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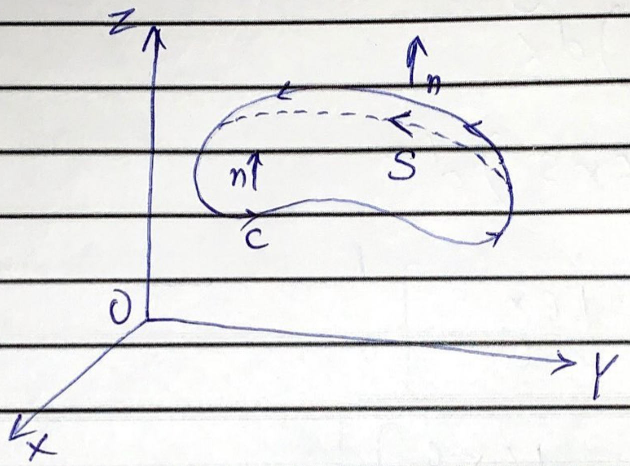
1:* Write the Statement of Stokes theorem.



$$\oint_C \vec{F} \cdot \vec{dr} = \iint_S (\nabla \times \vec{F}) \cdot \vec{dS}$$

Where:
C = closed curve
S = Surface bounded by C

F = A vector feild whose Components have Continious derivatives in open region of R^3 Containing S.

Roll No. 0201ite092 Page No.______ Date:______ Signature______

3.* The ratio of present age of Suresh and Mahesh is 7:5. If after 6 years their age will the ratio of 4:3. Find the present age of Mahesh?

let present age of Suresh is x

of Mahesh is y

$$\frac{x}{y} = \frac{7}{5} \Rightarrow x = \frac{7}{5}y$$

After 6 years $\frac{x+6}{y+6} = \frac{4}{3}$

$$3x + 18 = 4y + 24$$

$$\frac{21y}{5} - 4y = 6$$

$$21y - 20y = 30$$

$$\boxed{y = 30}$$

So, Mahesh is 30y

4.* Obtain half range sine series for $f(x) = e^x, 0<x<1$

$f(x) = e^x$

$$e^x = \sum_{x=1}^{\infty} a_n \sin n \cdot \pi x$$

$$a_1 = \int_0^{2\pi} e^x \sin n\pi (\pi x) \qquad \int e^x \sin x = I$$

$$= \frac{-e^{x} \cdot \cos \pi x}{\pi} + \frac{1}{\pi} \int e^{x} \cos \pi x$$

$$= -\frac{e^{x} \cos \pi x}{\pi} + \frac{1}{\pi} \left(e^{x} \frac{\sin \pi x}{\pi} - \int e^{x} \frac{(\sin \pi x)}{\pi} \right)$$

$$I \left[1 + \frac{1}{\pi^2} \right] = e^{x} \frac{\sin \pi x}{\pi^2} - e^{x} \frac{\cos \pi x}{\pi}$$

gim

$$a_1 = \frac{2\pi (1+e)}{\pi^2 + 1}$$

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Roll No. 0201ite097 Page No.______ Date:______ Signature______

4.* Obtain the half-range sine series for e^x in $0<x<1$

Sol:

$$e^x = \sum_{n=1}^{\infty} b_n \sin n\pi x, \text{ (Since } l=1)$$

$$b_n = 2\int_0^1 e^x \sin n\pi x \, dx = 2\left[\frac{e^x}{1+(n\pi)^2} - n\pi \cos n\pi x\right]_0^1$$

$$2\left[\frac{e}{1+(n\pi)^2}(-n\pi \cos n\pi) - \frac{1}{1+(n\pi)^2} - (n\pi)\right]$$

$$\frac{2}{1+n^2\pi^2}\left[-en\pi(-1)^n + n\pi\right] = \frac{2n\pi}{1+n^2\pi^2}\left[1-e(-e)(-1)^n\right]$$

hence:

$$e^x = 2\pi \sum_{n=1}^{\infty} \frac{n[1-e(-1)^n]}{1+n^2\pi^2} \sin n\pi x$$

5.* Find laplace transform of the function $f(t) = te^{-4t}\sin 3t$.

$$L\{t^k(f(t))\} = (-1)^k (d^k) \frac{L\{f(t)\}}{d} \cdot s^k$$

$$L\{\sin(at)\} = \frac{a}{(s^2+a^2)}$$

$$L\{e^{-bt}\sin at\} = \frac{a}{(s+b)+a^2}$$

Step-2

given function $f(t) = te^{-4t}(\sin 3t)$

Calculations:

