PHY556
Title: Coherence quantification and study in light-matter interactions

Shivang Pandey
200941

Supervisors: Prof. Debabrata Goswami, Prof. Satyajit Banerjee
(Dated: November 13, 2024)

Coherence is vital for scaling quantum computers, but its quantification remains a challenge. This project tries to compare established and newly proposed coherence measures, particularly a recent quantifier using only diagonal elements of the density matrix. We extend the analysis from 2-qubit to 3-qubit systems and beyond through numerical simulations. Additionally, we study coherence in light-matter interactions, analyzing how inter-qubit interaction, detuning, and coupling impact coherence. These findings can identify optimal conditions for coherent quantum computing and to enhance the performance of quantum algorithms in the future.

I. MOTIVATION

Coherence is one of the most important factors to consider while building quantum computers, and has been a major bottleneck in scaling of quantum computers. Looking at coherence as a resource, good quantifiers and good understanding of coherence can identify factors leading to decoherence and help in scaling of quantum computers.

II. OBJECTIVE

- There have been previous measures to quantify coherence using off-diagonal terms of the density matrix, which, however, are not directly experimentally measurable. A recent quantifier[1] has been proposed which only uses the diagonal elements of the density matrix. However, its comparison with existing quantifiers has only been demonstrated for 2-qubit system. My first objective is to reproduce these results for 2-qubit systems.
- Extending the previous work, compare the three quantifiers for multi-qubit systems - 3, 4, 5, 6 qubit systems - through numerical simulations, and hence, demonstrate (or disprove) the validity of the new quantifier.
- Study and simulate coherence in light-matter interaction and find out how factors such as inter-qubit interaction and detuning affect coherence, and find what could be the optimal conditions for coherent quantum computing.

III. METHODOLOGY

The two current major coherent quantifiers are - the relative entropy of coherence $C_{r,e}$, and the L-1 norm of coherence C_{L1}. The relative entropy of coherence is defined as

$$C_{r,e}(\rho) = S(\rho_{diag}) - S(\rho)$$

where S is the Von-Neumann entropy and ρ_{diag} is the diagonal matrix extracted by the density matrix ρ. The L-1 norm of coherence is defined as

$$C_{L1}(\rho) = \sum_{i,j,i\neq j} |\rho_{i,j}|$$

Coherence quantifiers should follow certain properties like (as defined in [2]) -

- $C(\rho) = 0$ iff $\rho = \sum_i \delta_i |i\rangle\langle i|$
- Monotonicity under incoherent completely positive trace preserving maps - $C(\rho) \geq C(\phi_{ICPTP}(\rho))$
- Non-increasing under mixing of quantum states (convexity) - $\sum_n p_n C(\rho_n) \geq C(\sum_n p_n \rho_n)$

Both $C_{r,e}$ and C_{L1} can be shown to satisfy the above properties. However, both these measures require the knowledge of off-diagonal terms of the density matrix, which are not experimentally measurable. In contrast, a new measure - principal diagonal difference of coherence C_{PDD} has been recently defined[1] which only depends on the diagonal elements of the density matrix.

$$C_{PDD}(\rho) = 1 - \frac{\sum_{i,j} |\rho_{ii} - \rho_{jj}|}{2(N-1)}$$

where N is the number of qubits in the system. However, it can be shown that this formula for C_{PDD} is only valid for pure states and not for mixed states. For mixed states, we use the following formula -

$$C_{PDD}^{mix} = A \sum_i p_i C_{PDD}(|\psi_i\rangle)$$

where p_i is the mixing probability of the pure state $|\psi_i\rangle$, and $A = \sqrt{\sum_i p_i^2}$ is the normalisation coefficient.

