Manoeuvre
The Career Signature

Trigonometry

➢ Trigonometric values of different angles

Angles (In Degrees)	0°	30°	45°	60°	90°	180°	270°	360°
Angles (In Radians)	0°	π/6	π/4	π/3	π/2	π	3π/2	2π
sin	0	1/2	1/√2	√3/2	1	0	-1	0
cos	1	√3/2	1/√2	1/2	0	-1	0	1
tan	0	1/√3	1	√3	∞	0	∞	0
cot	∞	√3	1	1/√3	0	∞	0	∞
csc	∞	2	√2	2/√3	1	∞	-1	∞
sec	1	2/√3	√2	2	∞	-1	∞	1

➢ Periodicity Identities (in Radians)

- $\sin(\pi/2 - A) = \cos A$ & $\cos(\pi/2 - A) = \sin A$
- $\sin(\pi/2 + A) = \cos A$ & $\cos(\pi/2 + A) = -\sin A$
- $\sin(3\pi/2 - A) = -\cos A$ & $\cos(3\pi/2 - A) = -\sin A$
- $\sin(3\pi/2 + A) = -\cos A$ & $\cos(3\pi/2 + A) = \sin A$
- $\sin(\pi - A) = \sin A$ & $\cos(\pi - A) = -\cos A$
- $\sin(\pi + A) = -\sin A$ & $\cos(\pi + A) = -\cos A$
- $\sin(2\pi - A) = -\sin A$ & $\cos(2\pi - A) = \cos A$
- $\sin(2\pi + A) = \sin A$ & $\cos(2\pi + A) = \cos A$

➢ Sum & Difference Identities

- $\sin(x+y) = \sin(x)\cos(y)+\cos(x)\sin(y)$
- $\cos(x+y) = \cos(x)\cos(y)-\sin(x)\sin(y)$
- $\tan(x+y) = (\tan x + \tan y)/ (1-\tan x \cdot \tan y)$
- $\sin(x-y) = \sin(x)\cos(y)-\cos(x)\sin(y)$
- $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
- $\tan(x-y) = (\tan x-\tan y)/ (1+\tan x \cdot \tan y)$

Double Angle Identities

- $\sin(2x) = 2\sin(x) \cdot \cos(x) = [2\tan x/(1+\tan^2 x)]$
- $\cos(2x) = \cos^2(x) - \sin^2(x) = [(1-\tan^2 x)/(1+\tan^2 x)]$
- $\cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$
- $\tan(2x) = [2\tan(x)]/ [1-\tan^2(x)]$
- $\sec (2x) = \sec^2 x/(2-\sec^2 x)$
- $\csc (2x) = (\sec x. \csc x)/2$

Triple Angle Identities

- $\text{Sin } 3x = 3\sin x - 4\sin^3 x$
- $\text{Cos } 3x = 4\cos^3 x - 3\cos x$
- $\text{Tan } 3x = [3\tan x - \tan^3 x]/[1-3\tan^2 x]$

Product identities

- $\sin x \cdot \cos y = (\sin(x+y) + \sin(x-y)) /2$
- $\cos x \cdot \cos y = (\cos(x+y) + \cos(x-y) /2$
- $\sin x \cdot \sin y = (\cos(x-y) - \cos(x+y)) /2$

Sum to Product Identities

- $\sin x + \sin y = 2\sin(x+y/2)\cos(x-y/2)$
- $\sin x - \sin y = 2\cos(x+y/2)\sin(x-y/2)$
- $\cos x + \cos y = 2\cos(x+y/2)\cos(x-y/2)$
- $\cos x - \cos y = -2\sin(x+y/2)\sin(x-y/2)$

Inverse Trigonometry Formulas

- $\sin^{-1} (-x) = - \sin^{-1} x$
- $\cos^{-1} (-x) = \pi - \cos^{-1} x$
- $\tan^{-1} (-x) = - \tan^{-1} x$
- $\text{cosec}^{-1} (-x) = - \text{cosec}^{-1} x$
- $\sec^{-1} (-x) = \pi - \sec^{-1} x$
- $\cot^{-1} (-x) = \pi - \cot^{-1} x$

