# -:- Numerical Methods -:-

* The limitations of analytical methods in practical applications have led scientists and engineers to evolve numerical methods.
* The input information is rarely exact since it comes from some measurement or the other hand the method also introduces some errors. As such the error in the final result may be due to an error in the initial data or in the method used or both. Our effort will be minimize these errors, so as to get the best possible results.
* We therefore begin by explaining various kind of approximations and errors which may occur in a problem.

**Approximate numbers** ⇒ There are two types of numbers

(i) exact

(ii) Approximate

* **exact numbers** ⇒ 2, 4, 9, 13, 7/2, 6.45 etc are exact numbers.

* **Approximate numbers** ⇒ There are numbers such as $\frac{4}{3} = (1.3333....)$, $\sqrt{2} = (1.414213.....)$ and $\pi = (3.141592.....)$ which can not be expressed by a finite number of digits. These numbers may be approximated by numbers 1.3333, 1.4142, 3.1416 respectively.

such numbers which represent the given numbers (2)
to a certain degree of accuracy are called approximate numbers.

Significant digits (figures) ⇒ The digits used to express a number are called significant digits (figures).

* Thus each of the numbers 7845, 3.589 and 0.4758 contains four significant digits.

* While the numbers 0.00386, 0.000587 and 0.0000296 contains only three significant figures since zeros only help to fix the positions of the decimal point.

* Similarly the numbers 45000 & 7300.00 have two significant figures only.

Rounding off ⇒ There are numbers with large number of digits, e.g. $\frac{22}{7} = 3.142857143$. In pratice, it is desirable to limit such numbers to a manageable number of digits such as 3.14 or 3.143. This process of dropping unwanted digits is called rounding off.

Rule to round off a number to n significant figures

*(i) Discard all digits to the right of the nth digit.

(ii) If this discarded number is

(a) less than half of a unit in the nth place, leave the nth digit unchanged.

(b) greater than half a unit in the nth place, increase the nth digit by unity.

(c) exactly half a unit in the $n^{th}$ place, increase the $n^{th}$ digit by unity if it is odd otherwise leave it unchanged.

For instance, the following numbers rounded off to three significant figures are

(i) 7.893 to 7.89

(ii) 12.865 to 12.9

(iii) 6.4356 to 6.44

(iv) 3.567 to 3.57 (0.003 < 0.005)

(v) 84767 to 84800

(vi) 5.8254 to 5.82

* Also numbers 6.284359, 9.864651 and 12.464762 are rounded off to four places of decimal as 6.2844, 9.8646, and 12.4648 respectively.

* Numbers thus rounded off to n significant figures (or n decimal places) are said to be correct to n significant figures (or n decimal places).

## Errors :-

In any numerical computation, we come across the following type of errors:

(1) Inherent errors ⇒ Errors which are already present in the statement of a problem before its solution, are called inherent errors.

* Such errors arise either due to the given data being approximate or due to the limitations of mathematical tables, calculators, or the digital computers.

* Inherent errors can be minimized by taking better data or by using high precision computing aids. (4)

(2) Rounding errors ⇒ This error arise from the process of rounding off the numbers during the computation. Such errors are unavoidable in most of the calculations due to the limitations of the computing aids.

* Rounding errors can, however, be reduced:

(i) By changing the calculation procedure so as to avoid subtraction of nearly equal numbers or division by a small number.

(ii) By retaining atleast one more significant figure at each step that given in the data and rounding off at the last step.

(3) Trancation errors ⇒ These errors are caused by using approximate results or on replacing an infinite process by a finite one.

* If we are using a decimal computer having a fixed word length of four digits, rounding off 13.658 gives 13.66 whereas trancation gives 13.65.

* For example, if

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \infty = X \text{ (say)}$$

is replaced by $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} = X'$ (say), then the truncation error is $X - X'$. 

* Truncation error is a type of algorithm error.

(4) <u>Absolute, Relative and Percentage errors</u> ⇒

If $X$ is the true value of a quantity and $X'$ is its approximate value, then $|X - X'|$ i.e $|\text{error}|$ is called the absolute error ($E_a$).

* The <u>relative</u> <u>error</u> is defined by

$$E_r = \left|\frac{X - X'}{X}\right| \quad \text{i.e} \quad \left|\frac{\text{Error}}{\text{True value}}\right|.$$

* The percentage error is $E_p = 100\, E_r$

$$= 100 \left|\frac{X - X'}{X}\right|.$$

* If $\bar{X}$ be such a number that $|X - X'| \leq \bar{X}$ then $\bar{X}$ is an upper limit on the magnitude of the absolute error and measures the absolute accuracy.

<u>Observation (1)</u> The relative and percentage errors are independent of the units used while absolute error is expressed in terms of these units.

<u>Observation (2)</u> If a number is correct to $n$ decimal places then the error $= \frac{1}{2} 10^{-n}$. For example, if the number is $3.1416$ correct to 4 decimal places, then error $= \frac{1}{2} \times 10^{-4} = 0.00005$