Manoeuvre
The Career Signature

Algebra

❖ Content

- Fundamental operations in Algebra, Expansion , Factorization , Quadratic Equations
- Progressions, Arithmetic, Geometric and Harmonic Progression , Relation between them
- Surds and Indices
- Logarithms
- Exponential and Log Series
- Permutations and Combinations

1. How many positive numbers less than 10,000 are such that the product of their digits is 210?
   a) 36 b) 42 c) 48 d) 54

2. Ten points are marked on a straight line and eleven points are marked on another straight line. How many triangles can be constructed with vertices from among the above points?
   a) 495 b) 550 c) 1045 d) 2474

3. α, β are the roots of the an equation $x^2 - 2x\cos\theta + 1 = 0$, then the equation having roots αn and βn is
   a) $x^2-2\cos n\theta x+1=0$

   b) $2x^2-2\cos n\theta x-1=0$

   c) $x^2+2\cos n\theta x+1=0$

   d) $x^2-2\cos n\theta x -1 =0$

4. The equation $(x-a)^3+(x-b)^3+(x-c)^3 = 0$ has

   a) All real roots

   b) One real and two imaginary roots

   c) Three real roots namely x=a, x=b, x=c

   d) None

5. Let α and β be the roots of the equation x^2-px +r =0and α /2 and β are the roots of the equation x^2-ax +r , then the value of r,
   a) 2/9(p-q)(2q-p)
   b) 2/9(q-p)(2q-p)
   c) 2/9(q-2p)(2q-p)
   d) 2/9(2p-q)(2q-p)

6. What should come at the place of question mark? 46080, 3840, 384, 48, 8, 2, ?
   a) 1
   b) 1/64
   c) 1/8
   d) None

7. The sum of third and ninth term of an A.P is 8. Find the sum of the first 11 terms of the progression.
   a) 44
   b) 22
   c) 19
   d) None

8. One-fifth of a number is equal to 58th of another number. If 35 is added to the first number, it becomes four times of the second number. Find the second number.
   a) 39
   b) 70
   c) 40
   d) 25

9. Simplification: $25^{(2.7)}\times5^{(4.2)}\div5^{(5.4)}$=?
   a) 5^4
   b) $5^{(3.2)}$
   c) $5^{(4.1)}$
   d) $5^{(4.2)}$

10. Sum of first positive integers will be
   a) 1200
   b) 1300
   c) 1375
   d) 1275

11. If log 2, log (2x - 1) and log (2x + 3) are in A.P, then x is equal to ____

a) 5/2
b) $\log_2 5$
c) $\log_3 2$
d) 32

12. Given √2 = 1.414 and the value of √8 + 2√32 – 3√128 + 4√50 is

a) 8.484
b) 8.526
c) 8.426
d) 8.8.76

13. If √15 = 3.88, then what is the value of √(5 / 3) ?

a) 1.293
b) 1.2934
c) 1.29
d) 1.295

14. If $9^n \times 3^5 \times (27)^3 / 3 \times (81)^4 = 27$, then the value of *n* is:

a) 0
b) 2
c) 3
d) 4

15. The term independent of x in the binomial expansion of is: $\left(\frac{3}{2}x - \frac{1}{3x}\right)^{10}$

a) -63/8
b) 63/8
c) 21/8
d) -21/8

16. **Set of 2 Questions(16-17):** Five students are to be arranged on five chairs for a photograph. Three of these are girls and the rest are boys. Find out the total number of ways in which three girls are together.

a) 36
b) 84
c) 100
d) 120

17. Find out the number of ways in which all three girls do not occupy consecutive seats.

a) 120
b) 36
c) 84
d) None

18. A six letter word is to be formed by using at least two vowels in it. How many such words can be formed (not necessarily meaningful) if all the letters in word are different?

a) 53349120
b) 53439120
c) 53431920
d) None

19. How many four digit numbers, which are divisible by 6, can be formed using the digits 0, 2, 3, 4, 6, such that no digit is used more than once and 0 does not occur in the left-most position?

a) 52
b) 49
c) 50
d) None

20. If $\log_2 X + \log_4 X = \log_{0.25}\sqrt{6}$ and $x > 0$, then x is

a) $6^{-1/6}$
b) $6^{1/6}$
c) $3^{-1/3}$
d) $6^{1/3}$

21. If a,b,c are in A.P. , b,c,d are in G.P. and c,d,e are in H.P. then a,c,e are in
    a) A.P.
    b) G.P.
    c) H.P
    d) No particular order

