CSP(Constraint Satisfaction Problem)

- **CSP** is a problem that requires its solution within some limitations or conditions also known as constraints.
- Constraint satisfaction means *solving a problem under certain constraints or rules.*
- It consists of the following:

V: A finite set of **variables** which stores the solution

(V = {V1, V2, V3,....., Vn})

D: It is a set of domains where the variables reside.

Each variable V has a nonempty domain (D = {D1, D2, D3,.....,Dn}) of possible values.

C: A finite set of **constraints** which are followed by the set of variables. (C = {C1, C2, C3,....., Cn})

CSP Continue..

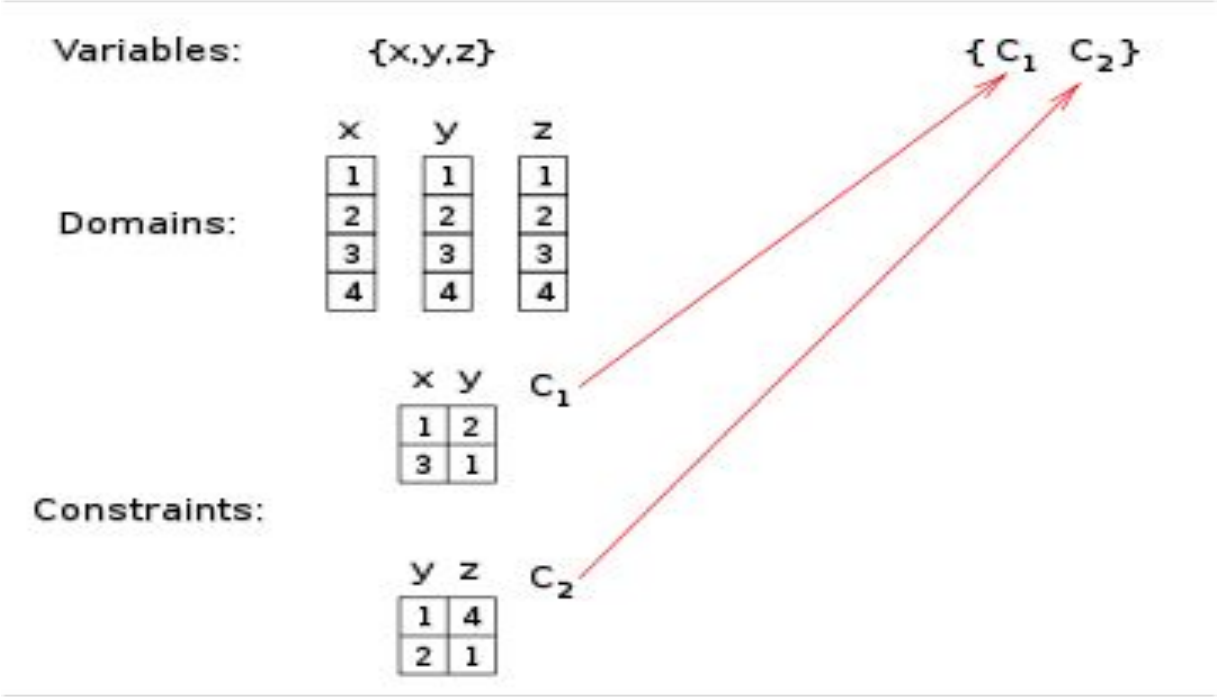
- The constraint value consists of a pair of **{scope, rel}**.
- The **scope:-** is a set of variables which participate in the constraint
- **rel:-** Relation which includes a list of values which the variables can take to satisfy the constraints of the problem

CSP Continue..

- Simple example:
- V = {V1} –
- Dom(V1) = {1,2,3,4}
- C = {C1,C2} – C1: V1 ≠ 2

  C2: V1 > 1
- All models for this CSP: {V1 = 3} ,{V1 = 4}

CSP Continue..



