Manoeuvre
The Career Signature

Differentiation and Its application

Q.1 $\frac{d(a^x+x^a)}{dx} = ?$

(a) $xa^{x-1} + xa^{a-1}$ (b) $\frac{a^x}{\log a} + \frac{x^a}{\log x}$ (c) $a^x \log a + x^a(\log x)$ (d) none

Q.2 $\frac{d}{dx}\left(\frac{1}{x^4 \sec x}\right) =$

(a) $\frac{x\sin x+4\cos x}{x^5}$ (b) $\frac{-(x\sin x+4\cos x)}{x^5}$ (c) $\frac{4\cos x-x\sin x}{x^5}$ (d) none of these

Q.3 $\frac{d}{dx}\left(\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right) =$

(a) ½ (b) -1/2 (c) 1 (d) -1

Q.4 If $f(x) = mx + c, f(0) = f'(0) = 1$ then $f(2) = ?$

(a) 1 (b) 2 (c) 3 (d) 4

Q.5 If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx} = ?$

(a) $\frac{\sin x}{2y-1}$ (b) $\frac{\cos x}{2y-1}$ (c) $\frac{\sin x}{2y+1}$ (d) $\frac{\cos x}{2y+1}$

Q.6 If $x^m y^n = (x+y)^{m+n}$ then $\left(\frac{dy}{dx}\right)_{x=1,y=2}$ is equal to

(a) ½ (b) 2 (c) 2m/n (d) m/2n

Q.7 If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots \infty}}}$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{1}{x(2y-1)}$ (b) $\frac{1}{x(y-1)}$ (c) $\frac{1}{x(y+1)}$ (d) $\frac{1}{x(2y+1)}$

Q.8 The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at $x = 1/2$ is

(a) -2 (b) 1 (c) 2 (d) 4

Q.9 Differentiate $\{-\log(\log x), x > 1\}$ with respect to x

(a) -1/(xlogx) (b) 1/logx (c) 1/x (d) xlogx

Q.10 If $x = at^2$ and $y = 2at$, then find $\frac{d^2y}{dx^2}$.

(a) $\frac{-1}{2at^2}$ (b) $\frac{-1}{2at^3}$ (c) $\frac{1}{2at}$ (d) $\frac{-1}{2at}$

Q.11 For the curve $x = t^2 - 1, y = t^2 - t$, tangent is parallel to x-axis where,

(a) t=0 (b) t=$\frac{1}{\sqrt{3}}$ (c) t=1/2 (d) t=$\frac{-1}{\sqrt{3}}$

Q.12 Find the angle of intersection of the curves $y = x^3$ and $6y = 7 - x^2$.

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) 0

Q.13 Find the local maximum/minimum points of $f(x) = (x-2)^2(x-3)$.

(a) x=11 (b) x=11/2 (c) x=11/4 (d) x=11/9

Q.14 The function $f(x) = \sin x + \sqrt{3}\cos x$ has a maximum value at x=?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) 0

Q.15 The equation of a normal to the curve $\sin y = x \sin\left(\frac{\pi}{3} + y\right)$ at x=0 is

(a) $2x+\sqrt{3}y=0$ (b) $2y-\sqrt{3}x = 0$ (c) $2y+\sqrt{3}x=0$ (d) $2x-\sqrt{3}y=0$

Q.16 The positive value of k for which $ke^x - x = 0$ has only one root is

(a) 1/e (b) 1 (c) e (d) $\log_e 2$

Q.17 The line x+y=2 is tangent to the curve $x^2 = 3 - 2y$ at its point

(a) (1,1) (b) (-1,1) (c) $(-\sqrt{3}, 0)$ (d) (3,-3)

Q.18 If normal to the curve y=f(x) is parallel to x-axis, then the correct statement is

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = 1$ (c) $\frac{dx}{dy} = 0$ (d) none of these

Q.19 Let p(x) be a real polynomial of least degree which has a local maximum at x=1 and a local minimum at x=3. If p(1)=6 and p(3)=2, then p'(0) is………….

(a) 3 (b) 6 (c) 0 (d) 9

Q.20 The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x|x^2 + 20x \leq 9x\}$ is........