22. In the expansion of $(x^2-2x)^{10}$ the coefficient of x^{16} is
    a) -1680
    b) 1680
    c) 3360
    d) 6720

23. If a and b are roots of equation 6x2-5x+1=0 then $\tan^{-1} a + \tan^{-1} b$ is
    a) π/2
    b) π/4
    c) 0
    d) 1

24. The number which exceeds its positive square root by 12 is
    a) 9
    b) 16
    c) 25
    d) None

25. If the interior angles of a polygon are in AP If the smallest angle be 120 and the common difference be 5 then the number of sides –
    a) 8
    b) 9
    c) 10
    d) 6

❖ Answer Keys:

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	6	A	11	A	16	A	21	B
2	C	7	A	12	A	17	C	22	C
3	A	8	C	13	A	18	B	23	D
4	B	9	D	14	C	19	C	24	B
5	D	10	D	15	A	20	A	25	B

❖ Solution:

1. 210=1*2*3*5*7=1*6*5*7.
   (Only 2*3 makes the single digit 6).
   So, four digit numbers with combinations of the digits {1,6,5,7} and {2,3,5,7} and three digit numbers with combinations of digits {6,5,7} will have the product of their digits equal to 210.
   {1,6,5,7} # of combinations 4!=24
   {2,3,5,7} # of combinations 4!=24
   {6,5,7} # of combinations 3!=6
   24+24+6=54.

2. We can get the triangles in two different ways.

   Taking two points from the line having 10 points

   (in $^{10}C2$ ways, i.e., 45 ways) and one point from the line consisting of 11 points (in 11 ways).
   So, the number of triangles here is 45×11=495.
   Taking two points from the line having 11 points (in $^{11}C2$, i.e., 55 ways) and one point from the line consisting of 10 points (in 10 ways), the number of triangles here is 55×10=550
   Total number of triangles, =495+550

   =1,045 triangles

3. The given quadratic equation $x^2 - 2x\cos\theta + 1 = 0$ having roots α and β.

   so $x = \dfrac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$

   $x = \cos\theta \pm i\sin\theta$
   then, $\alpha = \cos\theta + i\sin\theta$
   $\beta = \cos\theta - i\sin\theta$
   Now, $\alpha^n = \cos n\theta + i\sin n\theta$
   & $\beta^n = \cos n\theta - i\sin n\theta$
   Now $\alpha^n + \beta^n = 2\cos n\theta$
   $\alpha^n\beta^n = 1$
   Then quadratic equation whose roots are α^n and β^n are
   $x^2 - (\alpha^n + \beta^n)x + (\alpha\beta)^n = 0$
   $x^2 - (2\cos n\theta)x + 1 = 0$

4. Let $f(x) = (x - a)^3 + (x - b)^3 + (x - c)^3$.
   Then $f'(x) = 3\{(x - a)^2 + (x - b)^2 + (x - c)^2\}$
   clearly , $f'(x) > 0$ for all x.
   so, $f'(x) = 0$ has no real roots.
   Hence, $f(x) = 0$ has two imaginary and one real root

5. Answer

$$\alpha + \beta = p,\ \alpha\beta = r$$

and $\frac{\alpha}{2} + 2\beta = q$.

But $\alpha + \beta = p$ and $\alpha + 4\beta = 2q$

$$\Rightarrow \quad \beta = \frac{1}{3}(2q - p) \text{ and } \alpha = \frac{2}{3}(2p - q)$$

Thus $\alpha\beta = r$

$$\Rightarrow \quad \frac{2}{9}(2p - q)(2q - p) = r$$

6. Progression is 2*4=8 , 8* 6= 48, 48* 8= 384 and so on..

7. 3rd term = t3 , 6th term = t6 and 9th term = t9 and let d be difference between terms.
   so , t6 – 3d + t6 +3d = 8
   ⇨ 2t6 =8
   ⇨ t6=4
   ⇨ sum of 11 terms = t6* 11= 44

8. Let the first number be F, and the second number be S.

   One-fifth of the first number is equal to 5/8th of second number
   15 F = 58 S
   S = 825 F.
   If 35 is added to the first number, it becomes four times of the second number.
   F + 35 = 4 S
   S = 825 F.
   F + 35 = 4 S.
   S = 825 (4S - 35)
   25 S = 32 S – 8 * 35
   7 S = 280

S = 40
The second number is 40.

