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❖ Identities :

1. $a + b + c = 2$, $ab + bc + ca = 26$

$a^3 + b^3 + c^3 - 3abc = ?$

(a) -148 (b) -214 (c) -149 (d) 148

2. $x + \frac{1}{x} = 3$

$x^4 + \frac{1}{x^4} = ?$

(a) 47 (b) 48 (c) 49 (d) 5

3. $(a - b) = 3$ and $ab = 10$. Find $(a^3 - b^3) = ?$

(a) 117 (b) 127 (c) 328 (d) 119

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4. $a + b + c = 6$, $a^3 + b^3 + c^3 - 3abc = 219$

$ab + bc + ca = ?$

(a) 0 (b) $-\frac{1}{6}$ (c) 60 (d) 46

5. $a + b + c = 6$, $ab + bc + ac = 4$

$a^3 + b^3 + c^3 - 3abc = ?$

(a) 120 (b) 144 (c) 166 (d) 233

6. $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6}$, $x^2 + \frac{1}{x^2} = ?$

(a) 12 (b) 13 (c) 15 (d) 14

7. a + b + c = 13, ab + bc + ca = 54

$a^3 + b^3 + c^3 - 3abc = ?$

(a) 91 (b) 104 (c) 13 (d) 14

8.
$x^2 + y^2 + z^2 + 27 = 6(x + y + z)$, $\sqrt[3]{x^3 + y^3 - z^3} = ?$

(a) 3 (b) 13 (c) 27 (d) 54

9. x + y + z = 0

$\frac{x^2 + y^2 + z^2}{z^2 - yx} = ?$

(a) 2 (b) 3 (c) 5 (d) 4

10. $4x^2 - 6x - 1 = 0$

$8x^3 + \frac{1}{8x^3} = ?$

(a) 12 (b) 18 (c) 15 (d) 14

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11. $x^2 + x = 19$

$(x+5)^2 + \frac{1}{(x+5)^2} = ?$

(a) 12 (b) 13 (c) 81 (d) 79

12. $x - 5\sqrt{x} - 1 = 0$

$x^2 + \frac{1}{x^2} = ?$

(a) 727 (b) 729 (c) 529 (d) 343

13. $x^2 - 3x - 1 = 0$

$(x^2 + 8x - 1)(x^3 + x^{-1})^{-1} = ?$

(a) 4 (b) 1 (c) 2 (d) 3

14.

$x^2 + \frac{1}{(x)^2} = ?$

If $x + \frac{1}{x} = 5$

(a) 25 (b) 23 (c) 15 (d) 14

15. $a + b + c = 9$, $c^3 + a^3 = 35$, $a^3 + b^3 = 91$, $b^3 + c^3 = 72$

$ab + bc + ca = 26$, $b^3 + c^3 = 72$

$abc = ?$

(a) 28 (b) 33 (c) 25 (d) 24

16.

$a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, $b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, $(a+b)^2 - ab = ?$

(a) 990 (b) 99 (c) 98 (d) 44

17. $x + y = 10$, $xy = 4$, $x^4 + y^4 = ?$

(a) 8432 (b) 9232 (c)1235 (d) 2565

18. $(a - b) = 5$, $a^2 + b^2 = 45$, $ab = ?$

(a) 12 (b) 13 (c) 10 (d) 14

19. $x^4 + x^2y^2 + y^4 = \frac{21}{256}$

$x^2 + xy + y^2 = \frac{3}{16}$

$2(x^2 + y^2) = ?$

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $\frac{10}{8}$ (d) 1

20. $x^2 + \frac{1}{x^2} = 2$

$x - \frac{1}{x} = ?$

(a) -2 (b) 1 (c) 0 (d) -1

21. $x + y + z = 22$, $xy + yz + zx = 35$, $(x - y)^2 + (y - z)^2 + (z - x)^2 = ?$

(a) 793 (b) 681 (c) 758 (d) 751

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22. $\frac{x+y}{z} = 2, \frac{y}{y-z} + \frac{x}{x-z} = ?$

(a) 2 (b) 1 (c) 5 (d) 4

23. $a^4 + 1 = \frac{a^2}{b^2}(4b^2 - b^4 - 1), \quad a^4 + b^4 = ?$

(a) 2 (b) 3 (c) 5 (d) 0

24. $x^2 + \frac{1}{x^2} = 47$

$x + \frac{1}{x} = ?$

(a) 5 (b) 6 (c) 8 (d) 7

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25. $(x + 1/x)^2 = 3$

$x^{72} + x^{66} + x^{54} + x^{36} + x^{24} + x^6 + 1 = ?$

(a) 1 (b) 3 (c) 2 (d) 5

26.

