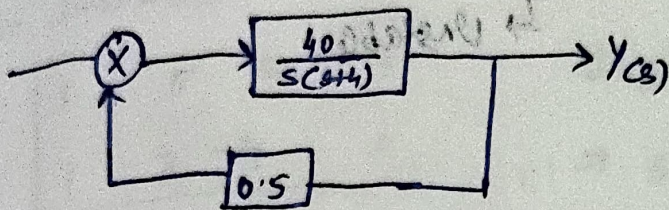


Assignment - 3

A1)



closed loop T/F $\Rightarrow T(s) = \frac{G(s)}{1 + G(s)H(s)}$

$$T(s) = \frac{\frac{40}{s(s+4)}}{1 + \frac{40}{s(s+4)} \times 0.5} = \frac{40}{s^2 + 4s + 20}$$

• Sensitivity w.r.t forward path transfer function

$$S_G = \frac{1}{1 + G(s)H(s)} = \frac{1}{1 + \frac{40}{s(s+4)} \times 0.5} = \frac{s^2 + 4s}{s^2 + 4s + 20}$$

Substitute $s = j1.3$

$$S_G = \frac{(j1.3)^2 + 4(j1.3)}{(j1.3)^2 + 4(j1.3) + 20} = \frac{-1.69 + j5.2}{18.31 + j5.2}$$

$$\boxed{|S_G| \approx 0.287}$$

• Sensitivity w.r.t feedback path transfer function

$$S_H = \frac{-G(s)}{1 + G(s)H(s)} = \frac{\frac{-40}{s(s+4)}}{1 + \frac{40}{s(s+4)} \times 0.5}$$

$$S_H = \frac{-40}{s(s+4) + 20}$$

Substituting $s = j1.3$ we get

$$\boxed{|S_H| \approx 2.10}$$

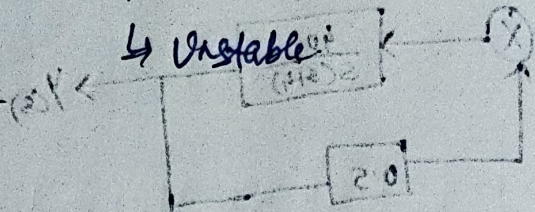
A2) (i) $s^3 + 4.5s^2 + 3.5s + 16 = 0$ *unstable*

→ s^3 | 1 $7\frac{1}{2}$ Sign changed twice

s^2 | $9\frac{1}{2}$ 16

s^1 | $-1/18$ 0

s^0 | 16



(2) $\omega = (2)T \leftarrow \omega T = 2001 \text{ } 6000$

(ii) $s^5 + 1.5s^4 + 2s^3 + 4s^2 + 5s + 10 = 0$

s^5 | 1 2 $50H$

s^4 | $3\frac{1}{2}$ 4 $10H + 2$

Sign Changed twice

→ unstable

s^3 | $-2\frac{1}{3}$ $-5\frac{1}{3}$ 0

s^2 | $-2\frac{1}{4}$ 10

s^1 | $-25/18$ 0

s^0 | 10

Stability reference loop because of two poles

$$\frac{1}{20 \times \frac{0H}{(H+2)} + 1} = \frac{1}{(0H)(H+1)} = 2$$

(iii) $s^5 + s^4 + 2s^3 + 2s^2 + 11s + 10 = 0$

$\epsilon \cdot 1 = 2$ substituted

s^5 | 1 2 $5 \cdot 2 + 10 \cdot 1 = 4$

s^4 | 1 2 10

Sign Changed twice

s^3 | e 1 0

→ unstable

s^2 | $-4e$ 10

s^1 | 1 0

s^0 | 10

Stability reference loop because of two poles

$$\frac{1}{20 \times \frac{0H}{(H+2)} + 1} = \frac{(2)N}{(2)H + 1}$$

for $\omega = 2$ substituted

$0.5 \times 10 = 5$

$$\frac{0H}{0H + (H+2)2} = 2$$

(iv) $s^4 + s^5 + 5s^4 + 3s^3 + 2s^2 - 4s - 8 = 0$
 All the coefficients are not +ve, so the system is unstable.

| | | | | | |
|-------|----------------|----|----|----|--|
| s^6 | 1 | 5 | 2 | -8 | |
| s^5 | 1 | 3 | -4 | 0 | |
| s^4 | 1 | 3 | -4 | | |
| s^3 | 2 | 3 | | | |
| s^2 | $3\frac{1}{2}$ | -4 | | | |
| s^1 | $2\frac{5}{3}$ | 0 | | | |
| s^0 | -4 | | | | |

Auxiliary eqⁿ
 $\Rightarrow s^4 + 3s^2 - 4 = 0$
 $\xrightarrow{\frac{dA}{ds}} 4s^3 + 6s = 0$

(v). $\Delta(s) = 1 + G(s)H(s)$

$$= s(s+2) + e^{-sT}$$

$$= s(s+2) + \frac{1 - sT}{1 + sT}$$

$$= s(s+2)(1+sT) + (1-sT)$$

$$= (s^2 + 2s)(1+sT) + (1-sT)$$

$$= s^2 + 2s + s^3T + 2s^2T + 1 - sT$$

$$= s^3T + s^2[2T+1] + [2-T]s + 1$$

| | | |
|-------|------------------------|-------|
| s^3 | T | $2-T$ |
| s^2 | $2T+1$ | 1 |
| s^1 | $\frac{2T^2T+1}{2T+1}$ | 0 |
| s^0 | 1 | |

System is stable
 when $T > 0$

$$(vi) s^3 + 10s^2 + 18s + k = 0 \text{ about } s = -1$$

$$s' = s - 1$$

$$\Rightarrow (s' - 1)^3 + 10(s' - 1)^2 + 18(s' - 1) + k = 0$$

$$\Rightarrow (s')^3 - 3(s')^2 + 3s' - 1 + 10[(s')^2 - 2s' + 1] + 18[s' - 1] + k = 0$$

$$\Rightarrow (s')^3 + 7(s')^2 + s' + (k + 27) = 0$$

| | | |
|-------|---------------------|----------|
| s^3 | 1 | 1 |
| s^2 | 7 | $k + 27$ |
| s^1 | $\frac{-k - 27}{7}$ | 0 |
| s^0 | $k + 27$ | |

System is stable

when $-27 < k < -20$

$$(OH)(CO) + 1 = COA$$

$$T^2 \cdot 0 + (s + 2)2 =$$

$$\frac{T^2 \cdot 1}{T^2 H} + (s + 2)2 =$$

$$(T^2 - 1) + (T^2 + 1)(s + 2)2 =$$

$$(T^2 - 1) + (T^2 + 1)(2s + 4) =$$

$$T^2 - 1 + T^2 \cdot 2s + T^2 \cdot 4 + 2s + 4 =$$

$$1 + 2[T^2 - 1] + [2 - 1 + T^2]2 + T^2 \cdot 2 =$$

stable is not

$0 < T < \infty$

| | | |
|-----------|-------------------------------|-------|
| $T^2 - 1$ | T | s |
| 1 | $1 + T^2$ | -2 |
| 0 | $\frac{1 + T^2 + 1}{1 + T^2}$ | $1/2$ |
| | 1 | 0 |