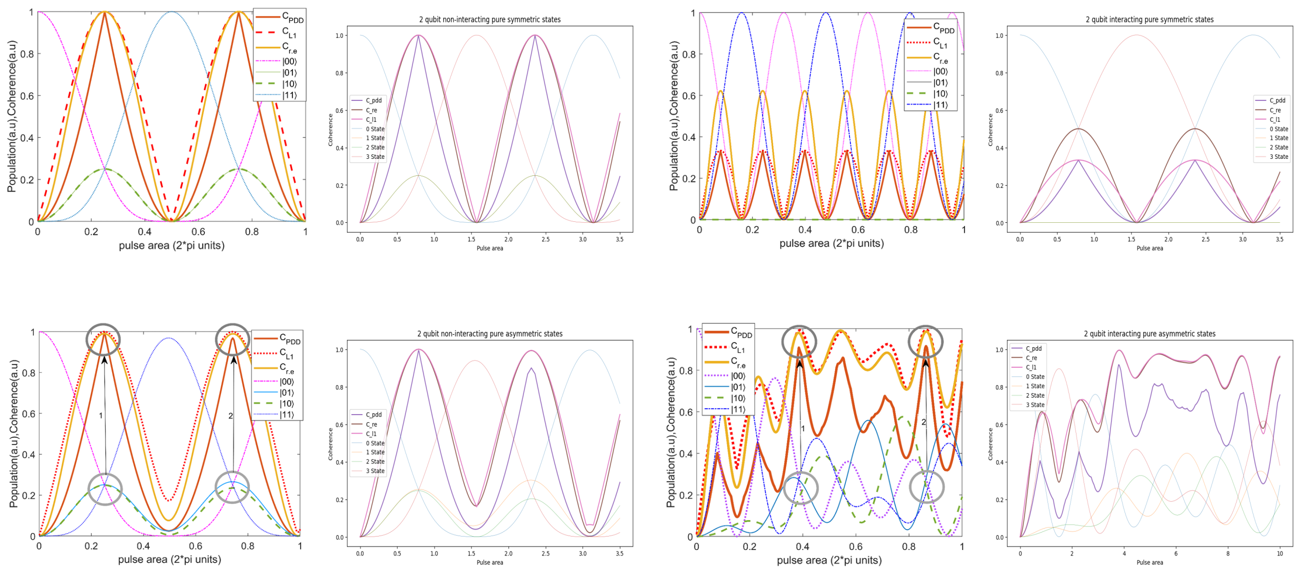


FIG. 1: Published results (left) vs my results for 2 qubit pure symmetric-asymmetric (top-bottom), and interacting-noninteracting (left-right)

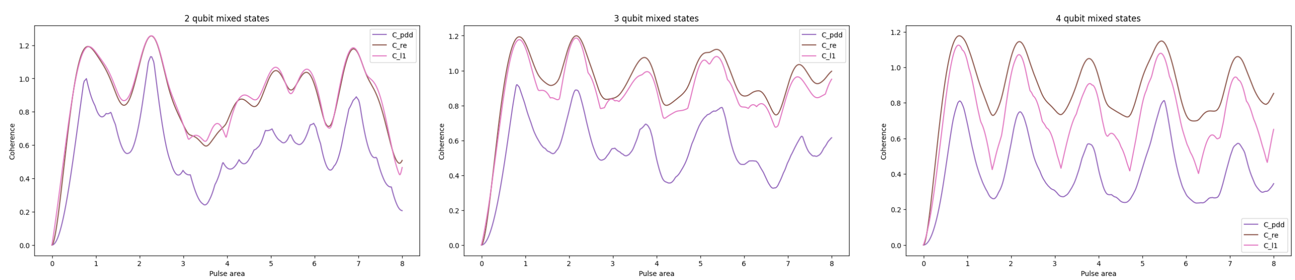


FIG. 2: Coherence evolution for 2, 3, 4 qubit mixed states

We compare these quantifiers in a two level systems, as defined in [3]. We use a density matrix approach by numerically integrating the Liouville equation,

$$\frac{d\rho}{dt} = \frac{i}{\hbar}[\rho, H]$$

for a Hamiltonian in the rotating frequency modulated FM frame of reference. $\rho(t)$ is a density matrix whose diagonal elements represent populations in the ground and excited states and off-diagonal elements represent coherent superposition of states. The Hamiltonian for the simple case of a two-level system under the effect of an applied laser field can be written in the FM frame as

