Co-ordinate Geometry

1. Two medians PS and RT of ΔPQR intersect at G at right angles. If PS = 9 cm and RT = 6 cm, then the length of RS in cm is

a)10 b)6 c) 5 d)3

2. The length of side AB and side BC of a scalene triangle ABC are 12 cm and 8 cm respectively. The value of angle C is 59°. Find the length of side AC.
   a) 12 b)10 c)14 d)16

3. The coordinates of the in centre of the triangle whose sides are $3x - 4y = 0$, $5x + 12y = 0$ and $y - 15 = 0$, are
   a) (1,8) b) (1,-8) c) (2,8) d) (2,-8)

4. G is the centroid of the equilateral ΔABC. If AB = 10 cm then length of AG is
   a) $(5\sqrt{3}) / 3$ cm b) $(10\sqrt{3}) / 3$ cm c) $5\sqrt{3}$ cm d)none

5. If the three medians of a triangle are same then the triangle is
   a)Equilateral b)isosceles c)right angle d)obtuse

6. The circle $x^2 + y^2 - 6x - 10y + k = 0$ does not touch or intersect the coordinate axes. If the point (1, 4) does not lie outside the circle, and the range of k is (a, b] then a + b is
   a) 34 b) 54 c) 20 d) None

7. Find the equation of straight line passing through (2, 3) and perpendicular to the line $3x + 2y + 4 = 0$
   a) y=5/3x- 2 b) 3Y=2x+5 c) 3Y=5x-2 d) None

8. Find the coordinates of the point which will divide the line joining the points (3, 5) and (11, 8) externally in the ratio 5: 2.
   a) (5/3, 1/3) b) (3/49, 1/10) c) (49/3, 10) d) None

9. What is the slope of the line passing through the points J (-2, 3) and (2, 7)?
   a) 1 b) 2 c) 4/3 d) 3/4

10. Find the equation of the line passing through (2, -1) and parallel to the line $2x - y = 4$.
    a) $y = 2x - 5$
    b) Y=2x+6
    c) Y=√2x +7
    d) 4.y=2x+5

11. Find the coordinates of the circum-centre of the triangle whose vertices are (0, 0), (8,0) and (0,6). Find the Circum-radius also.
    a) (4,3) , 6 b) (3,4) , 5 c) (4,3,) , 5 d) (4,3,) , 3

12. Find the area of the region that comprises all points that satisfy the two conditions x2 + y2 + 6x + 8y ≤ 0 and 4x ≥ 3y?

13. What is the equation of a set of points equidistant from the lines y = 5 and x = –4?
    a) $x + y = -1$
    b) $x - y = -1$
    c) $x + y = 1$
    d) $-x + y = -1$

14. The shortest distance from the point (-4,3) to the circle $x^2 + y^2 = 1$ is __________.

15. The equation of the straight line passing through the point M (-5,1), such that the portion of it between the axes is divided by the point M in to two equal halves, is

a) 10y - 8x = 80
b) 8y + 10x = 80
c) 10y + 8x = 80
d) 8y + 10x + 80 = 0

16. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 31), and (31, 0) is

a) 435
b) 465
c) 450
d) 464

17. The points (1, 3) and (5, 1) are the opposite vertices of a rectangle. The other two vertices lie on the line y=2x+c, then the value of c will be ___________.

18. N is the foot of the perpendicular from a point P of a circle with radius 7 cm, on a diameter AB of the circle. If the length of the chord PB is 12 cm, the distance of the point N from the point B is
a) $6\frac{5}{7}$
b) $12\frac{2}{7}$
c) $3\frac{5}{7}$
d) $10\frac{2}{7}$

19. On the parabola $y = x^2$, the point least distance from the straight line $y = 2x - 4$ is ____________.
a) (1,1) b) (1,-2) c) (1,-1) d) none

