Probability & Statistics

1. A and B take part in a duel. A can strike with an accuracy of 0.6. B can strike with an accuracy of 0.8. A has the first shot, post which they strike alternately. What is the probability that A wins the duel?
   a) 7/10 b) 15/23 c) 2/3 d) 11/17

2. Doctors have devised a test for leptospirosis that has the following property: For any person suffering from lepto, there is a 90% chance of the test returning positive. For a person not suffering from lepto, there is an 80% chance of the test returning negative. It is known that 10% of people who go for testing have lepto. If a person who gets tested gets a +ve result for lepto (as in, the test result says they have got lepto), what is the probability that they actually have lepto?
   a) 7/10 b) 8/11 c) 1/3 d) 1/2

3. N is a 3-digit number that is a multiple of 7; what is the probability that it will be a multiple of 5?
   a) 1/5 b) 11/54 c) 13/64 d) 13/66

4. I roll a die four times. In how many outcomes do we have two throws have the same number and the other two something different?
   a) 720 b) 480 c) 360 d) 350

5. A bag contains 4 red and 3 black balls. A second bag contains 2 red and 3 black balls. One bag is selected at random. If from the selected bag one ball is drawn, then what is the probability that the ball drawn is red?
   a) 39/70 b) 41/70 c) 29/70 d) 17/35

6. A bag contains 4 blue, 5 white and 6 green balls. Two balls are drawn at random. What is the probability that one ball is white?

a) 10/21 b) 1/2 c) 3/4 d) 2/25

7. Two person A and B agree to meet 20 april 2018 between 6pm to 7pm with understanding that they will wait no longer than 20 minutes for the other. What is the probability that they meet?

a) 5/9 b) 7/9 c) 2/9 d) 4/9

8. A and B play a game where each is asked to select a number from 1 to 25. If the two number match, both of them win a prize. The probability that they will not win a prize in a single trial is :

a) 1/25 b) 24/25 c) 2/25 d) none

9. A and B are independent witness in a case. The chance that A speaks truth is x and B speaks truth is y. If A and B agree on certain statement, the probability that the statement is true is

10. In an entrance test there are multiple choice questions, with four possible answer to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If the student gets the correct answer to a question, then the probability that he as guessing is

a) 37/40 b) 1/37 c) 36/37 d) 1/9

11. A man is known to speak the truth 2 out of 3 times. He threw a dice cube with 1 to 6 on its faces and reports that it is 1. Then the probability that it is actually 1 is

a) 2/7 b) 1/7 c) 2/3 d) 5/6

12. If E_1 and E_2 are two events associated with a random experiment such that $P(E_2) = 0.35$, $P(E_1 \text{ or } E_2) = 0.85$ and $P(E_1 \& E_2) = 0.15$ then $P(E_1)$ is

a) 0.25 b) 0.35 c) 0.65 d) 0.75

13. If A and B are two events and $P(A \cup B)=5/6$ $P(A \cap B)=1/2$ then A and B are

a) Dependent b) Independent c) Mutually Exclusive d) Equally likely

14. The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively. The $P(X = 1)$ is

a) 1/32 b) 1/16 c) 1/8 d) ¼

15. If is the mean of distribution of x, then usual notation is $\sum_{i=1}^{n} f(x - xi)$ is

a) Mean deviation about mean b) Standard deviation c) 1 d) 0

16. There are 15 boys and 10 girls in a class. If three students are selected at random, what is the probability that 1 girl and 2 boys are selected?

a) 1/40 b) 1/2 c) 21/46 d) 7/41

17. In a box there are 10 apples and 2/5th of the apples are rotten. If three apples are taken out from the box, what will be the probability that at least one apple is rotten.

a) 3/4 b) 5/6 c) 9/10 d) 8/13

18. The names of 5 students from section A, 6 students from section B and 7 students from section C were selected. The age of all the 18 students was different. Again, one name was selected from them and it was found that it was of section B. What was the probability that it was the youngest student of the section B?

