Mensuration

❖ List of formulae

Formulas for Areas and Volumes

➢ Circle:

1. Diameter, D = 2R
2. Area = πR^2 sq. units
3. Circumference = $2\pi R$ units

➢ Square:

1. Area = a^2 sq. units
2. Perimeter = 4a units
3. Diagonal, d = $\sqrt{2}$ a units

➢ Rectangle:

1. Area = L*B sq. units
2. Perimeter = 2(L+B) units
3. Diagonal, d = $\sqrt{L^2+B^2}$ units

➢ Right Angled Triangle:

1. Area = (½)bxh sq. units
2. Perimeter = b + h + hypotenuse
3. Hypotenuse = $\sqrt{b^2+h^2}$ units

- **Equilateral Triangle:**

1. Area = $\sqrt{4}\ a^2$ sq. units
2. Perimeter = 3a units, where a = side of the triangle

- **Scalene Triangle:**

1. Area: $\sqrt{s(s-a)(s-b)(s-c)}$ sq. units; s = (a+b+c)/2
2. Perimeter = (a+b+c) units

- **Isosceles Triangle:**

1. Area = b/4 $\sqrt{4a^2-B^2}$ sq units
2. Perimeter = 2a + b units, where b = base length; a = equal side length

- **Cube:**

1. Volume = a^3 cubic units
2. Lateral Surface Area (LSA) = $4a^2$ sq. units
3. Total surface area (TSA) = $6a^2$ sq. units
4. Length of diagonal = $a\sqrt{3}$ units

- **Cuboid:**

1. Volume = (Cross section area * height) = L * B * H cubic units
2. Lateral Surface Area (LSA) = 2[(L+B)H] sq. units
3. Total surface area (TSA) = 2(LB+BH+HL) sq. units
4. Length of the diagonals = $\sqrt{L^2+B^2+H^2}$ units

- **Sphere:**

1. Volume = (4/3) πR^3 cubic units
2. Surface Area = $4\pi R^2$ sq. units
3. If R and r are the external and internal radii of a spherical shell, then its Volume = (4/3) $[R^3-r^3]$ cubic units

- **Hemisphere:**

1. Volume = (2/3) πR^3 cubic units
2. TSA = $3\pi R^2$ sq. units

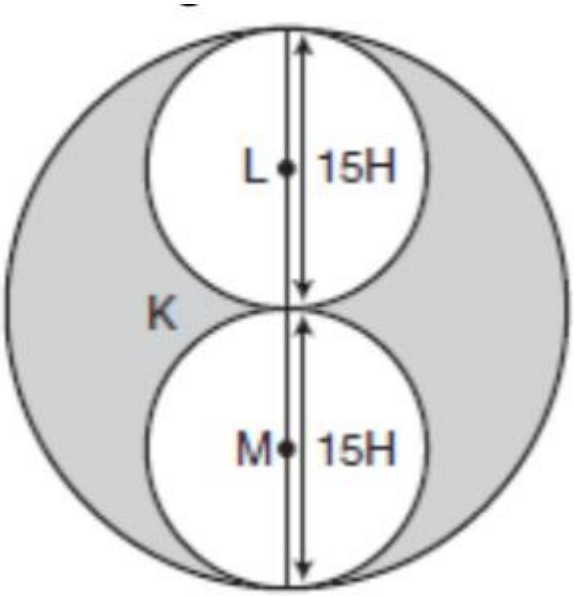
- **Cylinder:**

1. Volume = $\pi r^2 h$ cubic units
2. Curved surface Area (CSA) (excludes the areas of the top and bottom circular regions) = $2\pi Rh$ sq. units
3. TSA = Curved Surface Area + Areas of the top and bottom circular regions = $2\pi Rh + 2\pi R^2 = 2\pi R[R+h]$ sq. units

- **Cone:**

1. Volume = (1/3) $\pi R^2 h$ cubic units
2. Slant Height of cone, L = $\sqrt{R^2+H^2}$ units
3. CSA = πRL sq. units

1. For the given figure answer following questions



What is the area of the shaded figure?

a) 56.25π square feet
b) 112.5π square feet
c) 225π square feet
d) 337.4π square feet

2. What is the ratio of the area of Circle M and the area of Circle K?

a) 1:8
b) 1:4
c) 1:2
d) 1:1

3. A room has floor size of 15*6sq.cm. What is the height of the room , if the sum of the areas of the base and roof is equal to the sum of the areas of the four walls ?