$f(t) = te^{-4t}\sin(3t)$

$\therefore$

$L(f(t)) = L\{te^{-4t}\sin(3t)\}$

$(-1\ 1)\dfrac{d\{e^{-4t}(3t)\}}{ds}]$

$-\dfrac{d}{ds}\left[\dfrac{3}{(s+4)^2+9}\right]$

$-\dfrac{d}{ds}\left[\dfrac{3}{s^2+8s+25}\right]$

$-\dfrac{d}{ds}\left[\dfrac{3}{s^2+8s+16+9}\right]$

$-\dfrac{d}{ds}\left[\dfrac{}{}\right]$

$-3\dfrac{d(s^2+8s+25)^{-1}}{ds}$

$-3(-1)(s^2+8s+25)^{-2}\dfrac{d(s^2+8s+25)}{ds}$

By chain rule:-

$3(s^2+8s+25)^{-2}(2s+8)$

$L\{f(t)\} = \dfrac{6(s+4)}{(s^2+8s+25)^2}$

$f(t) = te^{-4t}\sin 3t$

$L\{f(t)\} = \dfrac{6(s+4)}{(s^2+8s+25)^2}$

6.* Find the lap lace transformation of

a) $t^2 \sin at$

$$\frac{a}{p^2+a^2}$$

$$L\{t^2 \sin at\} = (-1)^2 \frac{d^2}{dp^2} \cdot \frac{a}{p^2+a^2}$$

$$\Rightarrow \frac{(p^2+a^2)\cdot 0 - 2ap}{(p^2+a^2)^2}$$

$$\Rightarrow -\frac{2ap}{(p^2+a^2)^2}$$

$$\Rightarrow -\left[\frac{(p^2+a^2)^2 \cdot 2a - 2ap\left[2a(p^2+a^2)2p^2\right]}{(p^2+a^2)^4}\right]$$

$$\Rightarrow \frac{8ap^2(p^2+a^2) - 2a(p^2+a^2)^2}{(p^2+a^2)^4}$$

$$\Rightarrow \frac{6ap^2 - 2a^3}{(p^2+a^2)^3} =$$

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Roll No. 0201ite093 Page No.______ Date:______ Signature______

7.* Find the fourier series of $f(x) = x^3$ in $(-\pi, \pi)$

Soln:- $f(x) = x^3$ find $f(x)$ is even or odd

$f(x) = -(x)^3 = (-x)^3 \Rightarrow f(x) = -f(x)$

$f(x)$ is odd $\Rightarrow$ ($a_0 = a_n = 0$) b_n

forier series

$$f(x) \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2}$$

$$b_n = \frac{1}{L}\int_a^b (f(x)) \sin\left(\frac{n\pi x}{2}\right) dx \quad L = \frac{b-a}{2} = \frac{\pi}{2}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} x^3 . \sin nx . dx = \frac{2}{\pi}\int_0^{\pi} x^3 \sin nx . dx$$

Use Bernoullies rule

$$\int x . dx = u\int v . dx - u' \iint v\, dx\, dx + u'' \iint v\, dx\, dx$$

$$\frac{2}{\pi}\left[x^3 \frac{(-\cos nx)}{n} - 3x^2 \left(\frac{-\sin nx}{n^2}\right) + 6x . \frac{\cos nx}{n^3} - 6 . \frac{\sin nx}{n^4} \right]_0^{\pi}$$

$$\frac{2}{\pi}\left[\frac{-x^3 \cos nx}{n} + \frac{6x \cos nx}{n^3} \right]_0^{\pi}$$

Roll No. 0201 ite093 Page No.______ Date:______ Signature______

Note:- The line integral around S (the boundary curve) of F's tangential component is equal to the surface integral of the component of the curl of F.