(a) 9 (b) 7 (c) 5 (d) 0

Q.21 Find the second derivative of a $\sin^3 t$ with respect to a $\cos^3 t$ at $t = \frac{\pi}{4}$.

(a) $\frac{4\sqrt{2}}{3a}$ (b) $\frac{4\sqrt{3}}{3a}$ (c) $\frac{3}{a}$ (d) none of these

Q.22 Find the second derivative of x = ln t and y = t^3 + 1

(a) $3e^x$ (b) $9e^x$ (c) $9e^{3x}$ (d) none of these

Q.23 Let $y = a\sqrt{x} + bx$ be curve, $(2x - y) + \lambda(2x + y - 4) = 0$ be the family of lines. If the curve has slope -1/2 at (9,0) then a tangent belonging to the family of lines is

(a) x+2y-5=0 (b) x-2y+3=0 (c) 3x-y-1=0 (d) 3x+y-5=0

Q.24 The line y=x meets $y = ke^x$ for k≤ 0 at

(a) no points (b) one point (c) two points (d) more than two points

Q.25 For which interval, the function $(x^2 - 3x)/(x - 1)$ satisfies all the conditions of Rolle's theorem?

(a) $[0, 3]$ (b) $[-3, 0]$ (c) $[1.5, 3]$ (d) for no interval

1.(d)	2.(b)	3.(b)	4.(c)	5.(b)
6.(b)	7.(a)	8.(d)	9.(a)	10.(b)
11.(c)	12.(a)	13.(c)	14.(c)	15.(a)
16.(a)	17.(a)	18.(c)	19.(d)	20.(b)
21.(a)	22.(c)	23.(b)	24.(b)	25.(d)

- **Explanation :**

1. $\frac{d}{dx}(a^x + x^a) = a^x \log a + ax^{a-1}$

2. $\frac{d}{dx}\left(\frac{1}{x^4 \sec x}\right) = \frac{d}{dx}\left(\frac{\cos x}{x^4}\right) = \frac{x^4\frac{d}{dx}(\cos x) - (\cos x)\frac{d}{dx}x^4}{(x^4)^2} = \frac{x^4(-\sin x) - (\cos x)(4x^3)}{x^8}$

$$= \frac{-(x\sin x + 4\cos x)}{x^5}$$

3. Let $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan\frac{x}{2}}{1 + \tan\frac{\pi}{4}\tan\frac{x}{2}}\right) =$

$\tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) = \left(\frac{\pi}{4} - \frac{x}{2}\right)$

Now, $\frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -\frac{1}{2}$

4. We have $f'(x) = m \Rightarrow f'(0) = m \Rightarrow m = 1$

$f(x) = x + c \Rightarrow f(0) = c \Rightarrow c = 1$

$f(x) = x + 1 \Rightarrow f(2) = 2 + 1 = 3$

5. We have $y^2 = \sin x + y \Rightarrow 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1}$

6. Given $x^m y^n = (x+y)^{m+n}$

Taking logarithm on both the sides

We get $\log x^m + \log y^n = \log(x+y)^{m+n}$

$\Rightarrow$mlogx+nlogy=(m+n)log(x+y)

Differentiating both sides w.r. to x, we get

$$\Rightarrow \frac{m}{x}+\frac{n}{y}\frac{dy}{dx}=\frac{m+n}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{mx+my-mx-nx}{x(x+y)}=\left(\frac{my+ny-nx-ny}{y(x+y)}\right)\frac{dy}{dx}$$

$$\Rightarrow \frac{my-nx}{x(x+y)}=\frac{my-nx}{y(x+y)}\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{my-nx}{x(x+y)}\cdot\frac{y(x+y)}{my-nx}=\frac{y}{x}$$

Put x=1 and y=2, so we get $\frac{dy}{dx}=2$

7. Given $y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+\ldots\infty}}} \quad \ldots(1)$

Squaring both sides we've,

$$y^2=\log x+\sqrt{\log x+\sqrt{\log x+\ldots\infty}}$$

$\Rightarrow y^2=\log x+y$ (using 1)