20. The line $x - 1 = 0$ is the Directrix of the parabola, $y^2 - kx + 8 = 0$. Then, one of the values of k is ________.

a) k =8,2 b) k =-7,8 c) k= -8, 4 d) None

21. Let E be the ellipse $x^2 / 9 + y^2 / 4 = 1$ and C be the circle $x^2 + y^2 = 9$. Let P and Q be the points (1, 2) and (2, 1), respectively. Then

a) Q lies inside C but outside E

b) Q lies outside both C and E

c) P lies inside both C and E

d) P lies inside C but outside E

22. The foci of the ellipse $25 (x + 1)^2 + 9 (y + 2)^2 = 225$ are at ___________

a) (1,3) (1,6) b) (1,2)(1,6) c) (1,6)(1,2) d) (1,15)(1,3)

23. The equation of the ellipse whose latus rectum is 8 and whose eccentricity is $1 / \sqrt{2}$, referred to the principal axes of coordinates, is __________.

24. The circle $x^2 + y^2 = 8x$ and hyperbola $x^2/9 – y^2/4 = 1$ intersect at the points A and B. Find the equation of a common tangent with positive slope to the circle as well as to the hyperbola.

25. The circle passing through (1, -2) and touching the axis of x at (3, 0) also passes through the point:

(a) (–5, 2) (b) (2, –5) (c) (5, –2) (d) (–2, 5)

- **Solutions:**

1. PS = 9 cm
   $\Rightarrow$ GS = (1 / 3) × 9 = 3 cm
   RT = 6 cm
   $\Rightarrow$ RG = (2 / 3) × 6 = 4 cm
   $\therefore$, Rs = $\sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = 5$ cm
   Hence, option C is correct.

2. Given, AB = 12 cm, BC = 8 cm,
   $\angle C = 59°$
   Let $\angle A = \Theta$
   $\therefore$ $\angle B = 180° – (59° + \Theta) = 121° – \Theta$
   Now, let us see the choices. If AC = 12 cm, triangle would not be scalene. Hence, option A is ruled out. If AC = 10 cm, AB will become the largest side and ∠C the largest angle. But ∠C = 59°. Hence option B is ruled out. So, AC is either 14 cm or 16 cm. In any case, ∠B will be the largest angle and ∠A (say Θ) the smallest:
   Also, $\angle B = 180° – (59° + \Theta) = 121° – \Theta$
   By sine formula,
   (8 cm / sin Θ) = (12 cm / sin 59º) = AC / {sin(121º – Θ)}
   Thus, (8 cm / sin Θ) ≈ (12 cm / sin 60º)
   or, sin Θ ≈ (8 cm × sin 60º) / 12 cm
   $= (2/3) \times (\sqrt{3}/2) = (1/\sqrt{3}) = 0.577$
   $\therefore \cos\theta = \sqrt{\{1 – (1/3)\}} = \sqrt{(2/3)}$
   $\therefore$ sin (121° – Θ) ≈ sin (120° – Θ)
   = sin 120° cosΘ – cos120° sin Θ
   $= (\sqrt{3} / 2) \times \sqrt{(2 / 3)} – (–1 / 2) \times (1 /\sqrt{3})$
   $= (1 / \sqrt{2}) + (1 / 2\sqrt{3}) = 0.996$
   = Now, {AC / sin (121º – Θ)} / sin Θ
   or, AC = {8 cm sin (120º – Θ)} / sin Θ
   = {(8 × 0.996) / 0.0577} = 13.809 ≈ 14 cm
   Hence, option C is correct.

3. $3x - 4y \equiv 0$ ...(i)
$5x + 12y \equiv 0$ ....(ii)
$y - 15 \equiv 0$ ...(iii)
From (i) and (ii), A = (0, 0)
From (i) and (iii), B = (20, 15)
From (ii) and (iii), C = (–36, 15
$BC = \sqrt{\{(20 + 36)^2 + (15 - 15)^2\}} = 56$
$AB = \sqrt{\{(20)^2 + (15)^2\}} = 25$

$AC = \sqrt{(36^2 + 15^2)} = 39$
Let (α, β) be the incentre co-ordinates of ΔABC
$\alpha \{56 \times 0 + 39 \times 20 + 25 (-36)\} / 56 + 39 + 25$
$= (-900 + 780) / 120 = -120 / 120 = -1$
$\beta = \{56 \times 0 + 39 \times 15 + 25 (15)\} / 56 + 39 + 25 = 8$
$\therefore (\alpha\ \beta)\ (-1, 8)$
Hence, option B is correct.