Practice Questions

1. Minimum value of 5 $\sin^2\emptyset$+ 4 $\cos^2\emptyset$
   a)1 b)2 c)3 d)4

2. If sin (A-B) = ½ and cos(A+B) =1/2 , where A & B are positive acute angles then
a) A= 45°, B= 15° b) A= 15°, B= 45° c) A= 60°, B= 15° d) None

3. If tan A +cot A =4 then $\tan^4$ A+$\cot^4$ A is equal to
   a) 110 b) 191 c) 80 d) 194

4. If A > 0, B > 0 and A + B = , then the minimum value of tan A + tan B
   a) $\sqrt{3} - \sqrt{2}$ b) 4 – 2 $\sqrt{3}$ c) 2 - $\sqrt{3}$ d) none

5. If tan x = - 3/4 and 3π/2 < x < 2π, then the value of sin2x is
   a)7/25 b)-7/25 c)24/25 d)-24/25

6. If cosθ = 4/5 and cosϕ = 12/13, θ and ϕ both in the fourth quadrant, the value of cos(θ + ϕ)is
   a)-16/ 65 b)-33/65 c)33/65 d)16/65

7. Express (cos 5x – cos7x) as a product of sines or cosines or sines and cosines,
   a)2 cos4x cos x b)2sin4x sinx c)2 sin6x sin x d)2 cos 6x cosx

8. The value of is tan (45 + $A/2$)
   a) tan A- sec A b) tan A +sec A c) cot A –sec A d) cot A +sec A

9. The value of sin 10°sin 50°sin 70° is
   a) ½ b) 1/4 c) 3/4 d) 1/8

10. sin 15° + cos 105° =
    a) 2 sin 15° b) cos 15° + sin 15° c) 0 d) sin 15° – cos 15°

11. $\cos 1^o + \cos 2^o + \cos 3^o + \ldots + \cos 180^o =$

a) 0 b) -1 c) 2 d) 1

12. If tan A =1/2 and tan B= 1/3 then value of A+B

a) 45 b) 60 c) 30 d) 90

13. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 2$ is equal to (14)

a) 0 b) 2 c) -1 d) 1

14. $\tan^2 A + \cot^2 A$ is equal to (60)

a) ≥ 2 b) ≤ 2 c)) ≥ -2 d) none

15. Value of x for maximum value of x ($\sqrt{3}$ sinx +cos x)(67)

a) 30 b) 45 c) 60 d) 90

16. If $f(x) = \cos^2 x + \sec^2 x$ then (99)

a) $f(x) < 1$ b) $f(x) = 1$ c) $1 < f(x) < 2$ d) $f(x) \geq 2$

17. The minimum value of $3\cos^2 x + 2\sin^2 x$ is ?

a) 0 b) 3 c) 2 d) 1

18. The value $\cos 1^o \cos 2^o \cos 3^o \ldots\ldots\ldots \cos 179^o$ is ?

a) 0 b) 1/2 c) 1 d) ¼

19. sin A= 0.7 then cos A , 0 <A <90 is

a) 0.3 b) 0.7 c) $\sqrt{0.51}$ d) $\sqrt{0.9}$

20. If $\tan\theta + \cot\theta = 5$ then $\tan^2\theta + \cot^2\theta$ is

a) 23 b) 24 c) 25 d) 26

21. If x, y are acute angles, 0 <x+y <90 and sin(2x -20) = cos(2y +20) then the value of tan(x+y) is

a) 1 b) 0.5 c) 1.732 d) 0.866

22. If $\{(2 \sin \theta - \cos \theta) / (\cos \theta + \sin \theta)\} = 1$ then the value of $\cot \theta$ is
    a) ½ b) 1/3 c)3 d)2