Basics CSP Problems

- College or University Time Table with limited class room and teachers.
- Management of workers to achieve certain task.
- Course Planning and scheduling.
- Bus Route Planning.
- Resource Allocation

Popular Problems with CSP

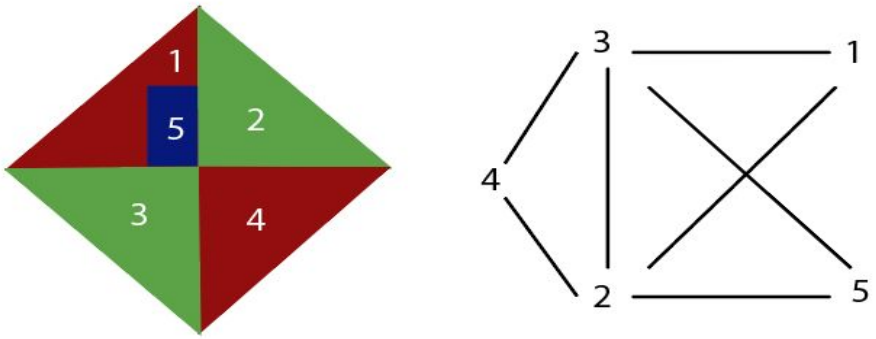
- **Map Coloring** (coloring different regions of map, ensuring no adjacent regions have the same color)
- **Sudoku** (a number grid)
- **N-Queen** (In an n-queen problem, n queens should be placed in an nXn matrix such that no queen shares the same row, column or diagonal.)
- **Crypt Arithmetic** (Coding alphabets to numbers.)

CSP-Converting Process

- A problem to be converted to CSP requires the following steps:
- **Step 1:** Create a variable set.
- **Step 2:** Create a domain set.
- **Step 3:** Create a constraint set with variables and domains (if possible) after considering the constraints.
- **Step 4:** Find an optimal solution.