4. AB = 10 cm
$\therefore BD = (AB / 2) = 5$ cm
$\angle ADB = 90°$
By Pythagoras theorem in ΔABD,
$\therefore AD = \sqrt{(AB^2 - BD^2)}$
$= \sqrt{(10^2 - 5^2)} = \sqrt{75} = 5\sqrt{3}$ cm
We know that,
$AG = (2 / 3)AD = (2 / 3) \times 5\sqrt{3}$
$= (10\sqrt{3}) / 3$ cm
Hence, option B is correct.

5. The median of an equilateral triangle are equal.
Hence, option A is correct.

6. $(x-3)^2+(y-5)^2=34-k$ also r> 5 thus 34-k >5 => k< 29
also for other axis $34-k < 3^2$
- ⇨ 34-k <9
- ⇨ k>34-9
- ⇨ k>25
- ⇨ range of [a,b]=[25,29] thus a+b =54

7. The given line is $3x + 2y + 4 = 0$ or $y = -3x / 2 - 2$

   Any line perpendicular to it will have slope = 2 / 3
   Thus equation of line through (2, 3) and slope 2 / 3 is
   $(y – 3) = 2 / 3 (x – 2)$
   $3y – 9 = 2x – 4$
   $3y – 2x – 5 = 0$.

8. The external division case will use the formula
   $x = (mx2 – nx1)/(m – n)$
   $y = (my2 – ny1)/(m – n)$
   where m:n is 5:2 in our case.
   Putting the values you will get points (49/3, 10).

9. $[(y2 - y1) / (x2 - x1)] = [(7-3) / (2-(-2))] = 4/4 = 1$

10. The given line is $2x – y = 4 \Rightarrow y = 2x – 4$ (Converting into the form of $y = mx + c$)
    Its slope = 2. The slope of the parallel line should also be 2.
    Hence for the required line
    m = 2 and (x1 , y1) = (2, -1).
    Equation = y - y1 /x - x1 = y2 - y1 /x2 - x1
    $\Rightarrow$ y - y1 /x - x1 = m
    $\Rightarrow$ y – y1 = m (x – x1) $\Rightarrow$ y – (- 1) = 2 (x – 2)
    $\Rightarrow$ y = 2x – 5.

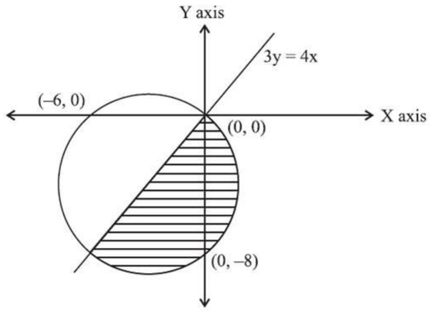
11. Circum-centre is the point of intersection of the perpendicular bisectors of the three sides of a triangle.
    Let S (x, y) be the circumcentre.
    SA = SB = SC. ∴ root of $x^2 + y^{2)}$ = root of $(x-8)^2 + (y-2)\ )^{2.}$
    $\sqrt{}(x^2 + y^2) =(\sqrt{}x-0)^2 + (y-6)\ )^2$.
    Squaring $x^2 + y^2 = x^2 – 16x + 64 + y^{2.}$ So x = 4.
    Also $x^2 + y^2 = x^2 + y^2 – 12y + 36$.
    So y = 3. Hence coordinates of the circumcentre is (4, 3).
    Circumradius = SA $=\sqrt{}\ x^2 + y\ ^2= \sqrt{16+9} = 5$.

12. $x^2 + y^2 + 6x + 8y < 0$

$x^2 + 6x + 9 – 9 + y^2 + 8y + 16 – 16 < 0$

$(x + 3)^2 + (y + 4)^2 < 25$

This represents a circular region with centre (–3, –4) and radius 5 units. Substituting x = y = 0, we also see that the in equation is satisfied. This means that the circle also passes through the origin. To find out the intercepts that the circle cuts off with the axes, substitute x = 0 to find out the y–intercept and y = 0 to find out x–intercept. Thus x–intercept = –6 and y–intercept = –8.