23. If $\cot \{(\pi / 2) - \theta\} = \sqrt{3}$ then the value of $\cos \theta$ is
    a) 0 b) 1/2 c) 1 d) ∞

24. The value of $(\sin 300^\circ \tan 330^\circ \sec 420^\circ / \cot 135^\circ \cos 210^\circ \operatorname{cosec} 315^\circ)$ is :
    a) $(\sqrt{3} / 2)$ b) $-(\sqrt{3} / 2)$ c) $(2 / 3)$ d) $-\sqrt{(2 / 3)}$

25. If $\tan\Theta + \cot\Theta = 16$, then find the ratio of $\tan^2\Theta + \cot^2\Theta$ to $\tan^2\Theta + \cot^2\Theta + 20 \tan\Theta.\cot\Theta$
    a) 64:65 b) 129:137 c) 27:29 d) 127:137

26. If $7 \sin^2 \theta + 3 \cos^2 \theta = 4$ and $0 \le \theta \le (\pi / 2)$, then the value of $\tan \theta$ is:
    a) $\sqrt{(3 / 7)}$ b) $\sqrt{(2 / 7)}$ c) $1 / \sqrt{3}$ d) $1 / \sqrt{7}$

27. If $\cos\Theta + \sec\Theta = 2$,then the value of $\cos^{68}\Theta + \sec^{68}\Theta$ equal to
    a)2 b)2 c)3 d)68

28. Find the value of $(\cos 60^\circ / \sin 30^\circ) + (\cos 65^\circ . \operatorname{cosec} 25^\circ / \tan 10^\circ . \tan 30^\circ . \tan 45^\circ . \tan 60^\circ . \tan 80^\circ)$
    a)1 b)-1 c)0 d)2

29. The value of $\cot (2\pi / 20) . \cot (4\pi / 20) . \cot (5\pi / 20) . \cot (6\pi / 20) . \cot (8\pi / 20)$ is
    a) 2 b) 0 c) 1 d) 3

30. The least value of $9 \operatorname{cosec}^2 A + 16 \sin^2 A$ is
    a)7 b)24 c)25 d)14

➢ Solutions:

1. Let f (x)= $5\sin^2 x + 4\cos^2 x = 4 + \sin^2 x$
   Thus f(x) $\geq 4 + 0$
   $\therefore$ Minimum value of f(x) = 4 (as $\sin^2 x \geq 0$)

2. Short cut: In such examples, substitute values from the options to check which one satisfy the condition
   Here A= 45 and B=15 satisfy both conditions.
   Also sin (A-B) =1/2 = sin 30 => A-B =30……i)
   and cos (A-B) = ½ = > A+B =60…………..ii)
   Solving for the two equations i) & ii) A= 45 and B=15

3. tan A +cot A= 4
   - ⇨ $\tan^2 A + \cot^2 A + 2\tan A \cot A = 16$
   - ⇨ $\tan^2 A + \cot^2 A = 14$
   - ⇨ $\tan^4 A + \cot^4 A + 2 = 196$
   - ⇨ $\tan^2 A + \cot^2 A = 194$

4. Solution : On differentiating
   x= tanA + tan(π/6-A)
   we get :
   dx/dA = sec²A-sec²(π/6-A)
   now putting dx/dA=0
   we get
   cos²(A) = cos²(π/6-A) so 0≤A≤π/6
   therefore
   A=π/6-A from here we get A = π/12 = B
   so minimum value of that function is
   2tanπ/12 which is equal to 2(2-√3)

5. Solution

   Given that $\tan x = \frac{-3}{4}$ and $\frac{3\pi}{2} < x < 2\pi$
   means x lies in fourth quadrant
   Then $\sin x = -\frac{3}{5}$ and $\cos x = \frac{4}{5}$
   Now $\sin 2x = 2\sin x.\cos x$
   $\sin 2x = -2 \times \frac{3}{5} \times \frac{4}{5}$
   $\sin 2x = \frac{-24}{25}$

6.

Given that $\cos\theta = \frac{4}{5}, \cos\phi \frac{12}{13}$

Given than θ, and ϕ both lies in fourth quadrant then

$\sin\theta = \frac{-3}{5}$ and $\sin\phi = \frac{-5}{13}$

Now, $\cos(\theta + \phi) = \cos\theta \cos\phi - \sin\theta \sin\phi$

$= \frac{4}{5} \times \frac{12}{13} - \left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right)$

$= \frac{48}{65} - \frac{15}{65} = \frac{33}{65}$

7.