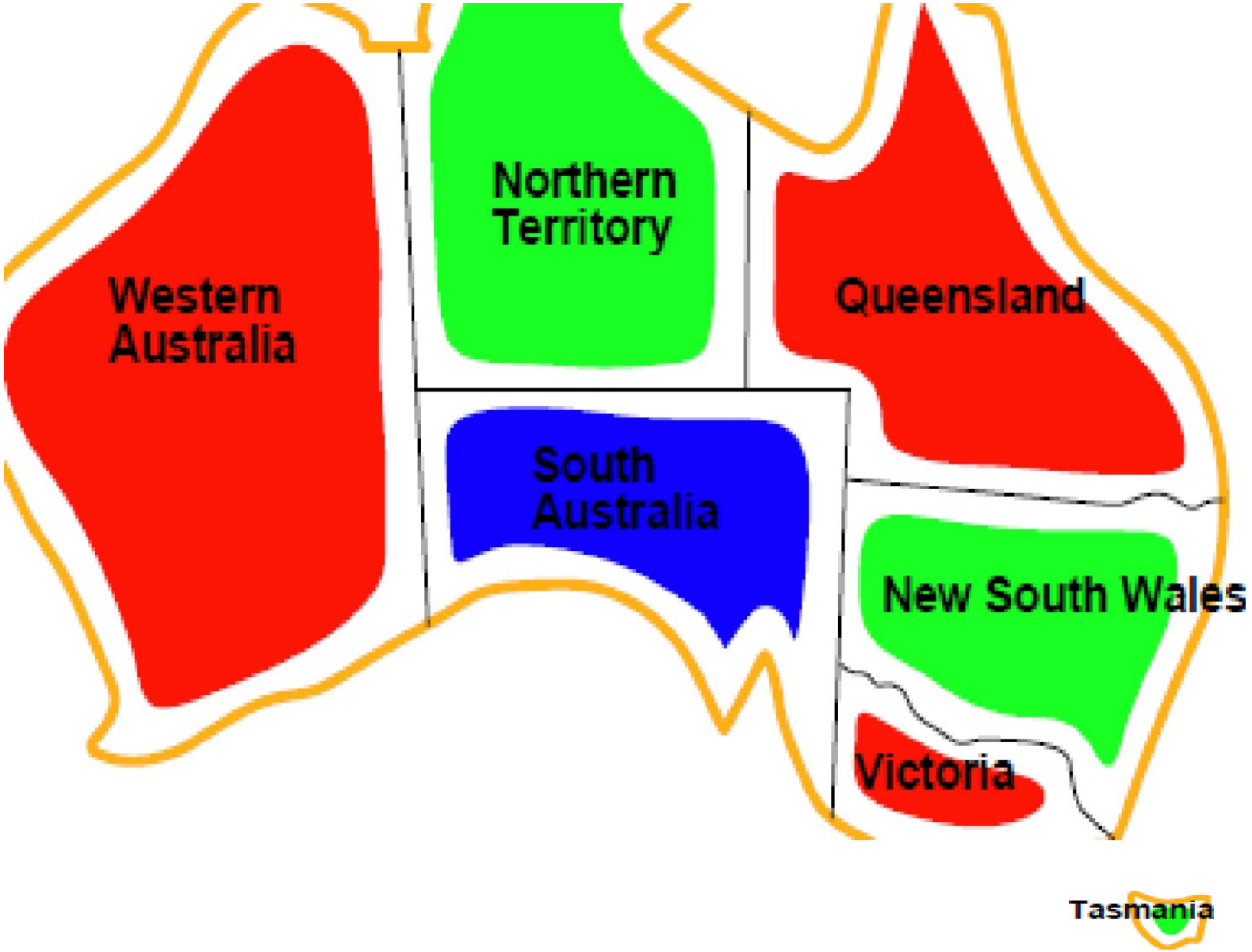
CSP Example

- **1.Graph Coloring:** The problem where the constraint is that no adjacent sides can have the same color.



Graph Coloring

Graph Coloring



Graph Coloring

- Variables *WA, NT', SA, Q, NSW, V, T*
- Domains *Dj —— (red, green, blue)*
- Constraints: adjacent regions must have different colors
- Solutions :- satisfying all constraints,

e.g., *WA— red, NT — gREEN,Q — red,*
N SW'— green,V — red, SA — blue,
T — green

CSP Example

- **Sudoku Playing:** The game play where the constraint is that no number from 0-9 can be repeated in the same row or column.

SUDOKU

4							5	9
2	6		5				3	
				9	2			
		2		6			1	
		3	8	1	9	7		
	7			3		5		
			3	4				
	3				6		2	7
5	9							6

Puzzle

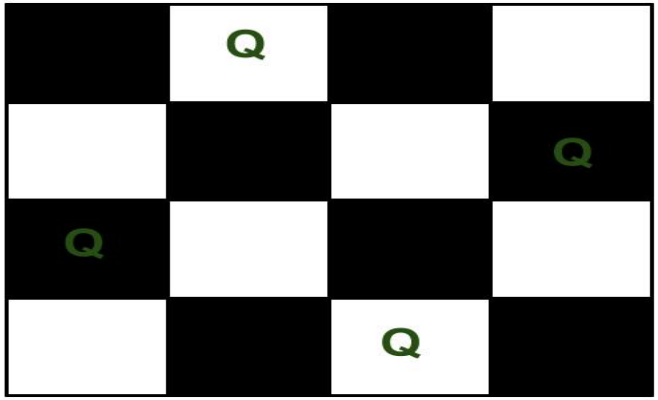
4	1	7	6	8	3	2	5	9
2	6	9	5	7	1	8	3	4
3	8	5	4	9	2	6	7	1
8	4	2	7	6	5	9	1	3
6	5	3	8	1	9	7	4	2
9	7	1	2	3	4	5	6	8
7	2	6	3	4	8	1	9	5
1	3	8	9	5	6	4	2	7
5	9	4	1	2	7	3	8	6

