class Test unit-1

1) $h(n) = u(n) - u(n-5)$

$$h(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

$y(n) = x(n) * h(n)$

$$y(n) = \sum_{m=0}^{4} x(n-m) = x(n) + x(n-1) + x(n-2) + x(n-3) + x(n-4)$$

substitute the given $x(n)$

$y(1) = x(1) = 2$

$y(2) = x(2) + x(1) = 1 + 2 = 3$

$y(3) = x(3) + x(2) + x(1) = 2 + 1 + 2 = 5$

$y(4) = x(4) + x(3) + x(2) + x(1) = 0 + 2 + 1 + 2 = 5$

$y(5) = x(5) + x(4) + x(3) + x(2) + x(1) = 0 + 0 + 2 + 1 + 2 = 5$

$y(6) = x(6) + x(5) + x(4) + x(3) + x(2) = 0 + 0 + 0 + 2 + 1 = 3$

$y(7) = x(7) + x(6) + x(5) + x(4) + x(3) = 0 + 0 + 0 + 0 + 2 = 2$

$y(n) = 0$

$$y(n) = \begin{cases} 2, & n = 1 \\ 3, & n = 2 \\ 5, & n = 3 \\ 5, & n = 4 \\ 5, & n = 5 \\ 3, & n = 6 \\ 2, & n = 7 \\ 0, & \text{otherwise} \end{cases}$$

2) $y(n) = 3x(n) + 4x(n-1) + 5x(n-2) + y(n-1)$

a) Impulse response h(n)

$n=0$: $h(0) = 3$

$n=1$: $h(1) = 3x(1) + 4x(0) + y(0) = 0 + 4 + 3 = 7$

$n=2$: $h(2) = 3x(2) + 4x(1) + 5x(0) + y(1)$

$= 0 + 0 + 5 + 7 = 12$

for $n \geq 3$ $x(\cdot) = 0$ so $h(n) = h(n-1)$

$h(n) = 12$ for $n \geq 2$

so,

$h(0) = 3$, $h(1) = 7$, $h(n) = 12$, for $n \geq 2$

$2(h(n)) = 3\delta(n)$

$+ 4\delta(n-1)$

$+ 12\,u(n-2)$

b) unit step response s(n)

$y(0) = 3$

$y(1) = 3 \cdot 1 + 4 \cdot 1 + y(0) = 3 + 4 + 3 = 10$

for $n \geq 2$, $x(n) = 1$ so the input part is

$3 + 4 + 5 = 12$ &

$y(n) = y(n-1) + 12$

$y(2) = 22,\ y(3) = 34$

$$s(n) = y(n) = \begin{cases} 3, & n = 0 \\ 12n - 2, & n \geq 1 \end{cases}$$

c) System function $H(z)$

$$Y(z) = (3 + 4z^{-1} + 5z^{-2})X(z) + z^{-1}Y(z)$$

so,

$$Y(z)(1 - z^{-1}) = (3 + 4z^{-1} + 5z^{-2})X(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$H(z) = \frac{3 + 4z^{-1} + 5z^{-2}}{1 - z^{-1}}$$

d) freq. response $H(e^{j\omega})$

$$H(e^{j\omega}) = \frac{3 + 4e^{-j\omega} + 5e^{-j2\omega}}{1 - e^{-j\omega}}$$

e) Magnitude response $H(e^{j\omega})$

$$N(\omega) = 3 + 4\cos\omega + 5\cos 2\omega - j(4\sin\omega + 5\sin 2\omega)$$

$$1 - e^{-j\omega} = 1 - \cos\omega + j\sin\omega$$

$$|1 - e^{j\omega}| = \sqrt{(1-\cos\omega)^2 + \sin^2\omega}$$

$$= 2|\sin(\omega/2)|$$

$$|H(e^{j\omega})| = \frac{\sqrt{(3+4\cos\omega + 5\cos 2\omega)^2 + (4\sin\omega + 5\sin 2\omega)^2}}{2|\sin(\omega/2)|}$$

f) phase response $\angle H(e^{j\omega})$

$$\angle H(e^{j\omega}) = \angle(3 + 4e^{-j\omega} + 5e^{-j2\omega}) - \angle(1 - e^{-j\omega})$$

use identity, $1 - e^{-j\omega} = 2je^{-j\omega/2}\sin(\omega/2)$

$$\angle(1 - e^{-j\omega}) = \frac{\pi}{2} - \frac{\omega}{2}$$

$$\angle H(e^{j\omega}) = \operatorname{atan2}(-[4\sin\omega + 5\sin 2\omega], 3 + 4\cos\omega + 5\cos 2\omega) - (\pi/2 - \omega/2)$$

9) $x(n) = e^{3n} u(n)$, find $y(n)$

$$Y(z) = H(z)X(z)$$
$$= \frac{3 + 4z^{-1} + 5z^{-2}}{(1-z^{-1})(1-e^{3}z^{-1})}$$

$$Y(z) = \frac{A}{1-z^{-1}} + \frac{B}{1-e^{3}z^{-1}}$$

At $z^{-1} = 1$, $A = \frac{3+4(1)+5(1)}{1-e3} = \frac{12}{1-e3}$

At $z^{-1} = e^{-3}$. $B = \frac{3+4e^{-3}+5e^{-6}}{1-e^{-3}}$

$$y(n) = A^{n} + Be^{3n}$$

$$y(n) = \frac{12}{1-e3} + \frac{3+4e^{-3}+5e^{-6}}{1-e^{-3}} e^{3n} \quad (n \geq 0)$$