$\cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{D-C}{2}\right)$

$\therefore \cos 5x - \cos 7x = 2 \sin\left(\frac{5x+7x}{2}\right) \sin\left(\frac{7x-5x}{2}\right)$

$= 2 \sin 6x \sin x$

8.

$\frac{1+\tan\frac{A}{2}}{1-\tan\frac{A}{2}}$

$= \frac{1+\frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}}{1-\frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}}$

$= \frac{\cos\frac{A}{2}+\sin\frac{A}{2}}{\cos\frac{A}{2}-\sin\frac{A}{2}}$

$= \frac{(\cos\frac{A}{2}+\sin\frac{A}{2})^2}{\cos^2\frac{A}{2}-\sin^2\frac{A}{2}}$

$= \frac{\cos^2\frac{A}{2}+\sin^2\frac{A}{2}+2\cos\frac{A}{2}\sin\frac{A}{2}}{\cos A}$

$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$

9.

$\sin 10° \sin 50° \sin 70°$
$= \sin 10° \cos(90°-50°) \cos(90°-70°)$.
$= 2\sin 10° \cos 10° \cos 40° \cos 20° / 2\cos 10°$
$= 2\sin 20° \cos 20° \cos 40/4 \ \cos 10°$
$= 2\sin 40° \cos 40/8 \cos 10°$
$= \sin 80/8 \cos 10$
$= \cos 10 / 8\cos 10$
$= 1/8$.

10. As cos (90+A) = -sin A
Thus sin 15 + cos105 = sin A + cos(90+15)
⇨ sin 15 – sin 15
⇨ 0

11. cos A + cos B = 2 cos (A+B) /2 * cos (A- B) /2
Now making pairs such that the sum of angles is 180 ex A =1 , B=179 ; A=2, B=178….
Thus cos 1 +cos 179 = 2 cos (180/2) * cos (178/2)
= 2 cos 90 * cos 86
=0 since cos 90 =0
All such pairs gives value as 0 remaining term is cos (180) = -1. Hence the ans. is -1

12. tan (A+B) = (tan A +tan B) / (1- tanA .tanB)
=(½ +1/3) /(1- ½.1/3)
=5/6 / 5/6
= 1 => A+B= 45

13. $\sin x + \sin^2 x = 1$, then the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 2$?
$\sin x + \sin^2 x = 1 \ = \sin x = 1 - \sin^2 x = \cos^2 x$
$\therefore \cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 2$
$\Rightarrow \sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x - 2$
$\Rightarrow (\sin^2 x)^3 + 3(\sin^2 x)^2 * \sin x + 3(\sin^2 x) * \sin x^2 + \sin^3 x - 2$
$\Rightarrow (\sin^2 x + \sin x)^3 - 2$
$\Rightarrow 1 - 2$ as $\sin x + \sin^2 x = 1$
$\Rightarrow -1$

14. As$\left(x - \frac{1}{x}\right)^2 = x^2 + \left(\frac{1}{x}\right)^2 - 2 \geq 0$ Put x = tan A in this equation

Thus $\tan^2 A + \cot^2 A - 2 \geq 0$

$\tan^2 A + \cot^2 A \geq 2$

15. x ($\sqrt{3}$ sinx +cos x) the maximum value of equation is 2 as $\sqrt{(3)+1}$ =2.