9. Let's recall a few properties of exponents,

$(a^m)^n = a^{m*n}$
$a^m * a^n = a^{m+n}$
$a^m / a^n = a^{m-n}$

$25^{2.7} \times 5^{4.2} \div 5^{5.4}$
$= (5^2)^{2.7} \times 5^{4.2} \div 5^{5.4}$
$= 5^{5.4} \times 5^{4.2} \div 5^{5.4}$
$= 5^{5.4+4.2-5.4}$
$= 5^{4.2}$

Therefore, $25^{2.7} \times 5^{4.2} \div 5^{5.4} = 5^{4.2}$

10. Sum of first n natural numbers which are in A.P Thus sum = 50/2(1+50)

11. log 2 , log (2x-1) , log (2x+3) are in A.P. implies that 2, (2x-1) and (2x+3) are in G.P.

When 3 three terms a, b, c are in G.P. b becomes the Geometric Mean of a, b

Therefore, (2x-1) is the Geometric Mean of 2 and (2x+3)

$(2x-1)^2 = 2 * (2x+3)$
$4x^2 + 1 - 4x = 4x + 6$
$4x^2 – 8x – 5 = 0$
$(2x - 5) (2x + 1) = 0$

x = 5/2 or x = -1/2
When x = -1/2, the second term becomes, log(-2). Since log(negative number) is not legal. => x = 5/2

12. $\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$

$= 2\sqrt{2} + 8\sqrt{2} - 3 \times 8\sqrt{2} + 4 \times 5\sqrt{2}$

$= (2 + 8 - 24 + 20)\sqrt{2}$

$= 6\sqrt{2} = 6 \times 1.414 = 8.484$

Hence, option A is correct.

13. Given, $\sqrt{15} = 3.88$

Now, $\sqrt{(5 / 3)} = \sqrt{(5 \times 3) / (3 \times 3)} = \sqrt{15} / 3$

$= 3.88 / 3 = 1.293$

Hence, option A is correct.

14. $\{9^n \times 3^5 \times (27)^3\} / 3 \times (81)^4$

$= 27 \Leftrightarrow \{(3^2)^n \times 3^5 \times (3^3)^3\} / 3 \times (3^4)^4$

$= 3^3 \Leftrightarrow (3^{2n} \times 3^5 \times 3^{(3x3)} / 3 \times 3^{(4\times4)} = 3^3$

$\Leftrightarrow 3^{2n+5+9} / 3 \times 3^{16}$

$= 3^3 \Leftrightarrow 3^{2n+14} / 3^{17}$

$= 3^3 \Leftrightarrow 3^{(2n+14-17)} = 3^3$

$\Leftrightarrow 3^{2n-3} = 3^3$

From the equation powers :

$\Leftrightarrow 2n - 3 = 3 \Leftrightarrow 2n = 6 \Leftrightarrow n = 3.$

Hence, option C is correct.

15.

The general term of the given binomial expansion is:

$$T_{r+1} = {}^{10}C_r \left(\frac{3}{2}x\right)^{10-r} \left(-\frac{1}{3x}\right)^r$$
$$= {}^{10}C_r \left(\frac{3^{10-2r}}{2^{10-r}}\right) x^{10-2r} (-1)^r$$

Now, for this term to be independent of x, we know that: $(10 - 2r) = 0$

$$\Rightarrow r = 5$$

$$\therefore T_6 = {}^{10}C_5 \left(\frac{3^{10-2.5}}{2^{10-5}}\right) x^{10-2.5} (-1)^5$$
$$= \frac{10!}{5!} \cdot \frac{3^{10-10}}{2^5} \cdot x^0 \cdot (-1)$$
$$= \left(-\frac{1}{2^5}\right) \cdot \frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1}$$
$$= -\frac{63}{8}$$

16. -17 As per the question, three girls can't occupy consecutive seats but two can.