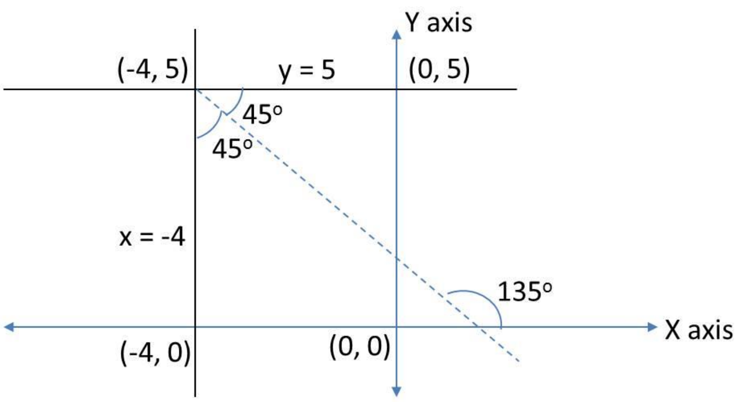
Now, the line 4x = 3y passes through the point (–3, –4). Or this line is the diameter of the circle. The area we are looking for is the area of a semicircle.

Required area = 25π/2

13. A set of points equidistant from the given two lines should lie on the dotted line as indicated. You can think of it as the perpendicular bisector to the base of an isosceles triangle formed by (–4, 5) and the two points on x = –4 and y = 5.

Or, the set of points equidistant from two lines form the angle bisector of the angle formed at the point of intersection of the two lines. The angle between these two lines is 900. Importantly, the lines are parallel to the axes. So, thinking of the line that is the angle bisector of this angle should not be too difficult.

This dotted line is at an angle of 135° with respect to the positive direction of x–axis and also passes through (–4, 5).



Slope = m = tan (135°) = –1.

Therefore, the equation is given by $(y - y_1) = m$

$(x - x_1)$ where (x_1, y_1) is $(-4, 5)$.
$(y - 5) = -(x + 4)$
$x + y = 1$

14. We can draw many lines to the circle that can be a secant or a tangent. We need to find the line which gives us the shortest distance of the point from the circle.
It is very evident that the line joining the center of the circle to the point (-4,3) will give us the shortest distance.
As we can see from the diagram, we have a right triangle with base length as 4 and height as 3.
Therefore the distance of the point (-4,3) from the center of the circle is the hypotenuse and its length is 5. (3,4,5 is a Pythagorean triplet)
Therefore, the distance of the point (-4,3) from the circle = 5 - radius of the circle = 5 - 1 = 4

15. The point P divides AB into 2 halves.
AP = PB
A is on the x axis, Hence Point A (h,0)
B is on the Y axis, Hence Point B (0,k)
We need to use midpoint formula.
[(h+0)2(h+0)2 , (k+0)2(k+0)2] = (-5 , 4)
h/2 = - 5
h = -10
k/2 = 4
k = 8
Hence the Y intercept is 8 and X intercept is - 10.
x−10x−10 + y8y8 = 1
-8x + 10y = 80
the equation is -8x + 10y = 80

16. For the point to be in interior of triangle we should consider x=1 , 2, 3.. upto 30 and y to 1,2,...29
Thus no of points = (29x30)/3 = 29x15= 435

17. Let ABCD be a rectangle. Given A (1, 3) and C (5, 1). Equation B and D lie on y=2x+c We know that the intersecting point of diagonal of a rectangle is the same or at the midpoint. So, midpoint of AC is (3, 2). Hence, y = 2x + c passes through (3, 2). Therefore, c = −4.