3) a) $f(1,2) = 5$

b) $f(3,2) = 6$

c) gray level resolution

distinct gray levels present $\{1,2,3,4,5,6,7,9\}$ → 8 levels

gray level resolution in bits $= \log_2 8 = 3$ bits

so, 8 gray levels (3 bit)

d) size of image

Dimension $= 3 \times 3 = 9$

e) zoom by factor 2

pixel replication, New size $= 3 \cdot 2 \times 3 \cdot 2 = 6 \times 6$

zoomed image by pixel replication:

$$\begin{bmatrix} 1 & 1 & 5 & 5 & 7 & 7 \\ 1 & 1 & 5 & 5 & 7 & 7 \\ 2 & 2 & 3 & 3 & 9 & 9 \\ 2 & 2 & 3 & 3 & 9 & 9 \\ 4 & 4 & 6 & 6 & 2 & 2 \\ 4 & 4 & 6 & 6 & 2 & 2 \end{bmatrix}$$

→ 6×6

f) zoom by factor 2 - zero order hold

zero order hold duplicates the value and holds it across the new pixels for factor 2 this is identical to pixel replication

- zoomed image by zero order hold:

$$\begin{bmatrix} 1 & 1 & 5 & 5 & 7 & 7 \\ 1 & 1 & 5 & 5 & 7 & 7 \\ 2 & 2 & 3 & 3 & 9 & 9 \\ 2 & 2 & 3 & 3 & 9 & 9 \\ 4 & 4 & 6 & 6 & 2 & 2 \\ 4 & 4 & 6 & 6 & 2 & 2 \end{bmatrix}$$

g) zoom by factor 2 - K-times zooming

so, for K=2 the zoomed image is again

$$\begin{bmatrix} 1 & 1 & 5 & 5 & 7 & 7 \\ 1 & 1 & 5 & 5 & 7 & 7 \\ 2 & 2 & 3 & 3 & 9 & 9 \\ 2 & 2 & 3 & 3 & 9 & 9 \\ 4 & 4 & 6 & 6 & 2 & 2 \\ 4 & 4 & 6 & 6 & 2 & 2 \end{bmatrix}$$

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(4) a) order of the image = 5 × 5

b) shrink the image with explaination and size of shrunk image

→ Nearest - neighbour subsampling

select rows 1, 3, 5, & column 1, 3, 5 :

f(1,1) = 1   f(1,3) = 9   f(1,5) = 9
f(3,1) = 4   f(3,3) = 2   f(3,5) = 7
f(5,1) = 9   f(5,3) = 2   f(5,5) = 4

so the shrunk image is

$$\begin{bmatrix} 1 & 9 & 9 \\ 4 & 2 & 7 \\ 9 & 2 & 4 \end{bmatrix}_{3\times 3}$$

5) sampling in image processing is selecting pixels from a continuous image grid to make a digital image

i) perfect sampling is
sampling exactly at Nyquist rate

(ii) critical sampling :-

- sampling exactly at the minimum rate required by Nyquist

iii) oversampling :-

- sampling at a rate much higher than Nyquist

iv) undersampling

- sampling below Nyquist rate

→ Standards / theorems used to remove Aliasing

i) Nyquist sampling theorem :-

- State: To reconstruct a signal / image without aliasing sampling freq. must be at least twice the maximum freq. present in the signal
- In image: pixel spacing ≤ half of the smallest feature size

ii) shannon's sampling theorem :-

- formalize the same concept in continuous signals / images

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Q-6 → Sol: Assumption of coordinate

- $p1 = (0,0)$
- $p2$ is to the right of $p1$ Assume $p2 (6,0)$
- $p3$ is vertically above $p2$ Assume $p3 = (6,4)$

→ formulas :-

1) Euclidean Distance :-
$$D_E(P1, P3) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2) city-block / D4 Distance :-
$$D4 (P1, P3) = |x_2 - x_1| + |y_2 - y_1|$$

3) chessboard / D8 distance :-
$$D8 (P1, P3) = \max(|x_2 - x_1|, |y_2 - y_1|)$$

$$D8 = \max(6, 4) = 6$$

→ step-2 : substitution

$x_1 = 0,\ y_1 = 0,\ x_2 = 6,\ y_2 = 4$

1) Euclidean Distance :-

$$DE = \sqrt{(6-0)^2 + (4-0)^2} = \sqrt{36+16} = \sqrt{52} \approx 7.21$$

2) city-block distance (D4) :-

$$D4 = |6-0| + |4-0| = 6+4 = 10$$

3) chessboard Distance :-

$$D8 = \max(6,4) = 6$$

→ ~~N4~~

7) N4(P) = 4-Neighbours

ND(P) = 4-diagonal neighbours of P

N8(P) = all 8 neighbours = N4(P) ∪ ND(P)

(1,1) = 0 (1,2) = 1 (1,3) = 1

(2,1) = 0 (2,2) = 1 (2,3) = 0

(3,1) = 0 (3,2) = 0 (3,3) = 1

Red pixel P = (2,2)

→ $N_4(P) = \{(1,2),(3,2),(2,1),(2,3)\}$ - but only the ones with value 1 are $\{(1,2)\}$

- $N_D(P) = \{(1,1),(1,3),(3,1),(3,3)\}$

so $N_D(p) = \{(1,3),(3,3)\}$

- $N_8(P) = N_4(p) \cup N_D(p)$
$= \{(1,2),(1,3),(3,3)\}$

→ final answer :-
$N_4(P) = \{(1,2)\}$, $N_D(P) = \{(1,3),(3,3)\}$
$N_8(P) = \{(1,2),(1,3),(3,3)\}$