$x^2 + \frac{1}{x^2} = 83$

$x^3 - \frac{1}{x^3} = ?$

(a) 764 (b) 750 (c) 756 (d) 760

27. $(x-5)^3 + (2x+6)^3 + (x-7)^3 = 3(x-5)(2x+6)(x-7)$. Find the value of x?

(a) 1 (b) 2.5 (c) 5 (d) 1.5

Answers :

1.a	2.a	3.a	4.b	5.b	6.d	7.a	8.a	9.d	10.a
11.b	12.d	13.b	14.a	15.c	16.a	17.c	18.c	19.ab	20.d
21.a	22.c	23.d							

Solution :

1. $a + b + c = 2$, $ab + bc + ca = 26$

$\because a^3 + b^3 + c^3 - 3abc = (a + b + c)[(a + b + c)^2 - 3(ab + bc + ac)]$

$\therefore \quad = 2 * [(2)^2 - 3(26)]$

$\therefore \quad = 2 * (4 - 78)$

$\therefore \quad = 2 * (-74)$

$\therefore \quad = -148$

So option (a) is correct.

2. Given that,

$x + \frac{1}{x} = 3$

$\therefore (x + \frac{1}{x})^2 = 3^2$ -------[Squaring both sides]

$\therefore x^2 + \frac{1}{x^2} + 2 * x * \frac{1}{x} = 9$

$\therefore x^2 + \frac{1}{x^2} + 2 = 9$

$\therefore x^2 + \frac{1}{x^2} = 7$

Now, $(x^2 + \frac{1}{x^2})^2 = 7^2$ --------[Squaring both sides]

$\therefore x^4 + \frac{1}{x^4} + 2 * x^2 * \frac{1}{x^2} = 49$

$\therefore x^4 + \frac{1}{x^4} + 2 = 49$

$\therefore x^4 + \frac{1}{x^4} = 47$

So option (a) is correct.

3. a – b = 3 and ab = 10.

$\because (a - b)^3 = a^3 - 3ab(a - b) - b^3$

$\therefore (a - b)^3 + 3ab(a - b) = a^3 - b^3$

$\therefore 3^3 + 3 * 10 * 3 = a^3 - b^3$

$\therefore 27 + 90 = a^3 - b^3$

$\therefore a^3 - b^3 = 117$

So option (a) is correct.

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4. $a + b + c = 6$, $a^3 + b^3 + c^3 - 3abc = 219$

$\because a^3 + b^3 + c^3 - 3abc = (a + b + c)[(a + b + c)^2 - 3(ab + bc + ac)]$

$\therefore 219 = 6 * [(6)^2 - 3(ab + bc + ac)]$

$\therefore 219 = 6 * [36 - 3(ab + bc + ac)]$

$\therefore 219 = 216 - 18(ab + bc + ac)$

$\therefore 219 - 216 = -18(ab + bc + ac)$

$\therefore 3 = -18(ab + bc + ac)$

$\therefore ab + bc + ac = -\frac{3}{18} = -\frac{1}{6}$

So option (b) is correct

5. a + b + c = 6, ab + bc + ca = 4

$\because a^3 + b^3 + c^3 - 3abc = (a + b + c)[(a + b + c)^2 - 3(ab + bc + ac)]$

$\therefore \quad = 6 * [(6)^2 - 3(4)]$

$\therefore \quad = 6 * (36 - 12)$

$\therefore \quad = 6 * (24)$

$\therefore \quad = 144$

So option (b) is correct.

6. $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6}$,

Squaring both sides, we get,

$x + \frac{1}{x} + 2 * x * \frac{1}{x} = 6$

$\therefore x + \frac{1}{x} + 2 = 6$

$\therefore x + \frac{1}{x} = 4$

Squaring both sides, we get,

$x^2 + \frac{1}{x^2} + 2 * x + \frac{1}{x} = 16$

$\therefore x^2 + \frac{1}{x^2} + 2 = 16$

$\therefore x^2 + \frac{1}{x^2} = 14$

So option (d) is correct.

7. a + b + c = 13, ab + bc + ca = 54

∵ $a^3 + b^3 + c^3 - 3abc = (a + b + c)[(a + b + c)^2 - 3(ab + bc + ac)]$

∴ $= 13 * [(13)^2 - 3(54)]$

∴ $= 13 * (169 - 162)$

∴ $= 13 * 7$

∴ $= 91$

So option (a) is correct.

