GALGOTIAS UNIVERSITY

All Programs Semester I CAT II – January 2022

Answer uploading Template

Enrolment / Admission No. of Student	21SCSE1010602	Name of Course	Multivariable Calculus
Name of Student	Shivam Dwivedi	Course Code	BBS01T1001
Program	B.tech CSE	Date of Examination	16.01.22
Semester	1st	Time	2:00 to 3:40PM
Signature of Student	Shivam		

Student shall start writing from below:

1) Ans Domain and range of $f(x,y) = \frac{2x}{y-x^2}$

$y - x^2 \neq 0$

So $D: \{(x,y) : y \neq x^2\}$

$R: \{(-\infty, 0) \cup (0, \infty)\}$

2) Ans $\int_0^2 \int_0^2 2x \, dy \, dx$

$= \int_0^2 dy \int_0^2 2x \, dx$

$= \int_0^2 dy \int_0^2 2x \, dx$

$= \int_0^2 dy \left[\frac{2x^2}{2}\right]_0^2$

$= \int_0^2 4 \, dy = [4y]_0^2$

$= 8$ Ans

3) Ans. By path ① $x = 0,\ y \to 0$

$$\lim_{y \to 0} \frac{2 \cdot 0 \cdot y}{0 + y^2} = 0$$

By path ② $y = 0,\ x \to 0$

$$\lim_{x \to 0} \frac{2 \cdot x \cdot 0}{x^2 + 0} = 0$$

By path ③ put $x = y$

$$\lim_{y \to 0} \frac{2y^2}{y^2 + y^2} = 1$$

limit by path ③ is not equal to path ① & path ② hence limit doesnot exist.

4) Solⁿ: for $\iint_R F(x,y)\,dA$

we get $\int_R\int (100-6x^2y)\,dx\,dy \quad \therefore R: 0 \le x \le 2, \; -1 \le y \le 1$

So,

$$\Rightarrow \int_{-1}^{1}\int_{0}^{2} (100-6x^2y)\,dx\,dy$$

$$\Rightarrow \int_{-1}^{1}\left[100x - \frac{6}{3}x^3y\right]_0^2 dy$$

$$\Rightarrow \int\left[200 - 16y\right] dy$$

$$\Rightarrow \left[200y - \frac{16}{2}y^2\right]_{-1}^{1}$$

$$\Rightarrow \left[200y - 8y^2\right]_{-1}^{1}$$

$$= 200 - 8 + 200 + 8$$

$$= 400 \text{ Ans.}$$

2) Sol:
$f(x,y) = -3x^2 + 3y^2 + 6xy - 2y^3$

$f_x = -6x + 6y, \quad f_y = 6y + 6x - 6y^2$

Now,

$f_x = 0 \Rightarrow -6x + 6y = 0 \Rightarrow -x + y = 0$

$f_y = 0 \Rightarrow 6y + 6x - 6y^2 = 0 \Rightarrow y(1-y) + y = 0$

$\Rightarrow y(2-y) = 0 \Rightarrow y = 0 / y = 2$

$\therefore (x,y) = (0,0)$ or $(2,2)$

$f_{xx} = -6, \quad f_{yy} = 6 - 12y, \quad f_{xy} = 6$

$D(x,y) = (-6)(6-12y) - 36$
$= 72y - 72$
$= 72(y-1)$

at $(0,0)$: $D(0,0) = 72(0-1) = -72 < 0$

at $(2,2)$: $D(1,2) = 72(2-1) = 72 > 0$

Saddle at $(0,0)$

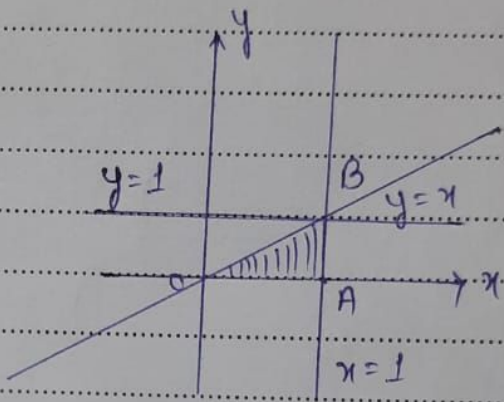
also, $D(x,y) > 0$ and $f_{xx} < 0$

$\therefore$ local maximum at $(2,2)$.

b) Ans $\cancel{I = \int_0^1 \int_y^1}$

$$I = \int_0^1 \int_y^1 x^2 e^{xy} \, dx \, dy.$$

x: y to 1
y: 0 to 1



ΔOAB is region of integration.
(for reversing the order of integration, we will first think for limit of y and then for limit of x)

$\therefore$ y: 0 to x
x: 0 to 1

$$\therefore \quad I = \int_0^1 \int_0^x x^2 e^{xy} \, dy \, dx$$

$$= \int_0^1 \left(x^2 \int_0^x e^{xy} \, dy \right) dx$$

$$= \int_0^1 \left(x^2 \left[\frac{e^{xy}}{x} \right]_{y=0}^{x} \right) dx$$

$$= \int_0^1 \left(\frac{x^2}{x} \right) \left(e^{x^2} - e^0 \right) dx$$

$$= \int_0^1 \left(x e^{x^2} - x \right) dx$$

$$I = \int_0^1 x e^{x^2} dx - \int_0^1 x \, dx$$

In 1st integral. Substitute $x^2 = t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \frac{1}{2} dt$

$$\therefore \int x e^{x^2} dx = \frac{1}{2} \int e^t \, dt$$

$$= \frac{1}{2} e^t + C$$

$$= \frac{1}{2} e^{x^2} + C$$

$$\therefore I = \frac{1}{2} \left[e^{x^2} \right]^1 - \left[\frac{x^2}{2} \right]_{x=0}^{1}$$

$$= \frac{1}{2} (e^1 - e^0) - \frac{1}{2} (1 - 0)$$

$$= \frac{e}{2} - 1.$$