Therefore, if we find the number of ways in which all three girls occupy consecutive seats and subtract this number from the total number of ways in which the five people can be arranged among themselves, we will get the required answer.
5 students can be arranged among themselves in ${}^5p_5$ ways = 120 ways.
Assume that the 3 girls are one entity. The total number of ways in which they can be arranged among themselves = 3! = 6
Also, the set of three girls and the other students can be arranged among themselves in 3! = 6 ways.
Thus, total number of ways in which three girls are together = 6 × 6 = 36
Thus, number of ways in which all 3 girls will not occupy consecutive seats = 120 – 36 = 84

17. Ans 84

18. Six letter words with at least two vowels can have 2, 3, 4 or 5 vowels as no letters can be repeated.
There are 21 consonants and 5 vowels.
All possible cases:
2 vowels and 4 consonants
3 vowels and 3 consonants
4 vowels and 2 consonants
5 vowels and 1 consonant
∴ Number of ways in which this can be done = ${}^5C_2 \times {}^{21}C_4 + {}^5C_3 \times {}^{21}C_3 + {}^5C_4 \times {}^{21}C_2 + {}^5C_5 \times {}^{21}C_1$
$= 10 \times 5985 + 10 \times 1330 + 5 \times 210 + 1 \times 21 = 74221$
In each of these cases, chosen 6 letters can arrange themselves in 6! Ways.
∴ Total number of ways in which this can be done = $6! \times 74221 = 720 \times 74221 = 53439120$

19. We have to find the no. of four digit numbers which are divisible by 6 that are to be formed using the digits 0, 2, 3, 4, 6.
We should know that 0 does not occur in the left most position. If the numbers are to be divisible by 6 it should be a multiple of 2 and 3.
So it should be an even number and also should be a multiple of 3.
$\Rightarrow 2 + 3 + 4 + 6 + 0 = 15$
Here we have two choices, drop something such that the number should be a multiple of 3 or we can drop a multiple of 3.
The following are the possibilities,
Drop 3 $\Rightarrow 2 + 4 + 6 + 0 = 12$ which is a multiple of 3.
Drop 6 $\Rightarrow 2 + 3 + 4 + 0 = 9$ which is a multiple of 3.
Drop 0 $\Rightarrow 2 + 3 + 4 + 6 = 15$ which is a multiple of 3.
We have to check the possible outcomes,
For 2 , 4 , 6 , 0 $\Rightarrow \underline{3} \times \underline{3} \times \underline{2} \times \underline{1} = 18$ ways.
For 2 , 3 , 4 , 0 $\Rightarrow \underline{3} \times \underline{3} \times \underline{2} \times \underline{1} = 18$ ways.
But out of these 18 outcomes there will be some possibilities where the last digit is 3.
4 such possibilities exist here which needs to be eliminated.
Therefore 18 – 4 = 14 ways.
For 2 , 3 , 4 and 6 if we have last digit as 3 ie. _ _ _ 3, we can rearrange 2 , 4 and 6 in 3! ways
Hence 2 , 3 , 4 and 6 can be arranged in 4! - 3! = 24 – 6 = 18 ways.
Therefore, in total we can form 18 + 14 + 18 = 50 four digit numbers.

20. $\log_2 x + \log_4 x = \log_{0.25}\sqrt{6}$
We can rewrite the equation as:
$\log_2 x$ + log2xlog24log2xlog24 = $\log_{0.25}\sqrt{6}$
$\log_2 x * 3/2 = \log_{0.25}\sqrt{6}$
=> $\log_2 x * 3 = 2\log_{0.25}\sqrt{6}$
=> $\log_2 x^3 = -\log_4 6$
=> $\log_2 x^3 = -\log 26 / \log 24$
=> $\log_2 x^3 = -\log 26/ 6$
=> $2\log_2 x^3 = -\log_2 6$
$2\log_2 x^3 + \log_2 6 = 0$
$\log_2 6X^6 = 0$
$6x^6 = 1$
$x^6 = 1/6$
$x = 6^{-1/6}$

21. For a,b, c in AP 2b= a+c
for b, c,d, in = 2ce/ (c+e)
Simplifying we get c^2= ae => thus in GP

22. Coefficient of $x^{16} = (x^2-2x)^{10}$
- ⇨ coefficient of x^6 int $(x-2)^{10}$
- ⇨ $^{10}C_4{}^2 C_4$
- ⇨ 210x16
- ⇨ 2260

23. a + b= 5/6 and a.b =1/6 then $\tan^{-1} a + \tan^{-1} b = \tan^{-1}((a+b)/\ 1-ab)$
- ⇨ $\tan^{-1} 1$

24. Trick : By inspection 16 exceeds its square root by 12.

25. Sum of angle = (n-2)*180
(n/2)(2*120 +(n-1)*5) = (n-2)180
(n-9)(n-16)=0
Fo n =16 gives angle as 195 which is invalid.
Thus n=9.