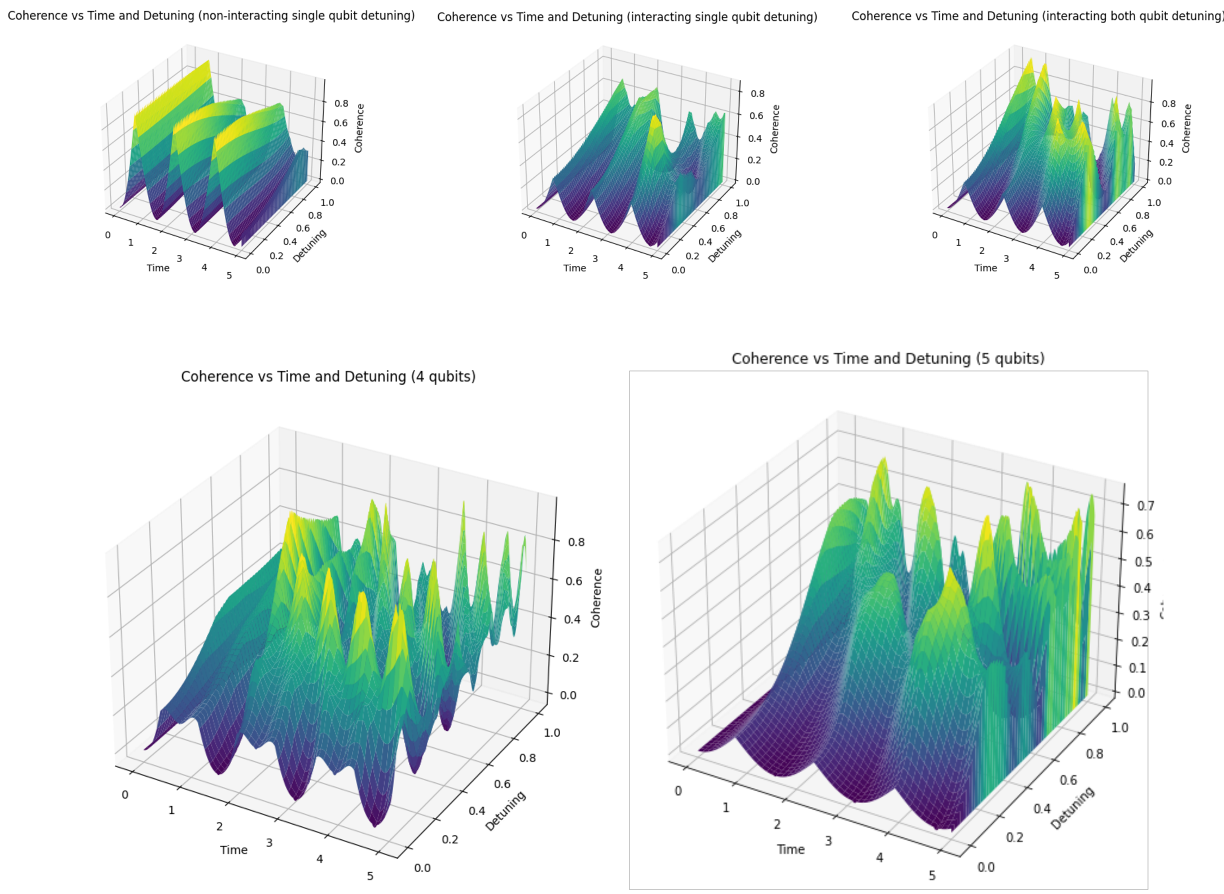
$$H_i = \begin{pmatrix} \Delta_i & \Omega_i/2 \\ \Omega_i^*/2 & 0 \end{pmatrix}$$

where Δ_i and $\Omega_i/2$ are detuning and Rabi frequency for the ith qubit. Detuning $\Delta = \omega - \omega_0$ refers to the difference between the frequency of the applied light (or laser pulse) ω and the natural resonant frequency of a molecular transition ω_0. The Rabi frequency Ω quantifies the rate at which population transitions occur between the two energy states of the system when it is driven by the light field.