a) 1/18 b) 1/15 c) 1/6 d) 1/12

19. The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1, 2, 6, then the mean deviation from the mean of the data is

a) 2.5 b) 2.6 c) 2.8 d) 2.4

20. For a given data the standard deviation is 20 . If 3 is added to each observation what is the new variance ?

a) 20 b) 23 c) 17 d) 60

21. For a given data the variance is 215 . If each observation is multiplied by 2 what is the new variance ?

a) 15 b) 30 c) 60 d) 7.5

22. Standard deviation of first 10 multipliers of 4 is

a) 7 b) 8 c) 11.5 d) 14

23. Let X and Y be two independent random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is **FALSE**?

a) E(XY) = E(X)E(Y) b) Cov(X,Y) = 0

c) Var(X+Y) = Var(X)+Var(Y) d) (D) $E(X^2Y^2) = (E(X))^2(E(Y))^2$

24. The mean of two samples of the sizes 250 and 320 were found to be 20,12 respectively. Their standard deviations were 2 & 5, respectively. Find the variance of combined sample of size 650.

a) 10.23 b) 32.5 c) 65 d) 26.17

25. The mean and Standard deviation of a sample were found to be 9.5 and 2.5, respectively. Later, an additional observation 15 was added to the original data. Find the S.D. of the 11 observation.

a) 2.6 b) 2.8 c) 2.86 d) 3.24

- **Solutions:**

1. A can win in the following scenarios:

   A strikes in first shot.

   A misses in the first shot, B misses in the second, A strikes in the third.

   A misses in the 1st shot, B misses in the 2nd, A misses the 3rd, B misses the 4th and A strikes the 5th.

   And so on…

   P(A in 1^{st} shot) = 0.6

   P(A in 3^{rd} shot) = 0.4 * 0.2 * 0.6 {A misses, then B misses and then A strikes}

   P(A in 5^{st} shot) = 0.4 * 0.2 * 0.4 * 0.2 * 0.6 {A misses, then B misses and then A misses, then B misses, then A strikes}

   Overall probability = Sum of all these

   = 0.6 + 0.4 * 0.2 * 0.6 + 0.4 * 0.2 * 0.4 * 0.2 * 0.6 + …

   Which is nothing but

   $0.6 + (0.4 * 0.2) * 0.6 + (0.4 * 0.2)^2 * 0.6 + (0.4 * 0.2)^3 * 0.6…$

   This is an infinite geometric progression with first term 0.6 and common ratio 0.4 * 0.2

   Required Probability = a/1−r = 0.6 /1−0.08 = 0.60/0.92 = 30/46 = 15/23

2. Let us draw the possibilities in this scenario.

   Prob (patient having lepto) = 0.9

   Prob (patient not having lepto) = 0.1

   Given that patient has lepto, Prob (test being positive) = 0.9

   Given that patient has lepto, (Prob test being negative) = 0.1

   Given that patient does not have lepto, Prob (test being negative) = 0.8

   Given that patient does not have lepto, Prob (test being positive) = 0.2

   Now, we are told that the test turns positive. This could happen under two scenarios – the patient has lepto and the test turns positive and patient does not have lepto and the test turns positive.

   Probability of test turning positive = 0.9 * 0.1 + 0.9 * 0.2 = 0.27.

   Now, we have not been asked for the probability of test turning positive. We are asked for the probability of patient having lepto given that he/she tests positive. So, the patient has already tested positive. So, this

0.27 includes the set of universal outcomes. Or, this 0.27 sits in the denominator.

Within this 0.27, which subset was the scenario that the patient does indeed have lepto?

This is the key question. This probability is 0.1 * 0.9 = 0.09.

So, the required probability = 0.09/ .27 = 1/3

So, if a patient tests positive, there is a 1 in 3 chance of him/her having lepto. This is the key reason that we need to be careful with medical test results.

3. N is a three digit multiple of 7.

   N could be 105, 112, 119, 126…..994.

   Or, 15 * 7, 16 * 7…..142 * 7.

   Or there are 142–14 = 128 numbers.

   Within these we need to locate the multiples of 5.

   Or, we need to isolate multiples of 35.

   Or, we need to see how many numbers there are in the list 105, 140, 175…..980.

   35 * 3, 35 * 4…35 * 28…

   Or, there are 26 such numbers.

