathbb{R}^n$, geometrically the feasible region of a LPP contains all its boundary points.

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  The above linear programming problem does not have an optimal solution.

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  Clearly the feasible region of this problem is the empty set. So this problem is called **infeasible**, and since it **does not** have a feasible solution it obviously does not have an optimal solution.

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  That is, if $\mathbf{x}_1$ and $\mathbf{x}_2$ are two optimal solutions of a LPP, then are $\mathbf{y}$'s of the form $\mathbf{y} = \lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2$, $0 \leq \lambda \leq 1$, **also** optimal solutions of the LPP?

- A nonempty set, $S \subseteq R^n$ is said to be a convex set if for all $\mathbf{x}_1, \mathbf{x}_2 \in S$, $\lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2 \in S$, for all $0 \leq \lambda \leq 1$.

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  $[\frac{2}{3}, 0]^T = \frac{1}{3}[0, 0]^T + \frac{2}{3}[1, 0]^T$ is a convex combination as well as a strict convex combination of $\mathbf{x}_1$ and $\mathbf{x}_2$.

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  The answer to this question is YES, due to Weierstrass, called the Extreme Value Theorem:

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- Given a LPP with a nonempty feasible region, $Fea(LPP) = S \subset \mathbb{R}^n$, an $\mathbf{x} \in S$ is called a corner point of S, if $\mathbf{x}$ lies at the point of intersection of n linearly independent hyperplanes defining S.

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- All points on the **boundary** of a disc are extreme points of the disc.
- A hyperplane, half space does not have any extreme point.

- Theorem: If $S = Fea(LPP)$ is nonempty, where $S = \{\mathbf{x} \in \mathbb{R}^n : \ \mathbf{a}_i^T\mathbf{x} \leq b_i, i = 1, \ldots, m, \quad -\mathbf{e}_j^T\mathbf{x} \leq 0, j = 1, \ldots, n\}$
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- The total number of extreme points of the feasible region of a LPP with $(m+n)$ constraints (including the non negativity constraints) is $\leq (m+n)_{C_n}$.
- Exercise: Think of a LPP $(m+n)$ constraints such that the number of extreme points of the $Fea(LPP)$ is equal to $\leq (m+n)_{C_n}$.

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