Here, we study two cases: (a) qubit-qubit non-interacting: $H = I \otimes H_1 + H_2 \otimes I$ (where I is the (2 $\times$ 2) identity operator), and (b) qubit-qubit interacting: $H = H_1 \otimes H_2$, keeping interaction strength "one." For each one of these cases, we examine both resonance ($\Delta = 0$) and detuning ($\Delta \neq 0$) conditions. We also extend this definition for systems having more than 2 qubits, and compare the three quantifiers.

IV. RESULTS

My first objective was to reproduce the plots in published [1] for 2 qubit case. My plots and the published ones are compared in FIG. 1. Simulations are done for pure - interacting and non-interacting, symmetric (detuning zero for both) and asymmetric cases; as well as for mixed states. The three quantifiers as shown in bold



(d) 4b case

(e) 5c case

FIG. 3: Coherence evolution vs detuning plots

- C_{L1} in pink, $C_{r,e}$ in brown and C_{PDD} in purple - while the state populations are shown in light. As we can see, my graphs match qualitatively with the already produced results. This confirms the correctness of my approach. We are also able to showed the sensitivity of C_{PDD} over $C_{r,e}$ and C_{L1} as was argued in the original paper.

Then I extended my code to do similar situations for $3, 4, 5, 6$ qubit cases which are shown in FIG. 4. In the 2 qubit case, there are only two possible cases of interaction - interacting or non-interacting, however, for multi-qubit case, multiple cases are possible, some of which are shown in FIG 4. These multiple interaction cases are described as per the table I.

Furthermore, I have plotted the evolution of coherence vs detuning as shown in the 3D plots in FIG. 3

V. DISCUSSION

The first and foremost inference that one can draw from the graphs is that all the coherence measures are matching qualitatively even for multi-qubit systems. This demonstrates the validity of C_{PDD}. Further, we can see the advantages of C_{PDD} being more sensitive, and more computationally efficient to compute.

The plots for multi-qubit systems in FIG 4 show that coherence decreases with increasing interaction. For example, if we look at the 6 qubit case, 6a (FIG. 4(g)) denotes the case when all qubits are non-interacting, and we see in this case the maxima is 1. 6b (FIG. 4(h)) is a system in which there are two interacting qubits while rest are non-interacting, while 6c (FIG. 4(i)) is a non-interacting four-particle subsystem interacting with a non-interacting two-particle subsystem, 6d (FIG. 4(j)) is a non-interacting four-particle subsystem interacting with a interacting two-particle subsystem, 6e (FIG. 4(k)) is a completely interacting four-particle subsystem and a two-particle non-interacting subsystem which do not interact, while 6f (FIG. 4(l)) is an interacting four-particle subsystem interacting with a non-interacting two-particle subsystem. From these descriptions we can see that the cases are in increasing order of level of interactions, and from the graphs we can see that the coherence maxima gets increasing lower with interaction. FIG. 4 shows that the same is valid for 4 and 5 qubit cases. Thus, there seems to be a negative correlation between max coher-