8. x + y + z = 0

∴ x + y = -z

∴ On squaring both sides, we get, $(x + y)^2$

$= (-z)^2$

∴ $x^2 + y^2 + 2xy = z^2$ ----- (1)

∴ $x^2 + y^2 + xy = z^2 - xy$------ (2)

Now,

$\frac{x^2+y^2+z^2}{z^2-xy} = \frac{x^2+y^2+x^2+y^2+2xy}{z^2-xy}$ -------From(1)

∴ $= \frac{2x^2+2y^2+2xy}{x^2+y^2+xy}$

∴ $= \frac{2(x^2+y^2+xy)}{x^2+y^2+xy}$

∴ $= 2 * 1$

∴ $= 2$

So option (a) is correct.

9. $x^2 + x = 19$

Let y = x + 5

➔ x = y – 5

Putting in the equation above, we get,

$(y-5)^2 + (y-5) = 19$

➔ $y^2 - 10y + 25 + y - 5 = 19$

➔ $y^2 - 9y + 1 = 0$

➔ $y^2 + 1 = 9y$

➔ $y + \frac{1}{y} = 9$ -------------[Dividing throught by y]

Squaring both sides, we get,

$y^2 + \frac{1}{y^2} + 2 * y * \frac{1}{y} = 81$

$\therefore y^2 + \frac{1}{y^2} + 2 = 81$

$\therefore y^2 + \frac{1}{y^2} = 79$

$\therefore (x+5)^2 + \frac{1}{(x+5)^2} = 79$

So option (d) is correct.

10. $x - 5\sqrt{x} - 1 = 0$

$\therefore x - 1 = 5\sqrt{x}$

∴ On squaring both sides, we get,$(x-1)^2$

$= (5\sqrt{x})^2$

$\therefore x^2 - 2x + 1 = 25x$

$\therefore x^2 + 1 = 27x$

Dividing throught by x, we get,

$x + \frac{1}{x} = 27$ -----(1)

$\therefore x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2$

$\therefore$ $= 27^2 - 2$ ------From(1)

$\therefore$ $= 729 - 2$

$\therefore$ $= 727$

So option (a) is correct.

11. $x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2$

$\therefore$ $= 5^2 - 2$

$\therefore$ $= 25 - 2$

$\therefore$ $= 23$

So option (b) is correct.

12. $a + b + c = 9$ -------- (1)

$c^3 + a^3 = 35$ ------(2)

$a^3 + b^3 =$ ------(3)
91

$b^3 + c^3 = 72$ ------(4)

$ab + bc + ca = 26$ (5)

Adding equations (2),(3) & (4), we get,

$2a^3 + 2b^3 + 2c^3 = 35 + 91 + 72 = 198$

$\therefore a^3 + b^3 + c^3 = 99$ (6)

As per the identity,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)[(a + b + c)^2 - 3(ab + bc + ca)]$

$\therefore$ 99 – 3abc = 9[9^2 – 3(26)] From(6),(1) & (5)

$\therefore$ 99 – 3abc = 9(81 – 78)

$\therefore$ 99 – 3abc = 9(3)

$\therefore$ 99 – 3abc = 27

$\therefore$ 3abc = 99 – 27

$\therefore$ 3abc = 72

$\therefore$ abc = 24

So option (d) is correct.

13. $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ & $b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

After rationalising,

$a = (\sqrt{3} + \sqrt{2})^2 = 5 + 2\sqrt{6}$ & $b = (\sqrt{3} - \sqrt{2})^2 = 5 - 2\sqrt{6}$

Now,

$(a + b)^2 - ab = (5 + 2\sqrt{6} + 5 - 2\sqrt{6})^2 - (5 + 2\sqrt{6})(5 - 2\sqrt{6})$

$\therefore$ $= 10^2 - [5^2 - (2\sqrt{6})^2]$

$\therefore$ $= 100 - (25 - 24)$

$\therefore$ $= 100 - 1$

$\therefore$ $= 99$

So option (b) is correct.

14. $x + y = 10$, $xy = 4$,

$x^4 + y^4 = (x^2 + y^2)^2 - 2x^2y^2$

$\therefore = [(x + y)^2 - 2xy]^2 - 2x^2y^2$

$\therefore = (10^2 - 2 * 4)^2 - 2(4)^2$

$\therefore = (100 - 8)^2 - 2(16)$

$\therefore = 92^2 - 32$

$\therefore = 8464 - 32$

$\therefore = 8432$

So option (a) is correct.

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15. $a - b = 5$, $a^2 + b^2 = 45$,

$a^2 + b^2 = (a - b)^2 + 2ab$

$\therefore 45 = 5^2 + 2ab$

$\therefore 45 = 25 + 2ab$

$\therefore 2ab = 20$

$\therefore ab = 10$

So option (c) is correct.

16.

