Assignment-4

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Control System

① Draw Root locus for the following OLTF:

$$G(s)H(s) = \frac{K}{s(s+2)(s^2+2s+5)}$$

char. equation

$$1 + G(s)H(s) = 0$$

$$s(s+2)(s^2+2s+5) + K = 0 \quad \text{--- (1)}$$

no. of Poles $P = 4$, no. of zeros $Z = 0$; no. of Asymptotes $= P - Z = 4$

Poles $\Rightarrow 0, -2, -1 \pm 2j$,

Angle of Asymptotes $= \frac{(2q+1)\cdot 180^\circ}{P-Z}$ $\{q = 0, 1, 2, 3\}$

$= 45^\circ, 135^\circ, 225^\circ, 315^\circ$

Centroid of Asymptotes $= \frac{\Sigma \text{Real Part of OLP} - \Sigma \text{real Parts of OLZ}}{P - Z}$

$= \frac{-4}{4} = -1$

Break away Point from eqn (1)

$$K = -\{(s^2+2s)(s^2+2s+5)\}$$

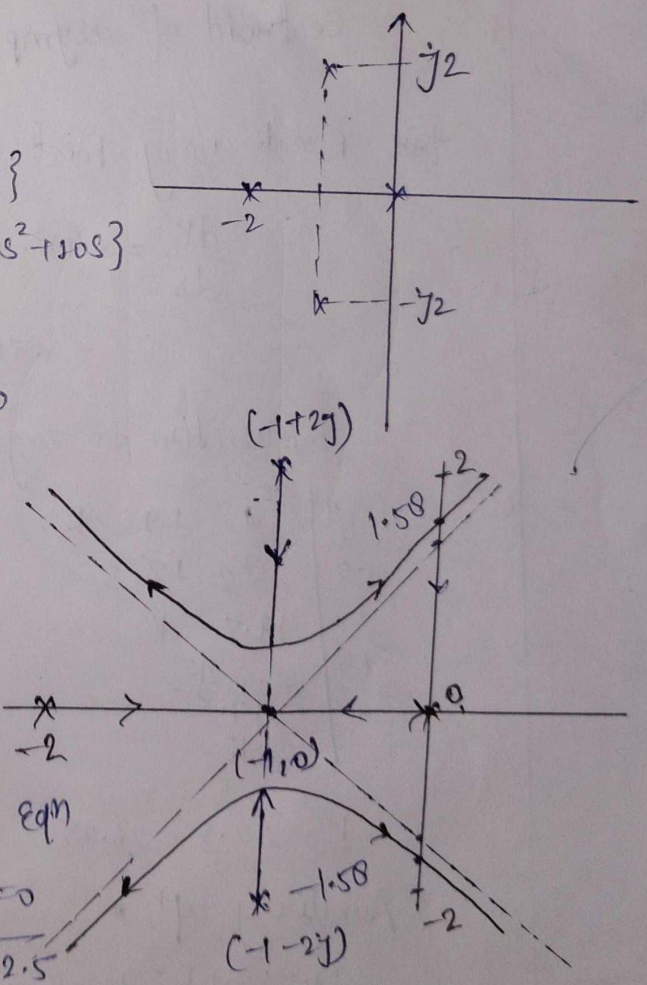
$$K = -\{s^4+2s^3+5s^2+2s^3+4s^2+10s\}$$

$$K = -s^4 - 4s^3 - 9s^2 - 10s$$

$$\frac{dK}{ds} = -4s^3 - 12s^2 - 18s - 10 = 0$$

$$4s^3 + 12s^2 + 18s + 10 = 0$$

$$s = -1, \; -1.0 \pm 1.22j$$



Char eqn

$$s^4 + 4s^3 + 9s^2 + 10s + K = 0$$

s^4	1	9	K
s^3	2	10	
s^2	6.5	K	
s^1	$\frac{32.5-2K}{6.5}$		
s^0	K		

$(K = 65/4)$

Auxiliary eqn

$$6.5s^2 + K = 0$$

$$s = \pm\sqrt{-2.5}$$

$$s = \pm 1.58j$$

(2) $G(s)H(s) = \dfrac{K}{s(s+4)(s^2+4s+3)}$

char. eqn →

$1+G(s)H(s)=0$

$s(s+4)(s^2+4s+3)+K=0$

$(s^2+4s)(s^2+4s+3)+K=0$

$s^4+8s^3+19s^2+12s+K=0$ — (I)

no. of Poles (P) = 4, no. of zeroes (Z) = 0

no. of Asymptotes = 4 − 0 = 4

Poles = 0, −4, −1, −3

Angle of Asymptotes

$$\theta = \frac{(2q+1)180^\circ}{P-Z} = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