Now differentiating both sides with respect to x we get,

(2y-1)$\frac{dy}{dx}=\frac{1}{x}$

Or $\frac{dy}{dx}=\frac{1}{x(2y-1)}$

8. Let $u=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$=\cos^{-1}(2x^2-1)$

let $x=\cos\theta$

$=\cos^{-1}(2\cos^2\theta-1)$

$=\cos^{-1}(\cos 2\theta)$

$=2\theta$

$=2\cos^{-1}x$

Now $\frac{du}{dx}=\frac{-2}{\sqrt{1-x^2}}$

v= $\sqrt{1-x^2}$

$$\Rightarrow \frac{dv}{dx}=\frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$\Rightarrow \frac{dv}{dx}=\frac{-x}{\sqrt{1-x^2}}$$

Therefore $\frac{du}{dv}=\frac{du/dx}{dv/dx}=\frac{\frac{-2}{\sqrt{1-x^2}}}{\frac{-x}{\sqrt{1-x^2}}}=\frac{2}{x}$

Put x=1/2 then $\frac{du}{dv}=4$.

9. Let y= -log(logx), then $\frac{dy}{dx}=\frac{-1}{\log x}\frac{d(\log x)}{dx}=\frac{-1}{x\log x}$

10. We have $x=at^2$ and $y=2at$, on differentiating both sides with respect to t.

$\frac{dx}{dt}=2at$ and $\frac{dy}{dt}=2a$

Therefore $\frac{dy}{dx}=\frac{1}{t}$,

Now $\frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{dy}{dx}\right)\times\frac{dt}{dx}=\frac{d}{dt}\left(\frac{1}{t}\right)\times\frac{1}{2at}=\frac{-1}{t^2}\times\frac{1}{2at}=-\frac{1}{2at^3}$

11. Given $x=t^2-1$ and $y=t^2-t$

If tangent is parallel to x-axis then $\frac{dy}{dx}=0$

$\frac{dy}{dt}\times\frac{dt}{dx}=0\Rightarrow(2t-1)\times\frac{1}{2at}=0\Rightarrow t=\frac{1}{2}$.

12. The point of intersection is obtained by solving equation simultaneously $y=x^3$ and $y=\frac{7}{6}-\frac{x^2}{6}$

$x^3=\frac{7}{6}-\frac{x^2}{6}\Rightarrow 6x^3=7-x^2\Rightarrow 6x^3+x^2-7=0$

$\Rightarrow$ (x-1)$(6x^2+7x+7)=0$

This gives x=1, therefore y=1,

Now from curve 1, $\frac{dy}{dx}=3x^2$, at $(1,1)$ $\frac{dy}{dx}=3$

From curve 2, $\frac{dy}{dx}=\frac{-2x}{6}$, at $(1,1)$ $\frac{dy}{dx}=\frac{-1}{3}$

Since the product of the slope is -1, hence the curve intersect at right angle.

13. If f'(x)=0 then x=2,2,11/4 are critical points.
f''(x)=2(x-2)(4x-11)+4$(x-2)^2$
f''(11/4)>0⇒x=11/4 is local minimum point.
f''(2)=0, therefore the test fails in second derivative test. x=2 is neither local maximum nor local minimum at x=2, hence x=11/4 is one and only local point of f(x).

14. f'(x)=0 for critical points, $\cos x-\sqrt{3}\sin x=0 \Rightarrow \cot x=\frac{\pi}{6}$
f''(x)=$-\sin x-\sqrt{3}\cos x \Rightarrow f''\left(\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}-\sqrt{3}\cos\frac{\pi}{6}<0.$
Hence f is maximum at point x=$\frac{\pi}{6}$.

15. Given value curve is $\sin y=x\sin\left(\frac{\pi}{3}+y\right)$
$\Rightarrow \cos y\frac{dy}{dx}=x\cos\left(\frac{\pi}{3}+y\right)\frac{dy}{dx}+\sin\left(\frac{\pi}{3}+y\right)$
$\Rightarrow\left(\frac{dy}{dx}\right)_{x=0}=\frac{\sqrt{3}}{2}\Rightarrow\left(\frac{-dx}{dy}\right)_{x=0}=\frac{-2}{\sqrt{3}}$
Therefore, equation of normal at (0,0) is
(y-0)=$\frac{-2}{\sqrt{3}}(x-0)\Rightarrow y=\frac{-2}{\sqrt{3}}x\Rightarrow 2x+\sqrt{3}y=0$