a) 1.12cm
b) 3.24cm
c) 4.29cm
d) 2.5cm

4. A circular wire of diameter 112 cm is cut and bent in the form of a rectangle whose sides are in the ratio of 9 : 7. The smaller side of the rectangle is
   a) 77
   b) 97
   c) 67
   d) 87

5. What is the area between a square of side 10 cm and two inverted semi-circular, cross-sections each of radius 5 cm inscribed in the square ?
   a) 17.5
   b) 18.5
   c) 20.5
   d) 21.5

6. The perimeter of a rectangle having area equal to 144 cm2 and sides in the ratio 4 : 9 is
   a) 52
   b) 56
   c) 60
   d) 64

7. A lawn is in the form of a rectangle having its sides in the ratio 2:3 The area of the lawn is 1/6 hectares. Find the length and breadth of the lawn.
   a)10m b)20m c)20m d)50m

8. Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per metre?

   a) Rs. 1934.40 b) Rs. 1265.43 c) Rs. 1374.40 d) Rs. 1005.21

9. A sphere of radius r is cut by a plane at a distance of h from its center, thereby breaking this sphere into two different pieces. The cumulative surface area of these two pieces is 25% more than that of the sphere. Find h.

   a) $r/\sqrt{2}$
   b) $r/\sqrt{3}$
   c) $r/\sqrt{5}$
   d) $r/\sqrt{6}$

10. Cylindrical cans of cricket balls are to be packed in a box. Each can has a radius of 7 cm and height of 30 cm. Dimension of the box is l = 76 cm, b = 46 cm, h = 45 cm. What is the maximum number of cans that can fit in the box?
    a) 15
    b) 17
    c) 22
    d) 21

11. The area of a square field is 24200 sq m. How long will a lady take to cross the field diagonally at the rate of 6.6 km/hr?

    a) 3 minutes
    b) 2 minutes
    c) 2.4 minutes
    d) 2 minutes 40 seconds

12. A 5 cm cube is cut into as many 1 cm cubes as possible. What is the ratio of the surface area of the larger cube to that of the sum of the surface areas of the smaller cubes?

    a) 1 : 6
    b) 1 : 5
    c) 1 : 25
    d) 1 : 125

13. If the diagonal and the area of a rectangle are 25 m and 168 m^2, what is the length of the rectangle?

    a) 17 m
    b) 31 m
    c) 12 m
    d) 24 m

14. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is
    a) $4\pi r2$ sq. units
    b) $6\pi r2$ sq. units
    c) $3\pi r2$ sq. units
    d) $8\pi r2$ sq. units

15. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be
    a) 12 cm
    b) 10 cm
    c) 13 cm
    d) 5 cm

16. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is
    a) 1:2
    b) 1:4
    c) 1:6
    d) 1:8

17. The total surface area of a hemi-sphere is how much times the square of its radius.
    a) π
    b) 4π
    c) 3π
    d) 2π

18. A smaller triangle is having three sides. Another big triangle is having sides exactly double the sides of the smaller triangle. Then what is the ratio of Area of Smaller triangle to Area of the bigger triangle?

    a) 1:2
    b) 2:1
    c) 1:4
    d) 4:1

19. A hollow cylindrical tube is made of plastic is 4 cm thick. If the external diameter is 18 cm and length of the tube is 59cm, then find the volume of the plastic?

    a) 10380 cm^3
    b) 10384 cm^3
    c) 10440 cm^3
    d) 10444 cm^3

20. The area of the Circular garden is 88704 m². Outside the garden a road of 7m width laid around it. What would be the cost of laying road at Rs. 2/m².

    a) Rs. 7,546
    b) Rs. 10,036
    c) Rs. 11,092
    d) Rs. 15,092

21. The circumference of the front wheel of a cart is 30 ft long and that of the back wheel is 36 ft long. What is the distance travelled by the cart, when the front wheel has done five more revolutions than the rear wheel?

    a) 20 ft
    b) 25 ft
    c) 750 ft
    d) 900 ft

22. A square and a rectangle have equal areas . If their perimeters are p1 and p2 –
    a) p1<p2
    b) p1=p2
    c) p1>p2
    d) none

23. Length of a rectangle is twice its breadth and one of its diagonals measures $10\sqrt{5}$. The perimeter of the rectangle is
    a) 60 cm
    b) 50cm
    c) 250cm
    d) None

24. How many iron rods each of length 7m and diameter 2cm can be made out of 0.88 cubic meter of iron ?
    a) 550
    b) 200
    c) 400
    d) None

25. Find the area of a rhombus one side of which measures 20 cm and one diagonal 24 cm.

a) 324 cm^2
b) 384 cm^2
c) 344 cm^2
d) 348 cm^2

- **Solutions:**

1. Area of Shaded region = 2 times the area of Circle M.
   Diameter of Circle M = 15 feet.
   Radius of Circle M = 7.5 feet
   Area of Circle M = $\pi (7.5)^2 = 56.25 \pi$ square feet.
   Area of Shaded region = 2 times the area of Circle M = 2 * 56.25 π square feet = 112.5 π square feet.
   Therefore, Area of shaded region is 112.5 π square feet.