It will be obviously at x= 60

16. Same as example no 14.

17. Answer: Option C

Solution:

Let $x = 2\sin^2\theta + 3\cos^2\theta$

$\Rightarrow x = 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$

$\Rightarrow x = 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta$

$\Rightarrow x = 2 + \cos^2\theta$ [since $\sin^2\theta + \cos^2\theta = 1$]

Therefore x will be the minimum when $\cos\theta = 0$. i.e. minimum value of x will 2

Alternative Solution:

$2\sin^2\theta + 3\cos^2\theta$

Minimum value is 2,

[If $x \sin^2\theta + y \cos^2\theta$, If x > y, then x will be always maximum value and y is minimum if y > x, vice versa will happen]

18. cos1° cos2° cos3° .... cos177° cos178° cos179°

= cos90°

= 0[0 will make whole series 0]

= 0

19. Answer: Option C

Solution:

sinθ=0.7⇒sin2θ+cos2θ=1⇒(0.7)2+cos2θ=1⇒0.49+cos2θ=1⇒cos2θ=1−0.49⇒cosθ= √0.51

20. Answer: Option A

    Solution:

    Given, $\tan\theta + \cot\theta = 5 \Rightarrow \tan\theta + \cot\theta = 5 \Rightarrow (\tan\theta + \cot\theta)^2 = 5^2$ (Squaring both sides)

    $\Rightarrow \tan 2\theta + \cot 2\theta + 2\tan\theta\cot\theta = 25$

    $\Rightarrow \tan 2\theta + \cot 2\theta = 25 - 2 [\because \tan\theta . \cot\theta = 1]$

    $\Rightarrow \tan 2\theta + \cot 2\theta = 23$

21. Answer: Option D

    Solution:

    $\sin(2x - 20^\circ) = \cos(2y + 20^\circ) \Rightarrow (2x - 20^\circ) + (2y + 20^\circ) = 90^\circ$

    [If $\sin A = \cos B$, then $A + B = 90^\circ$]

    $\Rightarrow 2(x+y) = 90^\circ \Rightarrow x + y = 45^\circ \therefore \tan(x+y) = \tan 45^\circ = 1$

22. Dividing numerator and denominator by $\sin\theta$

    $\Rightarrow [\{(2\sin\theta - \cos\theta) / \text{Sin}\,\theta\} / \{(\cos\theta + \sin\theta) / \sin\theta\}] = 1$

    $\Rightarrow \{(2 - \cot\theta) / (1 + \cot\theta)\} = 1$

    $\Rightarrow 2\cot\theta = 1 + \cot\theta$

    $\Rightarrow \cot\theta = 1 / 2$

23. $\cot\{(\pi / 2) - \theta\} = \sqrt{3}$

    $\Rightarrow \tan\theta = \sqrt{3}$

    $[\because \cot\{(\pi / 2) - \theta\} = \tan\theta]$

    $\therefore \theta = 60°$

    Hence, $\cos 60° = 1/2$.

24. $= [\{\sin(360^\circ - 60^\circ) \tan(360^\circ - 30^\circ) \sec(360^\circ + 60^\circ)\} / \{\cot(180^\circ - 45^\circ) \cos(180^\circ + 30^\circ) \text{cosec}(360^\circ + 45^\circ)\}]$

    $= \{(-\sin 60^\circ)(-\tan 30^\circ)(\sec 60^\circ)\} / \{(-\cot 45^\circ)(-\cos 30^\circ)(-\text{cosec}\, 45^\circ)\}$

    $= -[\{(\sqrt{3}) / 2 \times 1 / (\sqrt{3}) \times 2\} / \{1 \times (\sqrt{3}) / 2 \times \sqrt{2}\}] = -\sqrt{(2 / 3)}$

25. Given, $\tan\Theta + \cot\Theta = 16$

Squaring both sides, we get

$\tan^2\Theta + 2\tan\Theta \cdot \cot\Theta + \cot^2\Theta = 256$

or, $\tan^2\Theta + \cot^2\Theta = 256 - 2$

$\therefore$ $\tan^2\Theta + \cot^2\Theta = 254$

Now, $\tan^2\Theta + \cot^2\Theta + 20\tan\Theta.\cot\Theta$

$= (\tan^2\theta + \cot^2\theta + 20\tan\theta \times (1 / \tan\theta)$

$= 254 + 20 = 274$

$\therefore$ Reqd. ratio= $(254 / 274) = (127 / 137) = 127 : 137$

26. $7 \sin^2 \Theta + 3 \cos^2 = 4$

$\Rightarrow 7 \sin^2 \theta + 3(1 - \sin^2\theta) = 4 \Rightarrow \sin^2\theta = 1/4$, so, $\sin\theta = (1/2)$

