

Asymptotic Notation is a mathematic notation that is used to describe behaviour of a algorithm on basis of no. of inputs for eg if a ~~func~~ algorithm is represented as  $f(n)$  where  $n$  is a no. of input asymptotic notation  $O(n)$  for  $f(n)$  tells that upper bound for  $f(n)$  is  $n$  that is  $f(n) \leq c_1 n$  for  $n > n_0, c_1 > 0$

**Big O** - Big O notation gives asymptotic upper bound of an algorithm for eg if a algorithm is represented as  $f(n)$  where  $n$  is no. of input  $O(g(n))$  for  $f(n)$  tells us  $f(n) \leq c_1 g(n)$  for  $n > n_0, c_1 > 0$

**Omega** - Omega notation gives asymptotic lower bound of an algorithm for eg if a algorithm is represented as  $f(n)$  where  $n$  is no. of input  $\Omega(g(n))$  for  $f(n)$  tells us  $c_1 g(n) \leq f(n)$  for  $n > n_0, c_1 > 0$

**Theta** - Theta notation gives asymptotic upper as well as lower bound of an algorithm for eg if a algorithm is represented as  $f(n)$  (where  $n$  is no. of inputs)  $\Theta(g(n))$  for  $f(n)$  tells us  $c_1 g(n) \leq f(n) \leq c_2 g(n)$  for  $n > n_0, c_1 > 0, c_2 > 0$



Ans 2.

for ( $i=1, i \leq n, i++$ )  $\rightarrow (n+1)$  times

{  $i = i * 2$ ; }  $\rightarrow (n)$  times

3

$$n + n + 1$$
$$= 2n + 1 \text{ (times)}$$

time complexity =  $O(n)$

Any 3.  $TC(n) = 3T(n-1)$

$$T(n) = a \cdot T(n-2)$$
$$T(n) = 3^m T(n-m)$$
$$T(r) = 3^n T(0)$$
$$\tau(0) = 1$$
$$T(n) = 3^n \quad O(3^n)$$

Ans 4.  $T(n) = 2T(n-1) - 1$

$$T(n-1) = 2T(n-2) - 1$$
$$T(n) = 4T(n-2) - 1 - 2$$
$$2\pi(m) + (m-2) = 2\pi(m-3) - 1$$
$$T(n) = 8T(n-3) - 1 - 2 - 4$$
$$T(n) \leq 2^n T(n-n) = 1+2+4+8+\dots+2^{n-1}$$

ac-tu g p serer

$$a_n = a r^{n-1}$$
$$2^{n-1} = \phi(n^{k-1})$$
$$2^{n-1} = 2^{k-1}$$
$$K = n$$

Sun of sons

$$\frac{a(n^n - 1)}{n - 1}$$
$$\frac{1(2^n - 1)}{2 - 1}$$
$$= 2^m - 1$$



$$T(n) = 2^n - 2^n + 1$$

$$T(n) = 1$$

Ans 5.

~~i = 1~~  
~~1~~  
~~1~~  
~~1~~  
~~1~~

Series

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \dots k$$

$$\frac{k(k+1)}{2} = m$$

$$k(k+1) = 2m$$

$$k^2 + k - 2m = 0$$

$$k = \frac{-1 + \sqrt{1 + 4m}}{2}$$

while (s <= m) {

    i++      k--

    s = s + i;      k--

    printf("%d %d", i, k);

}

(matrix is balanced)

⊗

$$4k-1$$

$$\frac{2 \sqrt{1 + 4m}}{2} \approx 3$$

$$\frac{2 \sqrt{1 + 4m}}{2} \approx 3$$

$$(1+m) \times (1+m)$$

$$(1+m) \times (1+m)$$

$$O(m)$$

Ans 6.

$$k^2 \approx m$$

$$k = \sqrt{m}$$

$$(i-m)T + (1+m) + s + m + \dots = (m)T$$

for (i=1; i <= m; i++)  $\sqrt{m+1}$

$$(i-m)T + (1+m) + s + m + \dots = (m)T$$

{

    count++

$$\sqrt{m}$$

}

$$O(\sqrt{m})$$

$$O(\sqrt{m})$$

$$2\sqrt{m+1}$$



7.

④

1 2 4 8 16 ...  $2^k$

$$2^k = n$$

$$k = \log_2 n$$

$$\text{total } \log_2 n + 1$$

for ( $i = n/2; i \leq n; i++$ )  $n/2$

for ( $j = 1; j \leq n; j = j * 2$ )  $(\log_2 n + 2) \times n/2$

for ( $k = 1; k \leq n; k = k * 2$ )  $(\log_2 n + 2)^2 \times n/2$

$$\text{count} += (\log_2 n + 1)^2 \times \frac{n}{2}$$

$$n \times (\log_2 n + 2)^2$$

$$O(n(\log_2 n)^2)$$

8.

~~$$T(n) = n^2 + n + 2$$~~

function (int n)

{ if (n == 1) return;

for ( $i = 1$  to  $n$ ) {  $(n+1)$

for ( $j = 1$  to  $n$ ) {  $(n+1) \times (n+1)$

print(" \* ");  $(n+1) \times (n+1)$

}

}

function (n-3)

$$T(n) = 2n^2 + 4n + 2 + (n+1) + T(n-3)$$

$$T(n) = 2n^2 + 5n + 3 + T(n-3)$$

$$\frac{n-2}{3} = k$$

$$n = 1 + (k-1) \cdot 3$$

$$n = 3k + 2$$

$$1 \ 3 \ 6 \ 9 \ 12 = n$$

$$= 2n^2 + 2(n-3)^2 + 2(n-6)^2 + \dots + 2(n-3k)^2 + 5(n) + 5(n-3) + 5(n-6) + \dots + 5(n-3k) + T(n-3k)$$

Ans 8

~~$T(2) = 2 + 9 + 16 + 3 = 28$~~

$$= 2n^2 + 2(n-3)^2 + 2(n-6)^2 + 2(n-9)^2 \dots 2(n-3n) \\ + S(n) + S(n-3) + S(n-6) \dots S(\frac{n-3}{2}) \\ + 3 \frac{(n-2)}{2} \\ \circ \left( \frac{3^{2m}}{3^n} \right)$$

Ans 9.

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for (i = 1 to n)
    for (j = 1; j <= n; j = j + 1)
    {
        bubble (*)
    }
}

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j)  $1 + k_1 \tau^{2k+1} + 3k+3$  — — —  $n$   
 $t$  hours  
 ~~$t$  hours~~

$$m = 1 + (t-1)k$$

$$m = k + -k + 1$$

$$\frac{n+k-1}{k} = t$$

now  $K \subseteq I$

~~$n \times (n-1)$~~  + upper limit  $n = \frac{n+n-1}{2}$   
lower limit  $= \frac{2}{2+1-1}$

$$n + n+1 + n+2 + n+3 + \dots + 2n$$

$$O(n^2) = \frac{3n^2}{2}$$



10.  $m^k \geq c^m$

Omega notation is relation

$$m \gg 1$$

$$c < m$$

1 2 4 8 16 ...  $2^k$   
 $k = \log_2 m$   
 Total  $1 + 2 + 4 + \dots + 2^k$



$$10. \quad m^k \geq c^m$$

Omega is notation is relation

$$m > 1$$

$$c < m$$