   Probability 26/128 = 13/64

4. If we want to have two throws to have the same outcome and the other two being something different, we are essentially looking for three different outcomes in each throw.

   We can have three different outcomes in each throw in $^6C_3$ ways. We can select the one that is being repeated out of the three outcomes in $^3C_1$ ways.

   And then, they can be arranged in 4!/2! ways, since there are totally 4 throws out of which 2 are repeated. So, Totally , we can do this in $^6C_3$ * $^3C_1$ * 4!/2! = (4∗5∗6) /`6* 3 *12 = 20*3*12 = 720 ways.

5. Let Red balls be 'r' and brown balls be 'b'

   P(r) = p(b1) * p(r) + p(b2) * p(r)

   P(R) = 1/2 * 4/7 + 1/2 * 2/5

   P(R) = 17/35

6. Prob is 1W 1G or 1W 1B out of total balls 15 , 2 balls have to be selected thus $^{15}C_2$
   - ⇨ $(5x4 +5x6)/ {}^{15}C_2$
   - ⇨ 5x10 / 15X7
   - ⇨ 10/21

7. If the first person arrives at 6:00, then the probability of the two people meeting would be $\frac{20}{60} = \frac{1}{3}$. The probability of them meeting when the first person arrives between 6:00 and 6:40 is the same, since there is always 20 minutes after the first person for the second person to arrive.

   However, if the first person arrives between 6:40 and 7:00, the probability will range from $\frac{1}{3}$ and 0. The probability decreases linearly since the number of minutes for the second person to arrive decreases linearly. Thus, if the first person arrives between 6:40 and 7:00, the average probability of meeting is $\frac{\frac{1}{3}+0}{2} = \frac{1}{6}$.

   The probability of the first person arriving between 6:00 and 6:40 is $\frac{40}{60} = \frac{2}{3}$, while between 6:40 and 7:00 is $\frac{20}{60} = \frac{1}{3}$. We also have to double the probability since either could arrive first. Thus, the probability is:

   $$2 * (\frac{2}{3} * \frac{1}{3} + \frac{1}{3} * \frac{1}{6}) = \frac{5}{9}$$

8. The total number of ways in which numbers can be chose =25 x 25=625 The number of ways in which either players can choose same numbers = 25

   Probability that they win a prize = 25/625 = 1/25

   Thus, the probability that they will not win a prize in a single trial = 1 - 1/25 = 24/25

9. P(A speaks truth) = x

P(B speaks truth) = y

Since, both A and B agree on certain statement.

Hence, Total Probability =P(A)P(B)+P(A')P(B')

=xy + (1-x)(1-y)

If statement is true then it means both A and B speaks truth.

∴ Required Probability = xy / (xy+ (1-x)(1-y)

10. Solution

Let, $P(A)$ is Probability that a student knows the answer $=\frac{9}{10}$, $P(B)$ is probability that he guesses the answer $=\frac{1}{10}$ and $P(C)$ is probability he answers correctly.

$P(C/A)$ = He gives correct answer as he knows the answer = 1.

$P(C/B)$ = He guesses the answer correctly $=\frac{1}{4}$

as there are 4 choices.

We have to find $P(B/C)=\frac{P(B)P(C/B)}{P(B)P(C/B)+P(A)P(C/A)}$

$$=\frac{\frac{1}{10}\times\frac{1}{4}}{\frac{1}{10}\times\frac{1}{4}+\frac{9}{10}\times 1}=\frac{1}{37}$$

11. Probability of speaking truth =2/3

Probability of appearing 6 on face = 1/6

Required probability = (2/3 * 1/6) / (`2/3 *1/6 +1/3 *5/6 *1/5)

⇨ 2/3

12. 1Given that P(E2)=0.35 P(E1 or E2)= 0.85 and P(E1 & E2) =0.15

P(E1 or E2)= P(E1)+P(E2)- P(E1 & E2)

=> 0.85 =0.35 + P(E2)-0.15

⇨ P(E2)=0.65

13. From the definitions it can be proved that the two events are mutually exclusive.

14. np = 4

npq = 2

q = 1/2, p = 1/2, n = 8

p(X = 1) = 8C1 (1/2)(1/2)7

15. Mean Deviation from mean is Zero.

16. Total number of ways of selecting 3 students from 25 students = ${}^{25}C_3$

Number of ways of selecting 1 girl and 2 boys = selecting 2 boys from 15 boys and 1 girl from 10 girls