2. Circle K has twice the radius of circle M.
   Let's call the radius of Circle M as r.
   Radius of Circle K = 2r
   Area of Circle M = πr^2
   Area of Circle K = $\pi (2r)^2 = 4 \pi r^2$.
   Area of Circle K is 4 times the area of Circle M.
   Therefore, ratio of the area of Circle M and the area of Circle K is 1 : 4

3. Let l, b and h be length, breadth and height of room . Since room base and floor have equal area and opposite walls have equal areas, thus
   (2bh+2lh) =lb+lb
   2h(l+b) = 2lb
   h(l+b)= l*b
   h(15+6) =(15*6)
   h=90 /21= 30/7= 4.29

4. **Shortcut :** If the ratio of two sides of the gained rectangle is 9:7, the smaller side must be a multiple of 7. But if we closely look at the given options, none except option A; 77 is completely divisible by 7. Hence, we can immediately pick the option 'A' as our answer.

5. Given, each side of a square (a) = 10 cm
and, radius of each semi-circular (r) = 5 cm
Area between square and semi-circles = Area of square – 2 Area of semi-circle
$= a^2 - 2 \times \frac{1}{2}\pi r^2$
$= (10)^2 - 2 \times \frac{1}{2} \times \frac{22}{7} \times (5)^2$

$= 100 - 78.5 = 21.5 \text{ cm}^2$

6. Let, length of rectangle (l) = 4x and breadth of rectangle (b) = 9x

$\therefore$ Area of rectangle = l × b
$\Rightarrow$ $144 = 4x \times 9x = 36x^2$
$\Rightarrow$ $x^2 = 4$
$\Rightarrow$ $x = 2$
Now, $l = 4x = 4 \times 2 = 8$ cm and $b = 9x = 9 \times 2 = 18$ cm
$\therefore$ Perimeter of rectangle = $2(l + b) = 2(8 + 18) = 52 \text{ cm}^2$

7. Let length = 2x meters and the breadth = 3x meters.
Now, area =

$(\frac{1}{6} \times 10000)\text{m}^2 =$

$(\frac{5000}{3})\text{m}^2$

So, $2x \times 3x = \frac{5000}{3}$

$\Leftrightarrow x^2 = \frac{2500}{9}$

$\Leftrightarrow x = (\frac{50}{3})$.

So,

Length = $2x = \frac{100}{3}$ m = $33\frac{1}{3}$ m

and

Breadth = 3x = $(3 \times \frac{50}{3})$ m = 50 m.

8. Area of the carpet =
Area of the room =

(13×9) m^2

$\Rightarrow$ 117 m^2.

Length of the carpet = $(\frac{\text{Area}}{\text{Width}})$

$\Rightarrow (117 \times \frac{4}{3})$ m $\Rightarrow$ 156 m.

So, Cost of carpeting =
Rs. (156×12.40)
$\Rightarrow$ Rs. 1934.40.

9. Cumulative area of the two pieces = 25% more than that of sphere.
Area of 2 pieces = $1.25 \times 4\pi^2 = 5\pi r^2$
Extra area = πr^2
Extra area = area of two new circles that are now created circles.
Area each new circle = $\pi r^2/2$
Let radius of new circle be r_1.
Now, $\pi r_1^2 = \pi r^2/2$
$r_1 = r/\sqrt{2}$
Now, r_1 , h and r form a right angled triangle.
$h^2 + r_1^2 = r^2$
$h^2 + (r/\sqrt{2})^2 = r^2$
$h = r/\sqrt{2}$

10. Since, both the box and cans are hard solids; simply dividing the volume won't work because the shape can't be deformed. Each cylindrical can has a diameter of 14 cm and while they are kept erect in the box will occupy height of 30 cm
Number of such cans that can be placed in a row = l /Diameter = 76 /14 = 5 (Remaining space will be vacant)
Number of such rows that can be placed = Width/ Diameter46 /14 = 3
Thus 5 * 3 = 15 cans can be placed in an erect position.
However, height of box = 45cm and only 30 cm has been utilized so far
Remaining height = 15 cm > 14 cm (Diameter of the can)
So, some cans can be placed horizontally on the base.
Number of cans in horizontal row
= Length of box /Height of can
= 76 /30 = 2
Number of such rows = Width of box / Diameter of can =46/14 = 3
∴ 2 * 3 = 6 cans can be placed horizontally
∴ Maximum number of cans = 15+6 = 21

