

AI1110: Probability and Random Variables

Assignment 1

Shivanand Rachewad
BT22BTECH11014

12.13.5.9: Question: On a multiple-choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Solution:

- Let the probability of correct guess be p and incorrect be q which are $\frac{1}{3}$ and $\frac{2}{3}$ respectively
- X represents the number of correct answers by guessing in the set of 5 multiple choice questions.

The CDF of X is,

$$F(x) = \Pr(X \leq x) = \sum_{k=0}^x \binom{n}{k} p^k q^{n-k} \quad (1)$$

$$\Pr(x \geq 4) = 1 - \Pr(x \leq 3) \quad (2)$$

$$\implies 1 - F(3) \quad (3)$$

$$= 1 - \left(\binom{5}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^5 + \binom{5}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^4 + \binom{5}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^3 + \binom{5}{3} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 \right) \quad (4)$$

$$= 1 - \left(\frac{232}{243} \right) \quad (5)$$

$$= \frac{11}{243} \quad (6)$$

\therefore probability that a candidate would get 4 or more correct answers by guessing would be $\frac{11}{243}$