Differential Equations

1. The order of differential equation whose solution is $x^2+y^2+2gx+2fy+c=0$
   a) 1
   b) 2
   c) 3
   d) 4

2. The order of the differential equation of all circles of the radius r having centre on y axis and passing through the origin is
   a) 1
   b) 2
   c) 3
   d) 4

3. The order of differential equation whose solution is $y= a\cos x+b\sin x+c\, e^{-x}$ is
   a) 3
   b) 2
   c) 1
   d) None

4. The order of the differential equation of all circles of the radius r
   a) 2
   b) 3
   c) 4
   d) None

5. The degree and order of the differential equations of the family of all parabolas whose axis is x axis are respectively
   a) 2,1
   b) 2,3
   c) 3,2
   d) 2,3

6. The differential equation of all the lines in the xy plane is
   a) $\frac{dy}{dx} - x = 0$
   b) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$
   c) $\frac{d^2y}{dx^2} = 0$
   d) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

7. The differential equation whose solution is y = Asinx +B cos x
   a) $\frac{d^2y}{dx^2} + y = 0$
   b) $\frac{d^2y}{dx^2} - y = 0$
   c) $\frac{dy}{dx} + y = 0$
   d) $none$

8. Solution of the differential equation $(1+x^2)\frac{dy}{dx} = x$ is
   a) $y = \tan^{-1} x + c$
   b) $y = -\tan^{-1} x + c$
   c) $y = \frac{1}{2}\log(1+x^2) + c$
   d) $y = -\frac{1}{2}\log(1+x^2) + c$

9. Solution of the differential equation $\frac{dy}{dx} = \sec x(\sec x + \tan x)$ is
   a) y= sec x +tan x +c
   b) y= sec x -tan x +c
   c) y= sec x +cot x +c
   d) none

10. Solution of the differential equation (sin x+ cos x) dy +(cos x – sin x) dx =0
   a) e^x (sin x+cos x) +c =0
   b) e^y (sin x+cos x) = c
   c) e^y (- sin x+cos x) = c
   d) e^x (sin x -cos x) = c

11. Solution of the differential equation (1 + cos x) dy= (1- cosx) dx is

a) y = 2 tan x/2 – x+c
b) y = 2 tan x/2 + x+c
c) y = 2 tan x + x+c
d) y = x- 2 tan x/2 +c

12. Solution of the differential equation $x\frac{dy}{dx} + y = y^2$ is

a) y = 1+cxy
b) y = log (cxy)
c) y+1 = cxy
d) y = c+xy

13. Solution of the differential equation($1-x^2$)dy +xydx =x y^2dx is

a) $(y-1)^2 (1-x^2)=0$
b) $(y-1)^2 (1-x^2)=c^2y^2$
c) $(y-1)^2 (1+x^2)= c^2y^2$
d) None

14. Solution of the differential equation y'-y =1 , y(0) =-1 is given by y(x) is equal to

a) $-e^x$
b) $-e^{-x}$
c) -1
d) e^x -2

15. Integrating factor of the differential equation cos x $\frac{dy}{dx}$ + ysinx =1

a) cos x
b) tan x
c) sec x
d) sin x

16. Integrating factor of the differential equation $\frac{dy}{dx}$ = y tan x – y2 sec x is

a) tan x
b) sec x
c) –sec x
d) cot x

17. Solution of $\frac{dy}{dx} + y = e^{-x}$, y(0) = 0 is

a) $y = e^{-x}(x-1)$
b) $y= xe^{x}$
c) $y= xe^{-x} +1$
d) $y= xe^{-x}$

18. Equation of the curve through point (1,0) which satisfies the differential equation $(1+y^2)dx- xydy=0$ is

a) $x^2+y^2=1$
b) $x^2-y^2=1$
c) $2x^2+y^2=2$
d) None

19. Which of the following equation is non linear?

a) $\frac{dy}{dx}+\frac{y}{x}$ =log x
b) $y\frac{dy}{dx}$ +4x=0
c) dx+dy=0
d) $\frac{dy}{dx}$ -4x=0

20. Which of the following equation is linear?

a) $\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ =0
b) $(\frac{d^2y}{dx^2})^2+x\frac{dy}{dx}$ =0
c) $\frac{dy}{dx}$ +y/x = log x
d) *none*

➢ **Solution:**

1. Since there are 3 constants g,f, and c hence order of DE is 3.

2. Equation of family of circles of radius a is $(x-0)^2+(y-a)^2 = a^2$ or $x^2+y^2-2ay = 0$ which has a single constant hence has order 1

3. Since there are 3 constants a,b, and c hence order of DE is 3.

4. Equation of family of circles of radius a is $(x-h)^2+(y-k)^2 = a^2$ is a 2 parameter family of curves
   Hence the order is 2.