$\therefore \cos\theta = \sqrt{}(1-\sin^2\theta) = \sqrt{}(1 - 1/4) = (\sqrt{3} / 2)$

$\therefore \tan\theta = \sin\theta / \cos\theta = (1 / 2 \times 2\sqrt{3}) = 1/\sqrt{3}$

27. $\cos\Theta + \sec\Theta = 2$

or, $\cos\theta + 1/\cos\theta = 2$

or, $\cos^2\Theta + 1 = 2\cos\Theta$

or, $\cos^2\Theta - 2\cos\Theta + 1 = 0$

or, $(\cos\Theta - 1)^2 = 0$

$\therefore \cos\Theta = 1$

$\therefore \sec\theta = 1/\cos\theta = 1$

Now, $\cos^{68}\Theta + \sec^{68}\Theta = 1 + 1 = 2$

28. $(\cos 60^o / \sin 30^o) + (\cos 65^o . \operatorname{cosec} 25^o / \tan 10^o . \tan 30^o . \tan 45^o . \tan 60^o . \tan 80^o)$

$\{ \cos (90^o - 30^o) / \sin 30^o\} + \{ \cos 65^o . \operatorname{cosec} (90^o - 65^o) / \tan 10^o . \tan 30^o . \tan 45^o . \tan (90^o - 65^o). \tan (90^o - 10^o)\}$

$(\sin 30^o / \sin 30^o) + (\cos 65^o . \sec 65^o / \tan 10^o . \tan 30^o . \tan 45^o . \cot 30^o . \cot 10^o)$

$= 1 + \{ \cos 65 (1 / \cos 65^o) / \tan 10^o . \tan 30^o . \tan 45^o . \cot 30^o . \cot 10^o\}$

$= 1 + 1/2 = 2$

29. **Given expression:** $\cot(2\pi / 20) \cdot \cot(4\pi / 20) \cdot \cot(5\pi / 20) \cdot \cot(6\pi / 20) \cdot \cot(8\pi / 20)$

$= \cot(\pi / 10) \cdot \cot(\pi / 5) \cdot \cot(\pi / 4) \cdot \cot(3\pi / 10) \cdot \cot(2\pi / 5)$

$= \cot(180^\circ / 10) \cdot \cot(180^\circ / 5) \cdot \cot(180^\circ / 4) \cdot \cot\{(3 \times 180^\circ) / 10\} \cdot \cot\{(2 \times 180^\circ) / 5\}$

$= \cot 18^\circ \cdot \cot 36^\circ \cdot \cot 45^\circ \cdot \cot 54^\circ \cdot \cot 72^\circ$

[we know that (18, 72) and (36, 54) are complementary angles, so $\cot(90^\circ - \Theta) = \tan\Theta$ or, $\cot 54^\circ = \tan(90^\circ - 54) = \tan 36^\circ$]

$= (1 / \tan 18^\circ) \times (1 / \tan 36^\circ) \times \cot 45^\circ \cdot \tan 36^\circ \cdot \tan 18^\circ$

$\Rightarrow \cot 45^\circ = 1$

30. $9 \operatorname{cosec}^2 A + 16 \sin^2 A$

$= (9 / \sin^2 A) + 16 \sin^2 A$

$= (3 / \sin A)^2 + (4 \sin A)^2$

[$\because a^2 + b^2 = (a - b)^2 + 2ab$]

Let $a = (3 / \sin A)$, $b = 4 \sin A$

$= \{(3 / \sin A) - 4 \sin A\}^2 + 2 \times (3 / \sin A) \times 4 \sin A$

$= \{(3 - 4 \sin^2 A) / \sin A\}^2 + 24$

for the least value of $\{(3 - 4\sin^2 A) / \sin A\}$ should be 0.

$\therefore$ The least value will be 24.