⇒ Number of ways in which this can be done = ${}^{15}C_2 \times {}^{10}C_1$

=21/46

17. Let rotten apples $= 10 \times \frac{2}{5} = 4$, others $= 6$

If 1 apple is rotten + 2 apples are other

$= {}^4C_1 \times {}^6C_2 = 60$

If 2 apples are rotten + 1 apple is other

$= {}^4C_2 \times {}^6C_1 = 36$

If 3 apples are rotten

$= {}^4C_3 = 4$

Total outcomes $= {}^{10}C_3 = 120$

$$\text{Probability} = \frac{60 + 36 + 4}{120}$$

$$= \frac{100}{120} = \frac{5}{6}$$

18. The total number of students = 18

When 1 name was selected from 18 names, the probability that he was of section B

= 6/18= 1/3

But from the question, there are 6 students from the section B and the age of all 6 are different therefore, the probability of selecting one i.e. youngest student from 6 students will be 1/6.

19.

$$\overline{X} = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5} = 5$$

$$\sum_{i=1}^{5} X_i = 25 - - - -(1)$$

Also $\sigma^2 = 124$

$$\implies \frac{\sum X_1^2}{5} - (\overline{X})^2 = 124$$

$$\implies \frac{\sum X_1^2}{5} = 124 + 25 = 149$$

$$\implies (X_1^2 + X_2^2 + \ldots\ldots + X_5^2) = 745$$

$$\implies X_1^2 + X_2^2 = 704 - - - -(2)$$

$$by\ (1) X_1 + X_2 = 16 - - - -(3)$$

$$2X_1X_2 + 704 = 256$$

$$X_1X_2 = \frac{256 - 704}{2}$$

$$X_1X_2 = 128 - 352 = -224 - - - -(4)$$

$$\text{NOW } \frac{\sum |x_1 - 5|}{5} = \frac{|x_1 - 5| + |x_2 - 5| + 4 + 3 + 1}{5}$$

$$= \frac{8 + |x_1 - 5| + |11 - x_1|}{5}$$

$$= \frac{8 + 6}{5}$$

$$= 2.8\ ans$$

20. If 3is added to each data , the mean will be increase by 3 and standard deviation will be same.

variance = 20

21. New variance $=2^2 * 15$

$=4*15 =60$

22. Answer

First 10 multiples of 4 are 4,8,12...40.

This is an A.P.

sum=n/2(a+l)

= 10/2(4+40)

∴ sum=220.

Mean, u=sum/n

= 220/10 = 22

D1 = 4-22 = -18

D2 = 8-22 = -16

D3 = 12-22 = -10

D4 = 16- 22 = -8

Similarly we subtract multiple of 4 by 22 upto 10 terms we get

-18, -14, -10, -8...........18

S.D. = $\sigma^2 = \sum(D^2)/n$

$=[(-18)^2, (-14)^2, (-10)^2, (-6)^2 + (-2)^2 + (6)^2 + (10)^2 + (14)^2 + (18)^2]/10$

Solving this, we get

$\sigma = 11.5$

23. From the definitions and properties

24. Combined Mean = X = $(n_1x_1 + n_2x_2)/(n_1 + n_2) = (250 \times 20 + 320 \times 12)/650 = 13.6$

Let $d_1 = x_1 - X = 20\text{-}13.6 = 6.4$, $d_2 = x_2 - X = 12\text{-}13.6 = -1.6$

Variance = [250(6.4 + 40.96) + 320(13.6 + 2.56)]/650 = 26.17.

25. 1n $(\Sigma x_i) = 9.5$,

1n $(\Sigma [(x_i)^2 - (X)^2] = 6.25$, $\Sigma x_i = 95$ and 110 $\Sigma[(X)^2] = 96.5$

Corrected mean = (95 + 15)/11 = 10

Corrected Variance = [(1/11) x (965 + 225)] – 100 = 90/11

⇒ Standard deviation = √Variance = √(90/11) = 2.86.

Manoeuvre
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