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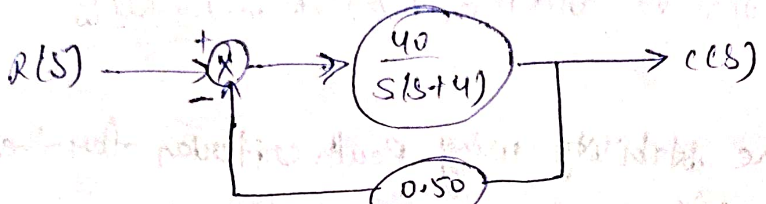
Assignment
control.

Ques-1 → Calculate the sensitivity of the closed-loop system shown in figure given below with respect to the forward path transfer function and feedback path transfer function at ω = 1.3 rad/sec



$G(S) = \frac{40}{S(S+4)}$, $H(S) = 0.50$, closed loop transfer func $T(S) = \frac{G(S)}{1+G(S)H(S)}$

$= \frac{40}{S^2+4S+20}$,

Sensitivity w.r.t. to forward path transfer function.

$$S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{1}{1+G(S)H(S)}$$

& $$S_H^T = \frac{\partial T/T}{\partial H/H} = \frac{-G(S)H(S)}{1+G(S)H(S)}$$ Now $s = j\omega|_{\omega=1.3}$

∴ (1) Forward Path $G(j1.3)$:

$$G(j1.3) = \frac{40}{j1.3(j1.3+4)} = \frac{40}{j1.3(4+j1.3)}, \quad \therefore |G(j1.3)| = \frac{40}{\sqrt{(1.3)^2+(4^2+(1.3)^2)}}$$

∴ $|G(j1.3)| \simeq 7.33$ & $\angle G(j1.3) = -90° - \tan^{-1}\left(\frac{1.3}{4}\right) = -90° - 18° = -108°$

(2) Feedback Path: $H(j1.3)$: $= 0.50$ (Purely real, no phase shift)

(3) loop gain: $G(j1.3)H(j1.3)$: $7.33 \times 0.50 = 3.66\angle -108°$

$$S_G^T = \frac{1}{1+G(j1.3)H(j1.3)} = \frac{1}{1+3.665\angle 108°} = \frac{1}{3.48} \simeq 0.287$$ ↳ mag.

$$S_H^T = \frac{-G(j1.3)H(j1.3)}{1+G(j1.3)H(j1.3)} = \frac{-3.665\angle -108^\circ}{3.48\angle -92^\circ} = 1.05\angle -16^\circ$$

$\therefore$ mag = 1.05

$\Rightarrow$ the closed loop system is more sensitive to changes in H(s) (105%) than to G(s) (28.7%) at $\omega = 1.3$ rad/s

Quesn → ② (a) Determine stability using Routh criterion for the following polynomials

(a) $s^3 + 4.5s^2 + 3.5s + 16 = 0$

s^3	1	3.5
s^2	4.5	16
s^1	-0.05	0
s^0	16	0

$\Rightarrow$ It's an unstable system as there is a -ve sign is present in the first column of routh array.

(b) $s^5 + 1.5s^4 + 2s^3 + 4s^2 + 5s + 10 = 0$

s^5	1	2	5
s^4	1.5	4	10
s^3	-0.667	-1.17	0
s^2	1.5	10	0
s^1	+12.2	0	0
s^0	10	0	0

$\Rightarrow$ It's an unstable system as there is a -ve sign is present in the first coloums of routh array.

(d) $s^6 + s^5 + 5s^4 + 3s^3 + 2s^2 - 4s - 8 = 0$

s^6	1	5	2	-8
s^5	1	3	-4	0
s^4	2	6	-8	
s^3	0	0		

$\Rightarrow$ Special case (zero row)

$\Rightarrow$ Auxiliary polynomial: $A(s) = 2s^4 + 6s^2 - 8$

$\therefore \frac{dA}{ds} = 8s^3 + 12s$

$\therefore$ updated routh array

s^6	1	5	2	-8
s^5	1	3	-4	0
s^4	2	6	-8	
s^3	8	12	0	
s^2	3	-8		
s^1	34.6	0		
s^0	-8			

$\Rightarrow$ It's an unstable system

(e) $G(s) = \frac{e^{-sT}}{s(s+1)}$, $H(s) = 1$.

(c) $s^5 + s^4 + 2s^3 + 2s^2 + 11s + 10 = 0$

s^5	1	2	11
s^4	1	2	10
s^3	$0(\varepsilon)$	1	0
s^2	$\frac{2\varepsilon-1}{\varepsilon}$	$\frac{10}{\varepsilon}$	
s^1	$\frac{2\varepsilon-1-10\varepsilon^2}{2\varepsilon-1}$		
s^0	10		

C-I $\rightarrow \lim_{\varepsilon \to 0} \frac{2\varepsilon-1}{\varepsilon} = 2 - \frac{1}{\varepsilon} = -\infty$

C-II $\rightarrow \lim_{\varepsilon \to 0} \frac{2\varepsilon-1-10\varepsilon^2}{2\varepsilon-1} = \frac{-1}{-1} = 1$

$\therefore$ unstable

(e) $G(s) = \frac{e^{-sT}}{s(s+2)}$ $\quad H(s) = 1$

characteristic Eq^n

$1 + G(s)H(s) = 0$

$1 + \frac{e^{-sT}}{s(s+2)} = 0,$

$s(s+2) + e^{-sT} = 0$

$(s^2+2s)(1+sT) + (1-sT) = 0$

$Ts^3 + (2T+1)s^2 + 2s + 1 - sT) = 0$

$Ts^3 + (2T+1)s^2 + s(2-T) + 1 = 0$

creating routh table

s^3	T	$2-T$
s^2	$2T+1$	1
s^1	$\frac{-2(T^2-T-1)}{1}$	0
		0
s^0	1	0

for system the stable

$$\frac{-2(T^2-T-1)}{2T+1} > 0 \Rightarrow \frac{T^2-T-1}{2T+1} < 0$$

Case-I : $T^2-T-1 > 0$ & $2T+1 < 0$

$T < -\frac{1}{2}$ and $T \in (-\infty, 1-\sqrt{5}) \cup (1+\sqrt{5}, \infty)$

so $T \in (-\infty, 1-\sqrt{5})$

C-II → $T^2-T-1 < 0$ and $2T+1 > 0$

$T \in (1-\sqrt{5}, 1+\sqrt{5})$ & $T > -\frac{1}{2}$

so, $T \in (-\frac{1}{2}, 1+\sqrt{5})$

(4) $s^3+10s^2+10s+K=0$ for $s=-1$

for $s=-1$ substitute s as $s-1$

$(s-1)^3+10(s-1)^2+10(s-1)+K=0$

$s^3-1-3s(s-1)+10(s^2+1-\cancel{1s})+10s-10+K=0$

$s^3-1-3s^2+3s+10s^2+10-20s+10s-10+K=0$

$s^3+7s^2+0-9+K=0$

Now

s^3	1	1	0
s^2	7	$-9+K$	0
s^1	$\frac{16-K}{7}$	0	
s^0	$-9+K$		

so $\frac{16-K}{7} > 0$

$16-K > 0$

$K < 16$

& $-9+K > 0$ $K > 9$

System stable for $K \in (9, 16)$