Similarly

$$\int_{x_0+(n-3)h}^{x_0+nh} y(x)\,dx = \frac{3h}{8}\left[y_{n-3}+3y_{n-2}+3y_{n-1}+y_n\right]$$

Adding above integrals, we get

$$\int_{x_0}^{x_3} y(x)\,dx + \int_{x_3}^{x_6} y(x)\,dx + \cdots + \int_{x_{n-3}}^{x_n} y(x)\,dx$$

$$= \frac{3h}{8}\left[y_0 + 3(y_1+y_2+y_4+y_5+\cdots+y_{n-2}+y_{n-1}) + 2(y_3+y_6+\cdots+y_{n-3}) + y_n\right]$$

$$\Rightarrow \boxed{\int_{x_0}^{x_0+nh} y(x)\,dx = \int_{x_0}^{x_n} y(x)\,dx = \frac{3h}{8}\left[y_0+3(y_1+y_2+y_4+y_5+\cdots+y_{n-2}+y_{n-1}) + 2(y_3+y_6+\cdots+y_{n-3})+y_n\right]}$$

Simpson's $\frac{3}{8}$ Three-eight rule

* While applying Simpson's Three-eight rule, the number of subintervals should be taken as a multiple of 3.

Ex ⇒ Evaluate the integral $\int_0^1 \frac{x^2}{1+x^3}$ using Simpson's $1/3$ rule. Compare the error with the exact value.

Solution ⇒ Let us divide the interval $(0,1)$ into 4 equal parts so that $h = 0.25$

Taking $y(x) = \frac{x^2}{1+x^3}$, we have

x	0	0.25	0.50	0.75	1.00
$y(x)$	0	0.06153	0.22222	0.39560	0.5
	y_0	y_1	y_2	y_3	y_4

By Simpson's 1/3 rule, we have

$$\int_0^1 \frac{x^2}{1+x^3}\,dx = \frac{h}{3}\left[y_0 + 4(y_1+y_3) + 2y_2 + y_4\right]$$

$$= \frac{0.25}{3}\left[0 + 4(0.06153 + 0.3956) + 2(0.22222) + 0.5\right]$$

$$= 0.23108$$

$$\int_0^1 \frac{x^2}{1+x^3}\,dx = \frac{1}{3}\log(1+x^3)\Big|_0^1 = \frac{1}{3}\log_e 2$$
$$= 0.23105$$

error $= 0.23108 - 0.23105 = 0.00003$

$\boxed{\text{Lecture-24}}$

Boole's Rule ⇒ Putting $n=4$ in Newton's cotes quadrature formula,

when $n=4$, then points will be (x_i, y_i),

$i = 0, 1, 2, 3, 4$,

Then

$$\int_{x_0}^{x_0+4h} y(x)\,dx = 4h\left[y_0 + 2\Delta y_0 + \frac{5}{3}\Delta^2 y_0 + \frac{2}{3}\Delta^3 y_0 + \frac{7}{90}\Delta^4 y_0\right]$$

$$= \frac{2h}{45}\left[7y_0 + 32y_1 + 12y_2 + 32y_3 + 7y_4\right]$$

Similarly

$$\int_{x_0+4h}^{x_0+8h} y(x)\,dx = \frac{2h}{45}\left[7y_4 + 32y_5 + 12y_6 + 32y_7 + 7y_8\right]$$

⋮

$$\int_{x_0+(n-4)h}^{x_0+nh} y(x)\,dx = \frac{2h}{45}\left[7y_{n-4} + 32y_{n-3} + 12y_{n-2} + 32y_{n-1} + 7y_n\right]$$

Adding all these integrals from x_0 to x_0+nh, where n is a multiple of 4, we get

$$\boxed{\int_{x_0}^{x_0+nh} y(x)\,dx = \frac{2h}{45}\left[7y_0+32y_1+12y_2+32y_3+14y_4+32y_5+12y_6+32y_7+14y_8+\cdots\right]}$$

This is known as <u>Boole's Rule</u>

* while applying Boole's rule, the number of sub-intervals should be taken as a multiple of 4.

<u>Example</u> ⇒ Use Boole's five-point formula to compute

$$\int_0^{\pi/2} \sqrt{\sin(x)}\,dx .$$

<u>Solution</u> ⇒ As, in question it is given that we have to take 5-points, so,

take $n=4$, points will be x_0, x_1, x_2, x_3, x_4
$\quad = 0, \pi/8, \pi/4, \frac{3\pi}{8}, \pi/2$

$$h = \frac{\pi/2 - 0}{4} = \pi/8$$

here $y(x) = \sqrt{\sin x}$

$$\Rightarrow \quad y(0) = 0 = y_0$$
$$y(\pi/8) = 0.61861 = y_1$$
$$y(\pi/4) = 0.84090 = y_2$$
$$y(\tfrac{3\pi}{8}) = 0.96119 = y_3$$
$$y(\pi/2) = 1.0 = y_4$$

$$I = \int_0^{\pi/2}\sqrt{\sin(x)}\,dx = \frac{2}{45}\left(\frac{\pi}{8}\right)\left[7y_0+32y_1+12y_2+32y_3+7y_4\right]$$

$$= \frac{\pi}{180}\left[7\times 0 + 32\times 0.61861 + 12\times 0.84090 + 32\times 0.96119 + 7\times 1.0\right]$$

$$= \quad = 1.18062 \quad \text{Ans}$$