Solution

CSP Example

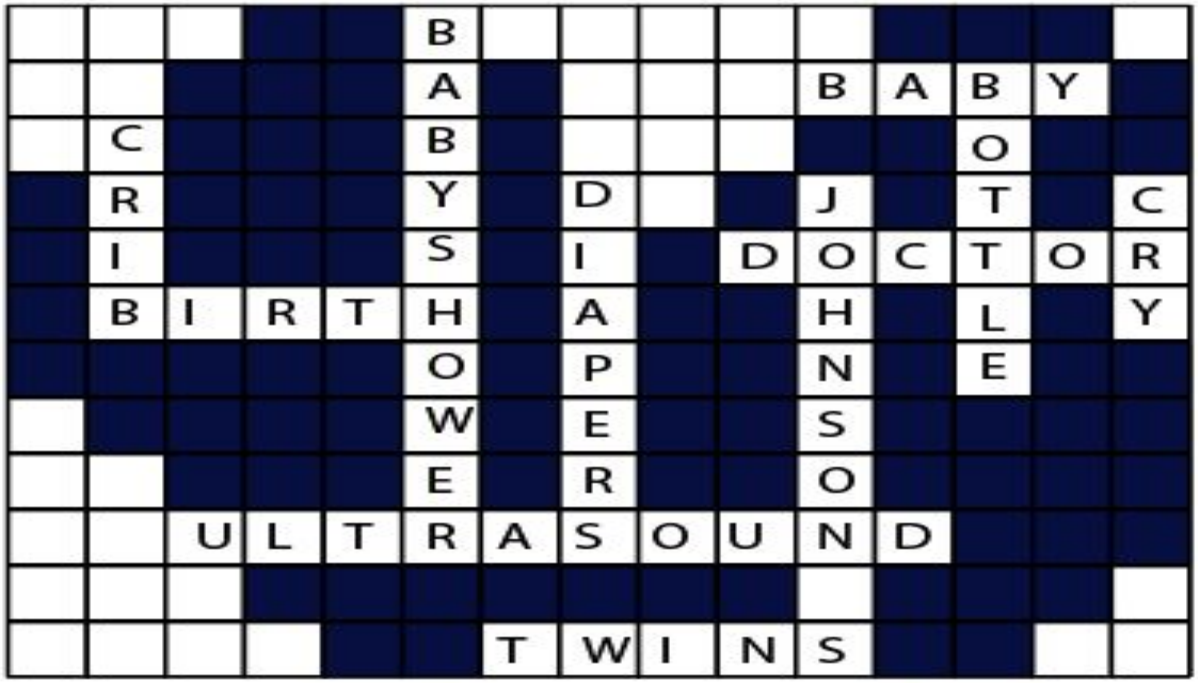
N-queen problem: In n-queen problem, the constraint is that no queen should be placed either diagonally, in the same row or column.

- For example, following is a solution for 4 Queen problem.



CSP Example

Crossword: In crossword problem, the constraint is that there should be the correct formation of the words, and it should be meaningful.



CRYPT-ARITHMETIC

- Crypt arithmetic Problem is a type of constraint satisfaction problem(CSP) where the game is about digits and its unique replacement either with alphabets or other symbols.
- The task in crypt arithmetic problem is to substitute each digit with an alphabet to get the result arithmetically correct.
- There are two words are given and another word is given an answer of addition for those two words.

The rules or constraints on a crypt arithmetic problem

- There should be a unique digit to be replaced with a unique alphabet or No two letter have same value
- The result should satisfy the predefined arithmetic rules, i.e., 2+2 =4, nothing else.
- The sum of Digits must be as shown in Result.
- Digits should be from **0-9** only.
- There should be only one carry forward, while performing the addition operation on a problem.
- The problem can be solved from both sides, i.e., **left-hand side (L.H.S), or right-hand side (R.H.S)**

CRYPT-ARITHMETIC
EXAMPLE:1

Example 1: T O 2 1

+ G O 8 1

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O U T 1 0 2

Letter	O	T	G	U
Values	1	2	8	0

2+G=U

2+9=11 WRONG BECAUSE 1 is already assign to O

2+8=10 SO G=8

CRYPT-ARITHMETIC EXAMPLE:2

- A B C D
- +E B C B
- ----------
- A F G A G

Letter	A	B	C	D	E	F	G
Values	1	6	5	7	8	0	3

```
 1 6 5 7
 9 6 5 6
-------------
1 0 3 1 3
```

D+B=G
6+5=11 CARRY=1 **WRONG** Because 1 already assign to A
7+6=13 carry 1 Right

CRYPT-ARITHMETIC EXAMPLE

- B A S E
- B A L L
- ---------------
- G A M E S

CRYPT-ARITHMETIC

```
    B  A  S  E
  + B  A  L  L
G  A  M  E  S
```

If G is allow Non zero

```
  7483
 +7455
 -------
 14938
```

Letter	A	B	E	G	L	M	S
Value	4	7	3	1	5	9	8

If G is allow zero

```
  2  4  6  1
  2  4  5  5
---------------
0 4  9  1  6
```

Letter	A	B	E	G	L	M	S
Values	4	2	1	0	5	9	6

CRYPT-ARITHMETIC EXAMPLE

```
    S E N D
+   M O R E
-------- ------
M O N E Y
```

S	9
E	5
N	6
D	7
M	1
O	0
R	8
y	2

CRYPT-ARITHMETIC EXAMPLE

2.

```
  YOUR
 +YOU
------
 HEART
------
```


Y	9
O	4
U	2
R	6
H	1
E	0
A	3
T	8