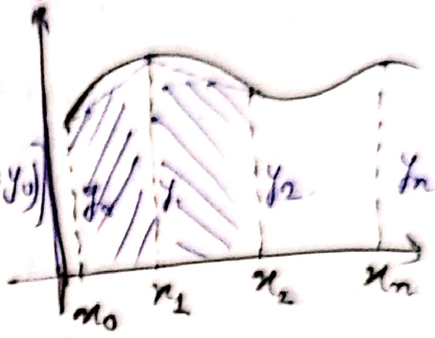
Also, from ...

Simpson's one-Third rule → putting $n=2$ in eq^n (ii)

Taking curve through (x_0,y_0), (x_1,y_1), (x_2,y_2) as a polynomial of the second order so that differences of order higher than the second vanish, we get

$$\int_{x_0}^{x_0+2h} y(x)\,dx = 2h\left[y_0 + \Delta y_0 + \frac{\Delta^2 y_0}{6}\right]$$



$$= 2h\left[y_0 + y_1 - y_0 + \frac{1}{6}(y_2 - y_1 - y_1 + y_0)\right]$$

$$= 2h\left[y_1 + \frac{y_2}{6} - \frac{y_1}{3} + \frac{y_0}{6}\right]$$

$$= 2h\left[\frac{2y_1}{3} + \frac{y_2}{6} + \frac{y_0}{6}\right]$$

$$= \frac{h}{3}\left[y_2 + 4y_1 + y_0\right]$$

$$\int_{x_0}^{x_0+2h} y(x)\,dx = \frac{h}{3}\left[y_0 + 4y_1 + y_2\right]$$

similarly

$$\int_{x_0+2h}^{x_0+4h} y(x)\,dx = \frac{h}{3}\left[y_2 + 4y_3 + y_4\right]$$

$$\vdots$$

$$\int_{x_0+(n-2)h}^{x_0+nh} y(x)\,dx = \frac{h}{3}\left(y_{n-2} + 4y_{n-1} + y_n\right)$$

Adding all these integrals,

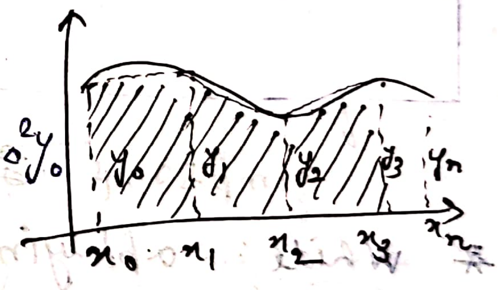
$$\boxed{\int_{x_0}^{x_0+nh} y(x)\,dx = \frac{h}{3}\left[y_0 + 4(y_1+y_3+\cdots+y_{n-1}) + 2(y_2+y_4+\cdots+y_{n-2}) + y_n\right]} \quad \text{(iv)}$$

$$\int_a^b y(x)\,dx = \int_{x_0}^{x_n} y(x)\,dx = \frac{h}{3}\left[y_0 + 4(y_1+y_3+y_5+\dots+y_{n-1}) + 2(y_2+y_4+\dots+y_{n-2}) + y_n\right] \quad \text{(iv)}$$

This is known as <u>Simpson's one-third rule</u> or <u>simply Simpson's rule</u>.

* While applying Simpson's one-third rule, the given interval must be divided into an even number of equal subintervals, since we find the area of two strips at a time.

<u>Simpson's three-eight rule</u> ⇒ Putting $n=3$, in Eq^n (ii) above and taking the curve through (x_i, y_i), $i = 0, 1, 2, 3$ as a polynomial of the third order so that above the third order differences vanish, we get



$$\int_{x_0}^{x_0+3h} y(x)\,dx = 3h\left[y_0 + \frac{3}{2}\Delta y_0 + \frac{3}{4}\Delta^2 y_0 + \frac{1}{8}\Delta^3 y_0\right]$$

$$\cancel{= \frac{3h}{8}\left[8y_0 + \frac{3}{4}\right.}$$

$$= \frac{3h}{8}\left[8y_0 + 12\Delta y_0 + 6\Delta^2 y_0 + \Delta^3 y_0\right]$$

$$= \frac{3h}{8}\left[8y_0 + 12(y_1-y_0) + 6(y_2-y_1-y_1+y_0) + y_3-y_2-2(y_2-y_1)+y_1-y_0\right]$$

$$= \frac{3h}{8}\left(y_0 + 3y_1 + 3y_2 + y_3\right)$$

Similarly

$$\int_{x_0+3h}^{x_0+6h} y(x)\,dx = \frac{3h}{8}\left[y_3 + 3y_4 + 3y_5 + y_6\right]$$

Similarly

$$\int_{x_0+(n-3)h}^{x_0+nh} y(x)\,dx = \frac{3h}{8}\left[y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n\right]$$

Adding above integrals, we get

$$\int_{x_0}^{x_3} y(x)\,dx + \int_{x_3}^{x_6} y(x)\,dx + \cdots + \int_{x_{n-3}}^{x_n} y(x)\,dx$$

$$= \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-2} + y_{n-1}) + 2(y_3 + y_6 + \cdots + y_{n-3}) + y_n\right]$$

$$\Rightarrow \boxed{\int_{x_0}^{x_0+nh} y(x)\,dx = \int_{x_0}^{x_n} y(x)\,dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-2} + y_{n-1}) + 2(y_3 + y_6 + \cdots + y_{n-3}) + y_n\right]}$$

Simpson's $\frac{3}{8}$ Three-eight rule

* While applying Simpson's Three-eight rule, the number of subintervals should be taken as a multiple of 3.