16. Here f'(x)=ke^x-1, substitute f'(x)=0⇒x=-logk
f''(x)=k$e^x \Rightarrow f''(-\log k)=1>0$
which implies that f(x) has one minimum at point x=-logk
since the equation has only one root we get f(-logk)=0⇒1+logk=0
k=1/e

17. The given curve is $x^2=3-2y$ (1)
Differentiating w. r. t x, we get $2x=0-2\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=-x$

The slope of the tangent of the curve is -x. From the given line, the slope is -1 and hence x=1 and from eq(1), y=1. Therefore, the coordinate of the point is (1,1).

18. The slpoe of the normal is $\frac{-1}{\frac{dy}{dx}}$, this is parallel to x-axis.

Therefore, $\frac{-1}{\frac{dy}{dx}} = 0 \Rightarrow \frac{dx}{dy} = 0$

19. Let p'(x)=k(x-1)(x-3). Then p(x)=k$\left(\frac{x^3}{3} - 2x^2 + 3x\right) + c$

Now p(1)=6⇒$\frac{4}{3}k + c = 6,$

Also, p(3)=2⇒c=2

So, k=3, and p'(0)=3k=9

20. f'(x)=6(x-2)(x-3)

so, f(x) is increasing in (3,∞). Also A=$\{4 \le x \le 5\}$. Therefore ,

$$f_{max} = f(5) = 7$$

21. let $y = a\sin^3 t, x = a\cos^3 t$

on differentiating with respect to t,

we get , $\frac{dy}{dx} = 3a\sin^2 t\cos t, \frac{dx}{dt} = -3a\cos^2 t\sin t$

$\therefore \frac{dy}{dx} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t} = \frac{-\sin t}{\cos t} = -\tan t,$

Again, differentiating with respect to x, we get

$$\frac{d^2y}{dx^2} = -\sec^2 t.\frac{dt}{dx} = \frac{-\sec^2 t}{-3a\cos^2 t\sin t} = \frac{1}{3a\cos^4 t\sin t}$$

$$\left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}} = \frac{1}{3a\left(\frac{1}{\sqrt{2}}\right)^4.\left(\frac{1}{\sqrt{2}}\right)} = \frac{(\sqrt{2})^5}{3a} = \frac{4\sqrt{2}}{3a}$$

22. we have $\frac{dx}{dt}=\frac{1}{t}$ and $\frac{dy}{dt}=3t^2$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{1/t}{3t^2}=3t^3$$

$\frac{d^2y}{dx^2}=9t^2\times\frac{dt}{dx}=9t^3$ since x=lnt, therefore $t=e^x$,

so $\frac{d^2y}{dx^2}=9e^{3x}$.

23. putting x=9 and y=0 in the given equation of curve, we have

$$0=3a+9b-\frac{1}{2}=\frac{a}{2\times3}+b\Rightarrow a=-3b.......(1)$$

$$\frac{dy}{dx}=\frac{a}{2\sqrt{x}}+b$$

$$\left(\frac{dy}{dx}\right)_{(9,0)}=\frac{a}{6}+b=\frac{-1}{2}..............(2)$$

Using eq (1) and (2) , we get b=-1 and a=3, therefore $y=3\sqrt{x}-x$

Point (1,2) lies on curves as well as it is point of intersection of family of lines. $\frac{dy}{dx}=\frac{3}{2\sqrt{x}}-1$,

$$\left(\frac{dy}{dx}\right)_{(1,2)}=\frac{1}{2}$$

$$y-2=\frac{1}{2}(x-1)\Rightarrow x-2y+3=0$$

24. The line y=x and the curve $y=ke^x(k\leq0)$ at exactly one point.

25. Here, $f(x)=\frac{x^2-3x}{x-1}\Rightarrow -\sin c=-\frac{2}{\pi}\Rightarrow c=\sin^{-1}\frac{2}{\pi}$

Obviously, it is not derivable at x=1, that is in (0,3). Also f(a)=f(b) does not hold for $[-3,0]$and $[1.5,3]$. Hence the answer is D.