5. $y^2 = 4a(x-h)$
   $2yy1 = 4a$
   $yy1 = 2a$
   $y1^2+yy2 = 0$
   Hence degree 1, order =2

6. Equation of all lines in xy plane is given by y= mx+c
   Differentiating twice with respect to x we get $\frac{d^2y}{dx^2} = 0$

7. y = Asinx +B cos x
   ⇨ $\frac{dy}{dx}$ = Acos x – B sin x
   ⇨ $\frac{d^2y}{dx^2}$ = - Asinx - B cos x= - (Asinx +B cos x)= -y
   ⇨ $\frac{d^2y}{dx^2} + y = 0$ is the answer.

8. $(1+x^2)\frac{dy}{dx} = x$
   $\frac{dy}{dx} = = \frac{x}{1+x2}$
   Integrating on both sides y = ½ log $(1+x^2)$ +c

9. $\frac{dy}{dx}$ = secx (sec x+ tan x)

$\frac{dy}{dx}$ = sec x^2 +secx tan x

Integrating on both sides y= sec x +tan x +c

10. sin x+ cos x) dy +(cos x – sin x) dx

thus $\frac{dy}{dx} = -\frac{cosx-sinx}{sinx+\cos x}$

dy= $= -\frac{cosx-sinx}{sinx+\cos x}$dx

Integrating both sides we get

y = - log (sinx +cos x)+ log C

y =log (c/ sinx +cos x)

e^y (sin x+cos x) = c

11. 1 + cos x) dy= (1- cosx) dx

thus$\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$ = $\tan^2$ x/2

dy = ($\sec^2$ x/2 - 1) dx

On integrating both sides we get ,

y = 2 tan x/2 – x+c

12. x $\frac{dy}{dx}$ + y = y^2

x $\frac{dy}{dx}$ = y^2 -y

dy /(y^2 -y) = dx/x

$[\frac{1}{1-y} \ \frac{1}{y}] = \frac{1}{x}$

On integrating log(y-1) – log y =log x +log c

y = 1+cxy

13. (1-x^2)dy +xydx =x y^2dx

(1-x^2)dy = xy (y-1) dx

dy /(y^2 -y) = dx/(1- x^2)

On integrating

log (y-1) – log y = -1/2 (1- $x^{2)}$ + log C

hence solution (y-1)^2 (1-x^2)=c^2y^2

14. y'-y =1 , y(0) =-1

thus $\frac{dy}{dx}$ -y =1

dy/ (y+1) =dx

On integrating we get,

log (1+y) = x+c

$1+y = e^x e^c$

as x=0 , y= -1puting values we get e^c =0

thus $1+y = e^x$ X 0 => y = -1

15. cos x $\frac{dy}{dx}$ + ysinx =1

hence $\frac{dy}{dx}$ +y tanx = sec x

thus integrating factor is $e^{\int tanx}$ = sec x

16. $\frac{dy}{dx}$ = y tan x – y2 sec x is

Dividing by y^2

$1/ y^2 * \frac{dy}{dx}$ -1/y tanx = -sec x

Put 1/y = Y thus $1/ y^2 * \frac{dy}{dx}$ = dY /dx

Hence equation Dy/Dx + Y tanx = sec x

Thus integrating factor is $e^{\int tanx}$ = sec x

17. $\frac{dy}{dx} + y = e^{-x}$, y(0) = 0

integrating factor = e^x

$y e^x = x+c$

as y(0) =0 => c=0

Hence solution is $y= xe^{-x}$

18. $(1+y^2)dx- xydy=0$ is

We have $dx/x = ydy / (1+y^2)$

Integrating we get

$\log x = ½ \log (1+y^2) + \log c$

$x = c \sqrt{(1+y^2)}$

It passes through (1, 0) so we get c=1

solution is $x^2-y^2=1$

19. A differential equation in which dependent variable and its differential coefficient occur only in first degree and are not multiplied together is a linear differential equation . Hence $y\frac{dy}{dx}$ +4x=0 is a non linear .

20. Linear equation is option c.