Label	Description
H0	Single particle hamiltonian with no detuning
H1	non_interacting_Hamiltonian(H0,H0)
H2	interacting_Hamiltonian(H0,H0)
H4	non_interacting_Hamiltonian(H1,H1)
H5	interacting_Hamiltonian(H2,H2)
3a	non_interacting_Hamiltonian(H1, H0)
3b	interacting_Hamiltonian(H1, H0)
3c	non_interacting_Hamiltonian(H2, H0)
3d	interacting_Hamiltonian(H2, H0)
4a	non_interacting_Hamiltonian(H1, H1)
4b	interacting_Hamiltonian(H1, H1)
4c	non_interacting_Hamiltonian(H1, H2)
4d	non_interacting_Hamiltonian(H2, H2)
4e	interacting_Hamiltonian(H1, H2)
4f	interacting_Hamiltonian(H2, H2)
5a	non_interacting_Hamiltonian(H4, H0)
5b	interacting_Hamiltonian(H4, H0)
5c	non_interacting_Hamiltonian(H5, H0)
5d	interacting_Hamiltonian(H5, H0)
6a	non_interacting_Hamiltonian(H4, H1)
6b	non_interacting_Hamiltonian(H4, H2)
6c	interacting_Hamiltonian(H4, H1)
6d	interacting_Hamiltonian(H4, H2)
6e	non_interacting_Hamiltonian(H5, H1)
6f	interacting_Hamiltonian(H5, H1)

TABLE I: Interaction for various cases

ence and interaction.

The 3D plots of coherence vs detuning present some interesting insights. Intuitively, one might feel that increasing detuning might negatively affect coherence. However, from the graphs we can see that it is not always the case, the maxima of coherence is obtained at some finite, non-zero detuning and pulse width. Past work ([4]) showed that in the interaction between a quantum harmonic oscillator and a classical fluctuating environment, the system's coherence can increase with detuning. This occurs because non-resonant conditions allow for revivals of nonclassical states, which correspond to increased coherence in the system, as analyzed in the context of decoherence time and non-classicality revival with varying detuning. Similarly the results in [5] showed that as detuning increases, the coherence tends to revive periodically, maintaining a high non-zero value. This means the qubits can remain in a more coherent state at larger detunings compared to resonance conditions, where coherence decays more rapidly. Similar results were shown previously in [6] and [7]. This knowledge, however, can help experimentalists maximise coherence of the system by using optimal detuning and pulse area parameters.

VI. FUTURE WORK

Although our findings suggest that coherence should always decrease with increasing interaction effects, however, as demonstrated in [6], certain quantum systems may maintain high coherence even with significant interaction. Similar studies can be done for other systems to find optimal conditions for qubit scaling.

Moreover, similar results and findings can be applied for better execution of existing quantum algorithms to improve the performance and scalability.

[1] M. Hazra and D. Goswami, Quantifying coherence with principal diagonal elements of density matrix, APL Quantum **1**, 036124 (2024), https://pubs.aip.org/aip/apq/article-pdf/doi/10.1063/5.0209604/20166783/036124_1_5.0209604.pdf.

[2] T. Baumgratz, M. Cramer, and M. B. Plenio, Quantifying coherence, Physical review letters **113**, 140401 (2014).

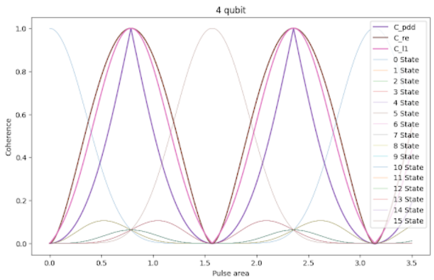
[3] D. Goswami, Probing coherence aspects of adiabatic quantum computation and control, The Journal of chemical physics **127** (2007).

[4] J. Trapani, M. Bina, S. Maniscalco, and M. G. A. Paris, Collapse and revival of quantum coherence for a harmonic oscillator interacting with a classical fluctuating environment, Phys. Rev. A **91**, 022113 (2015).