Stokes theorem expressed as:

$$\iint_S \text{curl}\,\vec{F}\cdot d\vec{s} = \int_{\partial s} \vec{F}\cdot d\vec{r}$$

2.* Sanjeev walks 10m towards:-

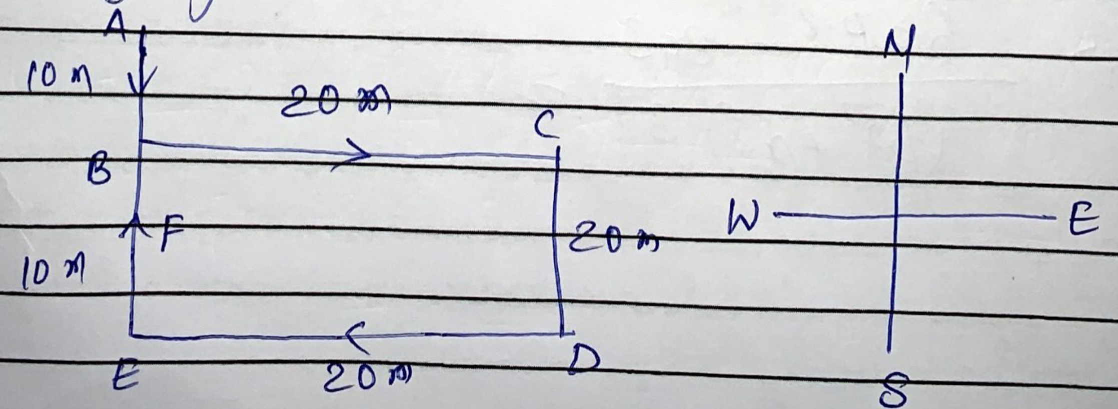
The movement of Sanjeev from A to F are as shown in fig:-

Sanjeev distance from starting point A

$AF = (AB + BF) = AB = (BE - EF) = AB + (CD - EF)$

$= [10 + (20 - 10)]\,m = (10 + 10) = 20m$

Also, F lies to the south of A. So, Sanjeev is 20m to the south of his starting point.



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Page No.

Date

9.* $F(t) = \begin{cases} \sin t & 0 < t < \pi \\ \sin 2t & \pi < t \end{cases}$

$F(t) = \sin t\, u(t-\pi) + \cos t\{u(t) - u(t-\pi)\}$

$= \cos t\, v(t) - (\sin t - \cos t)\, u(t-\pi)$

Sol^n: To find CF of $x^2y^2 + xy' + y = \log x^2$

$x^2 \frac{d^2x}{dx^2} + x\frac{dy}{dx} + y = \log x^2$

it is an homogeneous form

$[D(D-1) + D + 1]y = \log(e^t)^2$

taking LHS

CF

$[D^2 - D + D + 1]y = RHS$

$[m^2 + 1]y = 0$

$CF = e^{0t}[c_1 \cos t + c_2 \sin t]$

$m^2 = 1$

$m = \pm 1$

put $z = \log x$

$CF = c_1 \cos(\log x) + c_2 \log \sin(\log x)$

$$\frac{2}{\pi}\left[-\frac{\pi^3 \overset{(-1)^n}{\cos n\pi}}{n} + 6\pi \frac{\overset{(-1)^n}{\cos n\pi}}{n^3} - 0\right]$$

$$\frac{2}{\cancel{\pi}}\cancel{\pi}\left[\frac{-\pi^2(-1)^n}{n} + \frac{6(-1)^n}{n^3}\right]$$

$$2(-1)^n\left[-\frac{\pi^2}{n} + \frac{6}{n^3}\right]$$

Substitute b_n is fourier Series

$$f(x) = x^3 = 2\sum_{n=1}^{\infty} (-1)^n\left[\frac{-\pi^2}{n} + \frac{6}{n^2}\right]$$

3.*. ~~Find the particular differential~~ equation

~~$(D^2+4D+3)y = \sin(2x+3)$~~

10.* Introduce a woman Shashank "She is the mother of the only of my son. How that women is related to Shashank?

Woman is daughter in-law of Shashank.

* 9.

$$\vec{F} = 3xy\,\hat{i} - y^2\,\hat{j}$$

$$= 3x(2x^2)\hat{i} - (2x^2)^2 \cdot \hat{j}$$

$$6x^3\hat{i} - 4x^4\hat{j}$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$x\hat{i} + 2x^2\hat{j}$$

$$d\vec{r} = dx\,i + 4x\,dx\,\hat{j}$$

$$(\hat{i} + 4x\hat{j})\,dx$$

$$\vec{F}\cdot d\vec{r} = (6x^3 - 16x^5)\,dx$$

$$\Rightarrow \left[\frac{6x^4}{4} - \frac{16x^6}{6}\right]_0^1$$

$$\Rightarrow \left[\frac{6x^4}{4} - \frac{16x^6}{6}\right]_0^1$$

$$\Rightarrow \frac{6}{4} - \frac{16}{6}$$

$$\Rightarrow -\frac{7}{6}$$

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