Centroid of asymptotes $= \dfrac{-4-1-3}{4} = -2$ $(-2, 0)$

for Break away Point on differentiating eqn

$$\frac{dK}{ds} = -4s^3-24s^2-38s-12=0$$

$s = -3.58, -0.418, -2$

Intersection to img. axis

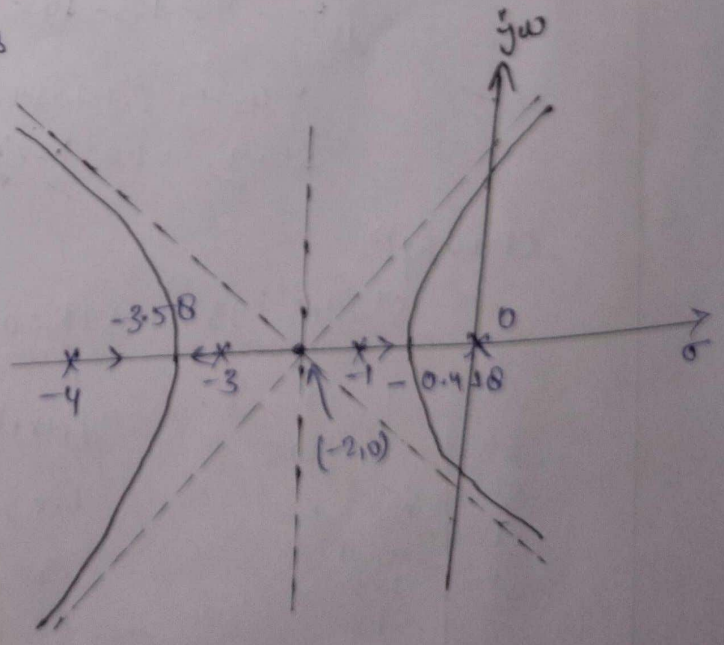
s^4	1	19	K
s^3	8	12	0
s^2	17.5	K	
s^1	$\frac{52.5-2K}{17.5}$		
s^0	K		

$K = \dfrac{52.5}{2} = 26.25$

Auxiliary eqn

$17.5s^2+K=0$

$s = \pm\sqrt{\dfrac{-26.25}{17.5}} = \pm 1.22j$



Q-3) $G(s)H(s) = \frac{K(s+3)}{(s^2-s-2)}$

Soln

no. of Poles $P=2$, & $Z=1$,

Poles $= 2, -1$ Zero $\Rightarrow s=-3$

no. of asymptotes $= P-Z = 2-1 = 1$

Angle of asymptotes $\theta = \frac{(2n+1)180^\circ}{P-Z} = 180^\circ$ //

centroid of asymptotes $= \frac{2-1+3}{1} = 4$

Char. eq. $1+G(s)H(s)=0$

$= s^2-s-2+(s+3)K=0$

$s^2+(K-1)s+3K-2=0$ — (I)