11. Let 'a' meters be the length of a side of the square field.
Therefore, its area = a^2 square meters. --- (1)
We know that the length of the diagonal 'd' of a square whose side is 'a' meters = $\sqrt{2}$ a –-- (2)

From (1) and (2), we can deduce that the square of the diagonal = $d^2 = 2a^2$
Or d = $\sqrt{2 \times area} = \sqrt{2 \times 24200} = \sqrt{48400} = 220$ meters.

The time taken to cross a length of 220 meters while traveling at 6.6 kmph is given by
$\frac{220\ meters}{6.6\ kmph} = \frac{220 \times 60}{6.6 \times 1000}$ (converting 1 km = 1000 meters and 1 hour = 60 minutes).

= 2 minutes

12. The volume of the larger cube = 53 = 125 cm3.

The volume of each of the smaller cubes = 13 = 1 cm3. Therefore, one would get 125 smaller cubes.

The surface area of the larger cube = 6a2 = 6(52) = 6 * 25 = 150

The surface area of each of the smaller cubes = 6 (12) = 6.

Therefore, surface area of all of the 125, 1 cm3 cubes = 125 * 6 = 750.

Therefore, the required ratio = 150 : 750 = 1 : 5

13. The diagonal d = 25m. and area A = 168 m2.

Let 'l' be the length and 'b' be the width of the rectangle.

Therefore, l2 + b2 = d2. and lb = A

We can therefore write (l + b)2 = d2 + 2A and (l - b)2 = d2 - 2A.

Substituting and solving we get, l + b = 31 and l - b = 17. Hence l = 24 and b = 7

14. $2\pi r^2 + 2\pi r^2 = 4\pi r^2$

15. $h = \sqrt{(l^2 - r^2)} = \sqrt{(169 - 25)} = 12$ cm

16. Ratio of two surface areas will be

Area1/Area2 = $\pi r_1^2 h_1 / \pi r_2^2 h_2$

⇨ $(r^2 h/2) / (r^2 h)$

⇨ ½ = 1:2

17. By formula Total surface area = $(3\pi r^2)$ thus 3π times

18. Smaller triangle sides = a, b, c

Area = √s(s-a) (s-b) (s-c);

s = a+b+c/2
= √(a+b+c)(b+c-a)(a+c- b)(a+b-c)/4
Bigger triangle =2a, 2b, 2c
Area = √(a+b+c)(b+c-a)(a+c- b)(a+b-c)
Ratio = 1:4

19. R = 9, r = 5
V = 22/7*59(9^2 - 5^2)
= 22/7*59(81 - 25)
= 10384

20. 88704 = 22/7*r^2
r = 168
Outer radius = 168+7 = 175
Outer area = 22/7*175^2 = 96250
Road area = 96250 – 88704 = 7546
Cost = 7546*2 = 15092

21. The circumference of the front wheel is 30 ft and that of the rear wheel is 36 feet.

Let the rear wheel make n revolutions. At this time, the front wheel should have made n+5 revolutions.
As both the wheels would have covered the same distance, n*36 = (n+5)*30
36n = 30n + 150
6n = 150
n = 25.
Distance covered = 25*36 = 900 ft.

22. From definitions it can side ^{2}= a^2=x*y
also p1= 4a and p2= 2(x+y)
a= p1/4 and p2/4 = (x+y)/2
thus $(p1/4)^2$ < $(p2/4)^2$ as GM <AM => p1<p2

23. Let length = 2x if breadth =x
then x^2+$(2x)^2$ = (100*5)= 500

- ⇨ $5x^2 = 500$
- ⇨ $x^2 = 100 = x = 10$
- ⇨ Perimeter = 2(2x+x) = 2* 3x = 60

24. vol of 1 rod = π * 7/(100 *100)
    = 11/5000
    = 0.88 cu. m
    No of rods = 0.88 * 5000/11 = 400

25. Let other diagonal be 2x
    Since halves of diagonal and one side form a right angled triangle with side as hypoteneous
    $20^2 = 12^2 + x^2$
    - ⇨ x= 16 cm
    - ⇨ other diagonal =32cm
    - ⇨ area of rhombus = ½ * (product of diagonals)
    - ⇨ A = ½ * (32)* (24)
    - ⇨ A= 384 cm^2