$(x^2 + y^2 + xy)(x^2 + y^2 - xy) = (x^2 + y^2)^2 - (xy)^2$

$\therefore \quad = x^4 + y^4 + 2x^2y^2 - x^2y^2$

$\therefore (x^2 + y^2 + xy)(x^2 + y^2 - xy) = x^4 + y^4 + x^2y^2$

$\therefore \quad \frac{3}{16} * (x^2 + y^2 - xy) = \frac{21}{256}$

$\therefore \quad x^2 + y^2 - xy = \frac{21*16}{256*3} = \frac{7}{16}$ ------(1)

& $\quad x^2 + y^2 + xy = \frac{3}{16}$ ------------(2)

Adding (1) & (2), we get,

$2x^2 + 2y^2 = \frac{5}{8}$

$\therefore 2(x^2 + y^2) = \frac{5}{8}$

So option (a) is correct.

17. $x^2 + \frac{1}{x^2} = (x - \frac{1}{x})^2 + 2$

$\therefore \quad 2 = (x - \frac{1}{x})^2 + 2$

$\therefore (x - \frac{1}{x})^2 = 0$

$\therefore x - \frac{1}{x} = 0$

So option (c) is correct.

18. $x + y + z = 22, \quad xy + yz + zx = 35,$

$(x - y)^2 + (y - z)^2 + (z - x)^2 = x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + x^2 + z^2 - 2xz$

$\therefore \quad = 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz$

$\therefore \quad = 2(x^2 + y^2 + z^2) - 2(xy + yz + xz)$

$\therefore \quad = 2[(x + y + z)^2 - 2(xy + yz + xz)] - 2(xy + yz + xz)$

$\therefore \quad = 2(x + y + z)^2 - 4(xy + yz + xz) - 2(xy + yz + xz)$

$\therefore \quad = 2(x + y + z)^2 - 6(xy + yz + xz)$

∴ $= 2(22)^2 - 6(35)$

∴ $= 2(484) - 210$

∴ $= 968 - 210$

∴ $= 758$

So option (c) is correct.

19. Given that, $\frac{x+y}{z} = 2$ ➔ $x + y = 2z$ ➔ $y = 2z - x$

∴ $\frac{y}{y-z} + \frac{x}{x-z} = \frac{2z-x}{2z-x-z} + \frac{x}{x-z}$

∴ $= \frac{2z-x}{z-x} - \frac{x}{z-x}$

∴ $= \frac{2z-2x}{z-x}$

∴ $= \frac{2(z-x)}{z-x}$

∴ $= 2$

So option (a) is correct.

20. . $x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2$

∴ $47 = (x + \frac{1}{x})^2 - 2$

∴ $(x + \frac{1}{x})^2 = 49$

∴ $x + \frac{1}{x} = 7$

So option (d) is correct.

21. $(x + \frac{1}{x})\ 2 = 3,$

$\therefore x + \frac{1}{x} = \sqrt{3}$

$\therefore x^3 + \frac{1}{x^3} = (\sqrt{3})^3 - 3(\sqrt{3}) = 0$ ------------[If $x + \frac{1}{x} = a$, then $x^3 + \frac{1}{x^3} = a^3 - 3a$]

$\therefore x^6 + 1 = 0 \Rightarrow x^6 = -1$

Now,

$x^{72} + x^{66} + x^{54} + x^{36} + x^{24} + x^6 + 1 = (x^6)^{12} + (x^6)^{11} + (x^6)^9 + (x^6)^6 + (x^6)^4 + x^6 + 1$

$\therefore \quad = 1 - 1 - 1 + 1 + 1 + 0$

$\therefore \quad = 1$

So option (a) is correct.

22. $x^2 + \frac{1}{x^2} = (x - \frac{1}{x})^2 + 2$

$\therefore \quad 83 = (x - \frac{1}{x})^2 + 2$

$\therefore (x - \frac{1}{x})^2 = 81$

$\therefore x - \frac{1}{x} = 9$

$\therefore x^3 - \frac{1}{x^3} = 9^3 + 3(9) = 729 + 27 = 756$ ---------[If $x - \frac{1}{x} = a$, then $x^3 - \frac{1}{x^3} = a^3 + 3a$]

So option (c) is correct.

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23. $(x - 5)^3 + (2x + 6)^3 + (x - 7)^3 = 3(x - 5)(2x + 6)(x - 7)$.

Let $x - 5 = a$, $2x + 6 = b$, $x - 7 = c$Then

$a^3 + b^3 + c^3 = 3abc$

This is the identity when $a + b + c = 0$

$\therefore x - 5 + 2x + 6 + x - 7 = 0$

$\therefore 4x - 6 = 0$

$\therefore x = 1.5$

So option (d) is correct.