Option Pricing using Binomial and Black-Scholes Models

Abstract

This report demonstrates the pricing of European-style options using the Binomial and Black-Scholes models. Real-world stock prices from Yahoo Finance are used to calculate expected return and volatility, which serve as inputs for the models. The performance of both models is analyzed with visualizations, showing the impact of increasing steps in the binomial model and comparing the results with the Black-Scholes formula. Additional graphs are presented to better understand stock price trends, volatility, and return distributions.

1 Introduction

Options are financial derivatives that derive their value from the price of an underlying asset. This project uses the `yfinance` library to fetch historical stock prices and applies two models for option pricing:

1. The Binomial Model
2. The Black-Scholes Model

Both models help us understand how the stock price and volatility affect the option's value over time.

2 Objective

The primary objectives of this project are:

- Implement and compare two option pricing models.
- Use real-world stock data to calculate option prices.
- Analyze how the number of steps impacts the binomial model's results.

3 Methodology

3.1 Fetching Historical Data

Using the `yfinance` library, we fetch Tesla Inc. (TSLA) stock data from February 1, 2024, to March 30, 2024. We extract the 'Close' prices to analyze the stock's behavior and compute returns and volatility.

3.2 Calculating Return and Volatility

The daily log returns are calculated as:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

The expected return (μ) and volatility (σ) are computed based on these returns over a 100-day window.

3.3 Binomial Model

The binomial model builds a tree of possible stock prices and computes the option price by working backward from the expiration date to the present. The formula for each step is:

$$u = e^{\sigma\sqrt{dt}} \tag{2}$$

$$d = e^{-\sigma\sqrt{dt}} \tag{3}$$

$$p = \frac{e^{r\cdot dt} - d}{u - d} \tag{4}$$

where u and d are the upward and downward movements, and p is the risk-neutral probability.

3.4 Black-Scholes Model

The Black-Scholes model is given by:

$$d_1 = \frac{\ln(S_0/K) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}} \tag{5}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{6}$$

The call option price is:

$$C = S_0 \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2) \tag{7}$$

where $N(\cdot)$ is the cumulative distribution function of the standard normal distribution.

4 Results and Analysis

4.1 Binomial Model Results

We compute the option price using the binomial model for different step sizes. The results are summarized in Table.

4.2 Black-Scholes Model Result

Using the Black-Scholes formula with $r = 0.05$ and the estimated volatility, we get the following call option price:

$$C = 39.71 \tag{8}$$

Table 1: Option Prices using Binomial Model

Steps (n)	Option Price
2	39.68
4	39.68
10	39.69
50	39.70
100	39.71
300	39.71
600	39.71

5 Conclusion

This project successfully demonstrates the use of the binomial and Black-Scholes models for pricing European options. The binomial model shows convergence with increasing steps, while the Black-Scholes model provides an analytical solution. The additional graphs help visualize the stock's behavior, including trends and volatility.