18. Radius = 7 cm

$\Rightarrow$ Diameter, AB = 14 cm

PB = 12 cm

$\angle APB = 90°$

[$\because$ angle in the semi circle]

In ΔAPB, By Pythagoras theorem

$AP = \sqrt{(AB^2 - PB^2)} = \sqrt{(142 - 122)} = \sqrt{52}$

Let, AN = x cm $\Rightarrow$ NB = (14 – x) cm

In ΔAPN, By Pythagoras theorem

$PN^2 = AP^2 - AN^2 = 52 - x^2$ ...(i)

Again, In ΔPNB, By Pythagoras theorem

$PN^2 = PB^2 - NB^2 = 144 - (14 - x)^2$ ...(ii)

From Equation (i) and (ii),

$52 - x^2 = 144 - 196 + 28x - x^2$

$28x = 104$

$X = 26 / 7$

Hence, option D is correct.

19. Given, parabola $y = x^2$ .....(i)

Straight line $y = 2x - 4$ .....(ii)

From (i) and (ii),

$x^2 - 2x + 4 = 0$

Let $f(x) = x^2 - 2x + 4$,

$f'(x) = 2x - 2$.

For least distance, $f'(x) = 0$

$\Rightarrow 2x - 2 = 0$

$x = 1$

From $y = x^2$, $y = 1$

So, the point least distant from the line is (1, 1).

20. The parabola is $y^2 = 4 * [k / 4] (x - [8 / k])$.

Putting $y = Y$, $x - 8k = X$, the equation is $Y^2 = 4 * [k / 4] * X$

The directrix is $X + k / 4 = 0$,i.e. $x - 8 / k + k / 4 = 0$.

But $x - 1 = 0$ is the directrix.

So, $[8 / k] - k / 4 = 1$

$\Rightarrow k = -8, 4$

21. The given ellipse is $[x^2 / 9] + [y^2 / 4] = 1$. The value of the expression $[x^2 / 9] + [y^2 / 4] - 1$ is positive for $x = 1$, $y = 2$ and negative for $x = 2$, $y = 1$. Therefore, P lies outside E and Q lies inside E. The value of the expression $x^2 + y^2 - 9$ is negative for both the points P and Q. Therefore, P and Q both lie inside C. Hence, P lies inside C but outside E.

22. $25(x + 1)^2/225 + 9(y + 2)^2/225 = 1$

Here, $a = \sqrt{[225 / 25]} = 15 / 5$, $b = \sqrt{[225/9]} = 15 / 3$

$e = \sqrt{[1 - 9 / 25]} = 4 / 5$

Focus $= (-1, -2 \pm [15 / 3] * [4 / 5])$

$= (-1, -2 \pm 4)$

$= (1, 2); (1, 6)$

23. Here,$2b^2/ a = 8$, $e = 1/ \sqrt{2}$

$a^2 = 64$, $b^2 = 32$

Hence, the required equation of ellipse is $x^2/64 + y^2/32 = 1$.

24. The equation of circle $x^2 + y^2 = 8x$ can be rewritten as $(x - 4)^2 + y^2 = 16$

Tangent to hyperbola is $y = mx + \sqrt{(9m^2-4)}$, $m> 0$.

Distance from center to the tangent is:

$$\left| \frac{4m + \sqrt{9m^2 - 4}}{\sqrt{1 + m^2}} \right| = 4$$

on solving above equation, we get

$m = 2/\sqrt{5}$

$\Rightarrow y = 2x/\sqrt{5} + 4/\sqrt{5}$

Or $2x - \sqrt{5}\,y + 4 = 0$

Therefore, $2x - \sqrt{5}\,y + 4 = 0$ is equation of a common tangent with positive slope to the circle as well as to the hyperbola.

25. The equation of circle passing through the point (a,b) and having radius r is $(x - a)^2 + (y - b)^2 = r^2$

Since given circle touches the x-axis at (3, 0) and passes through the point (1, -2).

Find radius of the circle using distance formula,

So, $(1 - 3)^2 + (r + 2)^2 = r^2$

$4 + r^2 + 4 + 4r - r^2 = 0$

$\Rightarrow r = 2$

So, circle is $(x - 3)^2 + (y + 2)^2 = 4$

Point (5, -2) satisfy the equation.