Formulation for vehicle routing problem with simultaneous delivery and pickups

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Parameters:

N : Total number of customers
DC/CC : Node 0 representing the distribution center/central depot
V : Total number of available vehicles
$c_{i,j}$: Travel distance (routing cost) between node i and j
$t_{i,j}$: Travelling time between node i and j
a_0 : Earliest start time of any vehicle from the distribution center
a_i : Earliest start time of service at customer i
b_0 : Latest arrival time of any vehicle to the central depot
b_i : Latest start time of service at customer i
C_k : Capacity of vehicle k (heterogeneous vehicles assumed)
s_i : Service time of customer i
p_i : Pickup quantity of customer i
d_i : Delivery quantity of customer i
fixed_k : Dispatching cost of vehicle k (fixed/overhead cost)
a : Constant representing the trade-off between dispatching cost and routing cost

Decision Variables:

$x_{i,j}$: Assigned the value 1 when the arc between customer i and j is selected as part of the routing pl
$\text{del}_{i,k}$: Assigned the value 1 when customer i is assigned to vehicle k, 0 otherwise
del_k : Assigned the value 1 when vehicle k is used in the solution, 0 otherwise
$\text{load}_{0,k}$: Load of vehicle k when it starts from the distribution center (DC)
$\text{start}_{0,k}$: Starting time of vehicle k when it starts from the DC
$\text{load}_{i,k}$: Load on vehicle after completing the service at customer i
dist_i : Total distance traveled up to customer i
start_i : Starting time of the service at customer i

Objective Function:

$$\text{Minimize } Z = \alpha \sum_{i=1}^{N} \sum_{j=1}^{N} (x_{ij} * c_{ij}) + \left((1-\alpha) \sum_{k=1}^{V} (\text{del}_k * \text{fixed}_k) \right)$$

Subject to the following constraints:

$$\sum_{k=1}^{V} \text{del}_{i,k} = 1, \quad \text{for } i = 1, 2, \ldots, N \tag{2}$$

$$\text{del}_{i,k} \leq \text{del}_k, \quad \text{for } i = 1, 2, \ldots, N \text{ and } k = 1, 2, \ldots, V \tag{3}$$

$$\text{del}_{i,k} - \text{del}_{j,k} \leq (1 - x_{ij}), \quad \text{for } i, j = 1, 2, \ldots, N,\ i \neq j,\ \text{and } k = 1, 2, \ldots, V \tag{4}$$

$$\sum_{j=1}^{N} x_{i,j} = 1, \quad \text{for } i = 1, 2, \ldots, N \tag{5}$$

$$\sum_{i=1}^{N} x_{i,j} = 1, \quad \text{for } j = 1, 2, \ldots, N \tag{6}$$

$$x_{i,i} = 0, \quad \text{for } i = 1, 2, \ldots, N \tag{7}$$

$$x_{i,j} + x_{j,i} \leq 1, \quad \text{for } i = 1, 2, \ldots, N-1,\ j = i+1, i+2, \ldots, N \tag{8}$$

$$\text{dist}_j \geq \text{dist}_i + c_{i,j} - M(1 - x_{i,j}), \quad \text{for } i, j = 1, 2, \ldots, N,\ i \neq j \tag{9}$$

$$\text{st}_i \geq \text{start}_{0,k} + t_{0,i}, \quad \text{for } i = 1, 2, \ldots, N \text{ and } k = 1, 2, \ldots, V \tag{10}$$

$$\text{st}_j \geq \text{st}_i + s_i + t_{i,j} - M(1 - x_{i,j}) \quad \text{for } i, j = 1, 2, \ldots, N,\ i \neq j \tag{11}$$

$$\text{load}_{0,k} = \sum_{i=1}^{N} \text{del}_{i,k} \cdot d_i, \quad \text{for } k = 1, 2, \ldots, V \tag{12}$$

$$\text{ld}_i \geq \text{load}_{0,k} + d_i + p_i \quad \text{for } i = 1, 2, \ldots, N \text{ and } k = 1, 2, \ldots, V \tag{13}$$

$$\text{ld}_j \geq \text{ld}_i + d_i + p_i - M(1 - x_{i,j}) \quad \text{for } i, j = 1, 2, \ldots, N,\ i \neq j \tag{14}$$

Constraints (15)–(19) represent the bounds for the respective decision variables:

$$a_0 \leq \text{start}_{0,k} \leq b_0, \quad \text{for } k = 1, 2, \ldots, V \tag{15}$$

$$a_i \leq \text{st}_i \leq b_i, \quad \text{for } i = 1, 2, \ldots, N \tag{16}$$

$$0 \leq \text{load}_{0,k} \leq C_k, \quad \text{for } k = 1, 2, \ldots, V \tag{17}$$

$$0 \leq \text{ld}_i \leq \max_{1 \leq k \leq V} C_k, \quad \text{for } i = 1, 2, \ldots, N \tag{18}$$

$$0 \leq \text{dist}_i \leq M_1, \quad \text{for } i = 1, 2, \ldots, N \tag{19}$$

Constraints

1. Constraint (2): Each customer should be allocated to only a single vehicle.
2. Constraint (3): A customer can be allocated to a vehicle only when the corresponding vehicle is used in the final solution.
3. Constraints (4): Customers belonging to the same route are allocated to the same vehicle.
4. Constraints (5) and (6): Each customer need to be visited only once.
5. Constraints (7)–(8): Restrictions to avoid subtours in the route sequence
6. Constraints (9): If the arc between two nodes is selected/available as part of the travel plan, then the travel distance should be equal to the distance between them.
7. Constraints (10) and(11): If the arc between customers i and j is selected in the routing plan, then the service time at customer j should be later than the service time at customer i.
8. Constraint (12): The load of a vehicle at any point in the route should be less than the capacity of the vehicle.
9. Constraint (13): Limits the initial load of the vehicle when it starts from the DC.
10. Constraints (14) : Represent the cumulative load of a vehicle after visiting a particular customer.