GALGOTIAS UNIVERSITY

All Programs Semester I CAT II – January 2022

Answer uploading Template

Enrolment / Admission No. of Student	21SCSE1410079	Name of Course	Multivariable Calculus
Name of Student	Ananya Kumar	Course Code	BBS 01T1001
Program	B.Tech CSE [AI]	Date of Examination	16 January, 2022
Semester	1	Time	2:00 pm - 3:30 pm
Signature of Student	Ananya Kumar		

Student shall start writing from below:

A1.

$$f(x,y) = \frac{2x}{y-x^2}$$

$f(x,y)$ will be defined when,

$$y - x^2 \neq 0$$

$$y \neq x^2$$

Hence, Domain $= \{(x,y) : y \neq x^2\}$

Range $= R^2$ (All Real numbers.)

A2.

$$\int_0^2 \int_0^2 2xdydx \Rightarrow \int_0^2 \int_0^2 2xdxdy$$

$$\int_0^2 \left[\frac{2x^2}{2}\right]_0^2 dy \Rightarrow \int_0^2 (4)dy$$

$$= [4y]_0^2 = 8 \text{ A}$$

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A3. $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$

Case 1: $x=0, y\to 0$

$L_1 = \lim_{(0,y)\to(0,0)} \frac{2(0)\times y}{(0)^2+y^2} = 0$

Case 2: $y=0, x\to 0$

$L_2 = \lim_{(x,0)\to(0,0)} \frac{2x(0)}{x^2+(0)^2} = 0$

Case 3: $x=y$

$L_3 = \lim_{(x,x)\to(0,0)} \frac{2(x)(x)}{x^2+x^2}$

$= \lim_{(x,x)\to(0,0)} \frac{2x^2}{2x^2} = 1$

$L_1 = L_2 \neq L_3$

Hence limit for function $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$ does not exist.

Hence proved

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A4. $\iint_R f(x,y)\,dA \quad f(x,y) = 100 - 6x^2y$

$R: 0 \le x \le 2, \quad -1 \le y \le 1$

$$\int_{-1}^{1}\int_{0}^{2} (100 - 6x^2y)\,dx\,dy$$

$$\int_{-1}^{1} \left[100x - \frac{6x^3}{3}y\right]_0^2 dy$$

$$\int_{-1}^{1} (200 - 16y)\,dy \Rightarrow \left[200y - \frac{16y^2}{2}\right]_{-1}^{1}$$

$= 200 - 8 + 200 + 8 = 400$ Ans

A5. $f(x,y) = -3x^2 + 3y^2 + 6xy - 2y^3$

Differentiation w.r.t x

$$\frac{\partial f}{\partial x} = -6x + 6y \quad \text{---(1)}$$

Differentiation w.r.t y $\quad \frac{\partial f}{\partial y} = 6y + 6x - 6y^2 \quad \text{---(2)}$

Equating eq^n (1) & (2)

$-6x + 6y = 0 \Rightarrow x = y$ ---(3)

$6y = 10x \Rightarrow 5x - 3y = 0$ ---(3)

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$6y + 6x = 6y^2$ — (4)

~~$xy = 0$~~ — (4)

Putting (3) in (4)

$6y + 6y = 6y^2$

$12y = 6y^2$

$2y = y^2$

$2y - y^2 = 0$

$(2-y)y = 0$

$y = 2, 0$

$x = 2, 0$

Critical points are $(0,0)$ & $(2,2)$

Second derivatives

$$r = \frac{\partial^2 f}{\partial x^2} = -6$$

$$s = \frac{\partial^2 f}{\partial x \partial y} = 6$$

$$t = \frac{\partial^2 f}{\partial y^2} = 6 - 12y$$

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We know, $rt - s^2$ ——— (5)

Putting values in eq (5)

$(-6)(6 - 12y^2) - (6)^2$

$-36 + 72y^2 - 36 \Rightarrow 72y - 72$

for (0,0)

$rt - s^2 = -72 < 0$ Saddle point

for (2,2)

$rt - s^2 = 144 - 72 = 72 > 0$

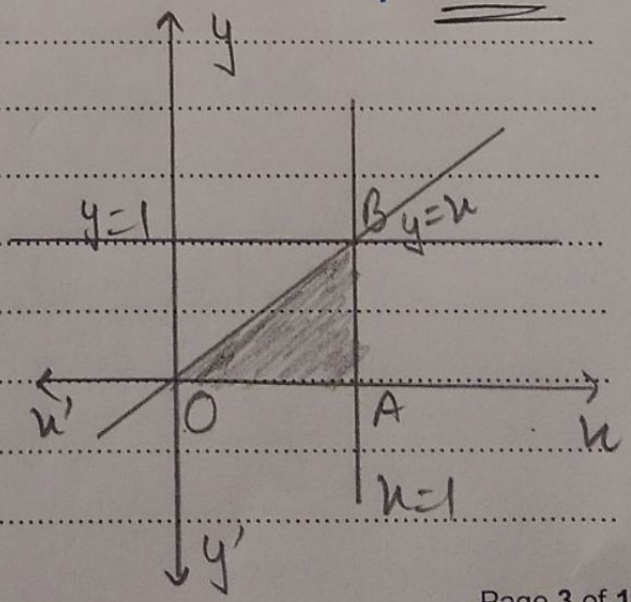
Here $r < 0$

Here (2,2) is maximum point.

A6 $\int_0^1 \int_y^1 x^2 e^{xy} dx dy$

x: y to 1

y: 0 to 1



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Region to be integrated in ΔOAB

For reversing order, limits will be changed
Here,

$y : 0 \text{ to } x$

$x : 0 \text{ to } 1$

$$I = \int_0^1 \int_0^x x^2 e^{xy} \, dx \, dy$$

$$= \int_0^1 \left[x^2 \int_0^x e^{xy} \, dy \right]_0^x dx$$

$$= \int_0^1 \left[x^2 \left[\frac{e^{xy}}{x} \right]_0^x \right]_0^x dx$$

$$= \int_0^1 \left[\frac{x^2}{x} \left[e^{x^2} - e^0 \right] \right]_0^x dx$$

$$= \int_0^1 \left[x e^{x^2} - x \right]_0^x dx$$

$$= \int_0^1 x e^{x^2} dx - \left[\frac{x^2}{2} \right]_0^1$$

$$I = \int_0^1 x e^{x^2} dx - \frac{1}{2} \quad \text{---} \quad (1)$$

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Let $x^2 = t$ $\quad x \to 0 \quad t \to 0$
$2x\,dx = dt$ $\quad x \to 1 \quad t \to 1$
$x\,dx = \frac{dt}{2}$ Putting values in (1)

$$I = \int_0^1 e^t \frac{dt}{2} - \frac{1}{2}$$

$$= \frac{1}{2}\left[e^t\right]_0^1 - \frac{1}{2}$$

$$= \frac{1}{2}\left(e^1 - e^0\right) - \frac{1}{2}$$

$$= \frac{1}{2}e - \frac{1}{2} - \frac{1}{2}$$

$$= \frac{e}{2} - 1$$