for breakaway Point

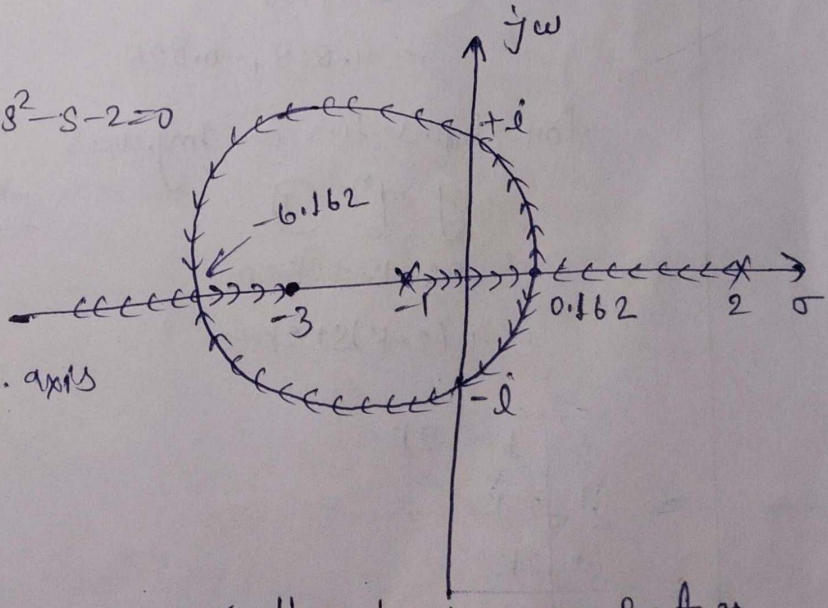
$K = -\frac{s^2+s+2}{s+3}$

$\frac{dK}{ds} = \frac{(s+3)(-2s+1)-(-s^2+2s+2)}{(s+3)^2} = 0$

$-2s^2+s-6s+3+s^2-s-2=0$

$s^2+6s-1=0$

$s = 0.162, -6.162$



for intersection on Imag. axis

s^2	1	$3K-2$
s^1	$K-1$	
s^0	$3K-2$	

$K_{marg} = 1$

Aux. eqn $\Rightarrow s^2+3K-2=0$

$s^2 = -1$

$s = \pm i$

So there break away Point is 0.162 and break in Point is -6.162.

Q → (4) $G(s) = \frac{K}{s}$, $H(s) = 1$, and time $\tau_D = 1$

Sol^n → Given → $G(s) = \frac{K}{s}$, $H(s) = 1$

Delay time $\tau_D = 1$

characteristic eq^n →

$$1 + G(s)H(s)e^{-s\tau_D} = 0$$

$$1 + \frac{K}{s}e^{-s} = 0$$

$$1 + \frac{K}{s}\left[\frac{(1-s/2)}{(1+s/2)}\right] = 0$$

→ origination Point $s = 0, -2$
termination Points $s = 2$

$$1 + \frac{K}{s}\left(\frac{2-s}{2+s}\right) = 0 \quad \text{— (I)}$$

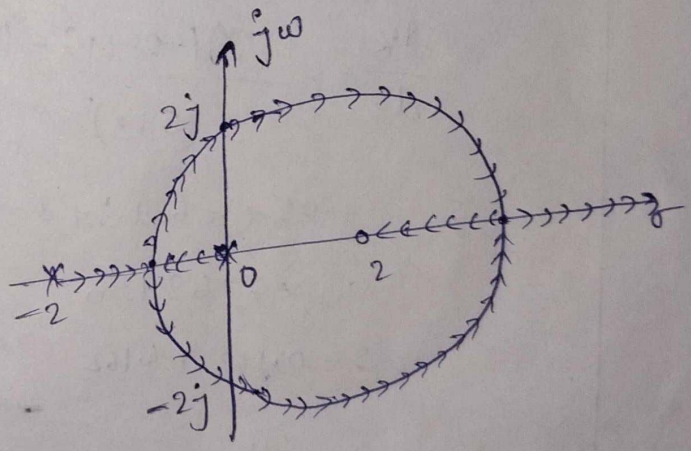
$$s(2+s) + K(2-s) = 0$$

$$\left(K = \frac{s(s+2)}{s-2}\right).$$

$$\frac{dK}{ds} = \frac{(s-2)(2s+2) - (s^2+2s)}{(s-2)^2} = 0$$

$$s^2 - 4s - 4 = 0$$

$$s = 4.828, -0.828$$



for Intersection on Img. axis's

by eq^n (I)

$$s^2 + 2s - Ks + 2K = 0$$

$$s^2 + (2-K)s + 2K = 0$$

s^2	1	$2K$
s^1	$2-K$	
s^0	$2K$	

$K = 2$

Auxiliary eq^n

$$s^2 + 2K = 0$$

$$s^2 = -4$$

$$s = \pm 2j$$