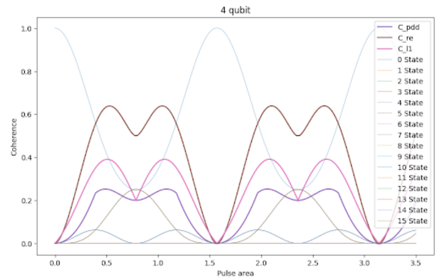
[5] L.-T. Shen, Z.-C. Shi, and Z.-B. Yang, Coherent state control to recover quantum entanglement and coherence, Entropy **21**, 10.3390/e21100917 (2019).

[6] A. Beckert, M. Grimm, N. Wili, R. Tschaggelar, G. Jeschke, G. Matmon, S. Gerber, M. Müller, and G. Aeppli, Emergence of highly coherent two-level systems in a noisy and dense quantum network, Nature Physics **20**, 472 (2024).

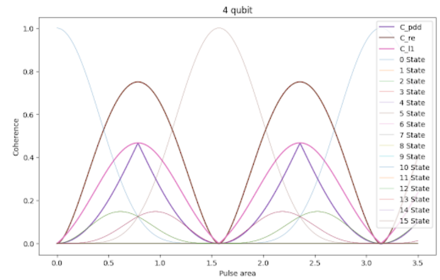
[7] B. T. Torosov, B. W. Shore, and N. V. Vitanov, Coherent control techniques for two-state quantum systems: A comparative study, Phys. Rev. A **103**, 033110 (2021).



