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A PREPRINT

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SOEN 6011
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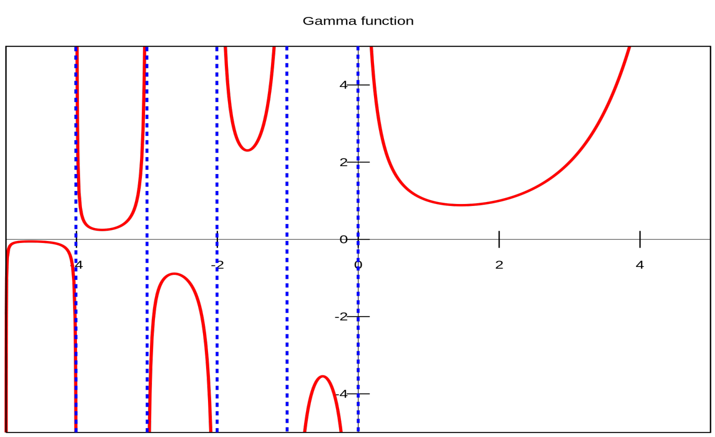
$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$$



Contents

PROBLEM 1

The gamma function is a continuous extension to the factorial function, which is only defined for the complex numbers and non-negative integers. The gamma function is defined as:

$$\Gamma(\alpha) = \int^{\infty} x^{\alpha-1} e^{-x} dx, \text{ where } \alpha > 0.$$

If p>0, then, integration by parts yields the formula (p+1)=p(p)(p+1)=p(p). Using this formula, one can extend the domain of definition inductively by setting first (p)=(p+1)p(p)=(p+1)p for 1<p<01<p<0.

- The gamma function does not satisfy any algebraic differential equation.
- It is applicable in the fields of probability and statistics, as well as combinatorics.
- The gamma function is an analytical function of, which is defined over the whole complex -plane with the exception of countably many points.
- The function has an infinite set of singular points, which are the simple poles with residues.
- The function does not have branch points and branch cuts.
- The function does not have periodicity.
- The function has mirror symmetry.
- The gamma function satisfies the recursive property:

$$\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1),$$

PROBLEM 2

Unique Identifier	**Requirements** **(FR-> Functional requirements)** **(NFR-> Non-functional requirements)**
FR1	Defined for only positive numbers. x>0
FR2	Integration is an asset so knowledge of integral is important aspect in order to calculate gamma function.
FR3	Integral function is required to calculate gamma function.
FR4	Power function is required to calculate gamma function.
FR5	Advance form of factorial is its main key as it supports recursion, knowledge of factorial is needed to calculate gamma function.It is recursive to real numbers but not to non-positive numbers.
FR6	Exponential function is required to calculate gamma function
NFR1 Scalability	The Gamma distribution has the scaling property so that if X is a random variable that follows a Gamma distribution, so does cX for c>0. To scale the distribution, multiply by c: cXGamma(k,c) want it to scale a distribution with k=2 and =2 to the range (0,1000)you could multiply by 1000.
NFR2 Efficiency	The Gamma distribution is not give exact values but they are very close to the approximations.
NFR3 Modifiability	There are multiple ways available to calculate the gamma distribution.
NFR4 Robust	The Gamma distribution is applied across various real world machines such as calculator.

PROBLEM 3

STRILING'S APPROXIMATION - GAMMA FUNCTION$gamma(x)$

for $j = 1$ **to** n
sqrt $(2 * \pi / x)$
power $((x/\exp), x)$
// Multiply 1 and 2.
gamma $= sqrt(2 * \pi / x) * ((x/\exp), x)$
gamma $(i/10.0)$

LANCZOS APPROXIMATION - GAMMA FUNCTION$gamma(x)$

for $j = 1$ **to** n
$g = 7$
double $[]p = 0.99999999999980993, ...$
if $(x < 0.5)$
$\pi/(sin(\pi * x) * gamma(1 - x))$
$x- = 1$
double $a = p[0]$
double $t = x + g + 0.5$
for $i = 1$ **to** $p.length$
$a+ = p[i]/(x + i)$
sqrt $(2 * \pi) * power(t, x + 0.5) * exp(-t) * a$
gamma $(i/10.0)$

The methods of Stirling and Lanczos share the common strategy of terminating an infinite series and estimating the error which results. Also in common among these two methods, is a free parameter which controls the accuracy of the approximation. Stirling's asymptotic series forms the basis for most computational algorithms for the gamma function and much has been written on its implementation. An example on the use of Stirling's series is given with a short discussion of error bounds. Although not quite as accurate as Lanczos' method using the same number of terms of the series and difficult to compute.

The conclusion is that each method has its merits and shortcomings, and the question of which is best has no clear answer. For a uniformly bounded relative error, Lanczos' method seems most efficient, while Stirling's series yields very accurate results for z of large modulus due to its error term which decreases rapidly with increasing |z|.

Out of the two implementation algorithms for the Gamma function, i will choose Lanczo's approximation method over Striling's approximation. Although Striling is easy to compute but still lacks accuracy where as Lanczo is more accurate, precise, efficient and can deal with relative error.

References

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