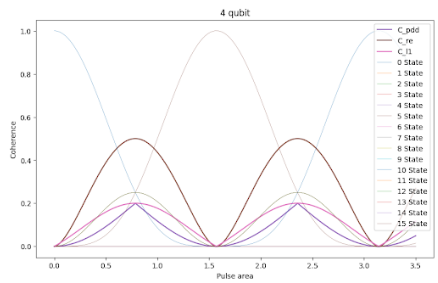
(a) 4a



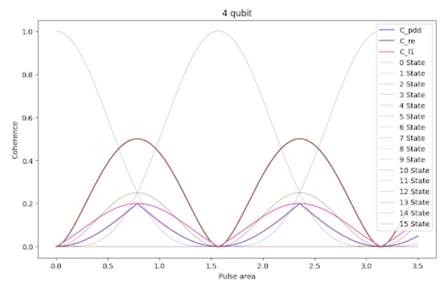
(b) 4b



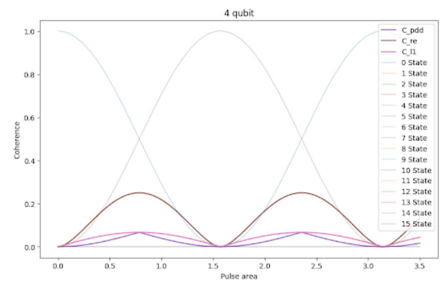
(c) 4c



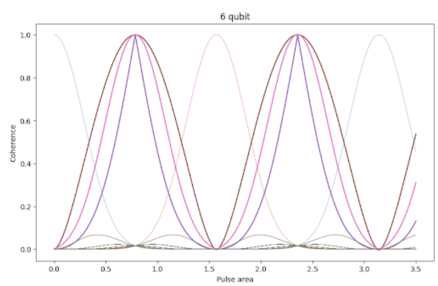
(d) 4d



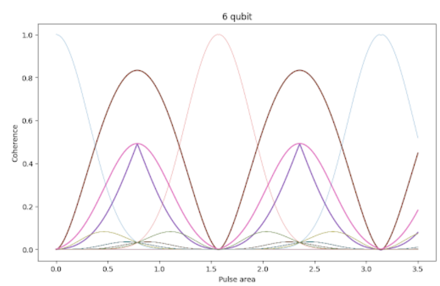
(e) 4e



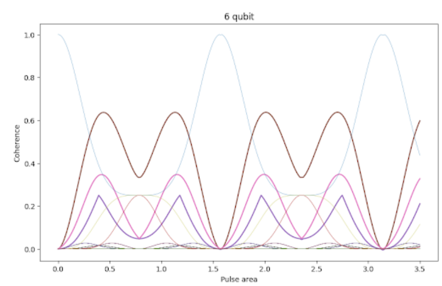
(f) 4f



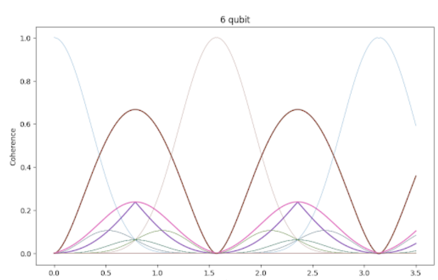
(g) 6a



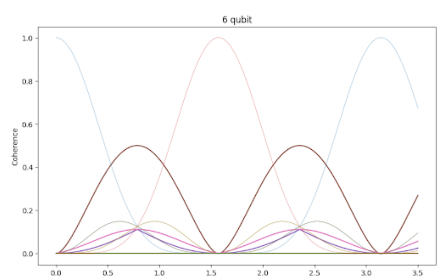
(h) 6b



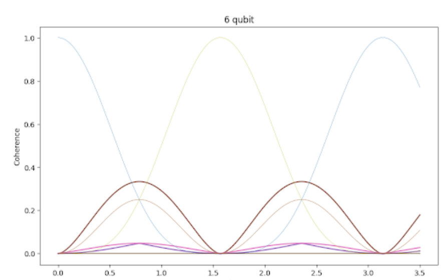
(i) 6c



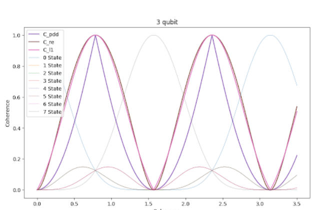
(j) 6d



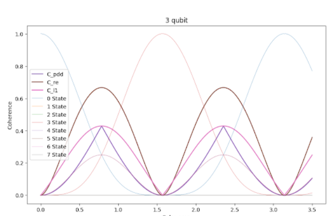
(k) 6e



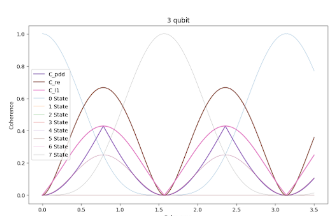
(l) 6f



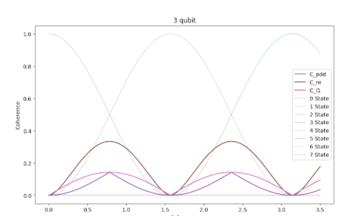
(m) 3a



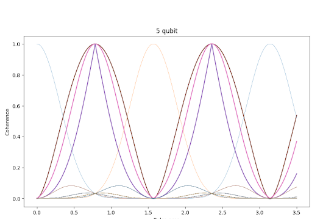
(n) 3b



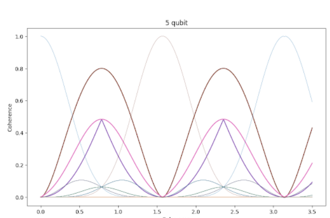
(o) 3c



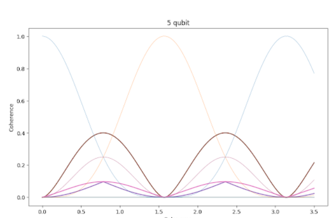
(p) 3d



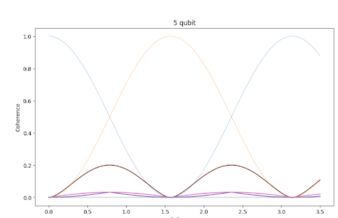
(q) 5a



(r) 5b



(s) 5c



(t) 5d

FIG. 4: Coherence evolution plots for different interaction cases for 3, 4, 5, 6 qubits