Modeling & Simulation: Output Analysis

Course: Modeling & Simulation - EEN14253

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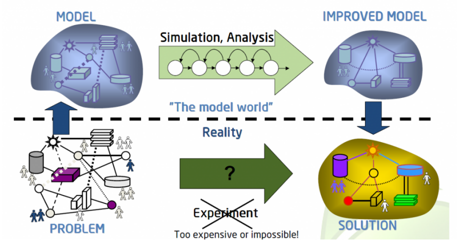

Verification & Validation of Models

Model Verification

- Building the model right
- Correct translation of the conceptual model to the simulation model and successful execution as a program.

Errors arise from the data, conceptual, specification models, and even during the programming stage. Common steps to perform verification are:

- Testing each sub-model, then the entire model
- Run the program under different conditions, then check if the output is reasonable
- Identify the state variables after event execution and cross-check the output with manual calculation after each run.

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Ex: For a queue simulation for a single-server bank system, customers arrive every 5 minutes, and the service takes 3 minutes on average. To verify the modeling process:

- Test the function generating arrivals and service times
- Verify intermediate outputs (e.g., arrival times, queue lengths).
- Manually calculate a few expected outcomes and compare them to the simulation results.

Verification & Validation of Models

Model Validation

- Building the right model
- Determine if the conceptual simulation model is an accurate representation.
- Valid if the output behavior is sufficiently accurate to fulfill the purpose.

Steps in a validation process:

- Empirically Test & compare with other models, e.g. analytical models
- Detail output data validation
  - Confidence Intervals
  - Statistical Studies

Model should be validated relative to the measures in the objective.
Ex: Simulating a bank with a single service counter. Validate whether the simulation accurately reflects the real bank operations.

- Collect Real-World Data
- Run simulations and compare with real-world data

What?

- Output analysis: examination of the data generated by a simulation
- Objective:
  1. Predict the performance of the system
  2. Compare the performance of two (or more) systems

The necessity of output analysis is :

- To figure out the number of observations required to achieve a desired error or confidence interval
- To estimate the standard error and/or confidence interval
- Challenges Arrival of subsequent packets may lack statistical independence i.e. no autocorrelation.
- Challenges: Initial conditions, e.g., the number of packets in a router at initial time would most likely influence the performance/delay of packets arriving later.

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Flowchart of the entire simulation loop can be given as:

Types of Simulations

Terminating Simulation

- Runs for some duration of time T_E, where E is a specified event that stops the simulation.
- Starts at time 0 under well-specified initial conditions.
- T_E may be known from the beginning or it may not
- Several runs may result in $T_E^1, T_E^2, \ldots$

Ex: Bank example: Opens at 8:30 am (time 0) with no customers present and 8 of the 11 teller working (initial conditions), and closes at 4:30 pm (Time T_E = 480 minutes).

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Non-Terminating Simulation

- Runs continuously or at least over a very long period of time.
- Runs for some analyst-specified period of time T_E
- Objective is to study the steady-state (long-run) properties of the system properties that are not influenced by the initial conditions of system, properties that are not influenced by the initial conditions of the model.

Ex: Examples: assembly lines that shut down infrequently, hospital emergency rooms, telephone systems, network of routers, Internet.

Whether a simulation is considered to be terminating or non-terminating depends on both:

- The objectives of the simulation study
- The nature of the system

Question / Scenario	**Type of Simulation**
Simulate the operations of a bank during working hours (9 AM to 5 PM)	t
Evaluate the average customer waiting time in a 24/7 call center	nt
Study the performance of an emergency department during a night shift	t
Analyze the long-term inventory levels in a warehouse	nt
Model the boarding and deboarding process of passengers at an airport gate for one flight	t
Assess the behavior of a power grid operating continuously throughout the year	nt
Simulate a car wash service for a single day to test new staff training	t
Measure the steady-state throughput of a production line that runs continuously	nt

Transient and Steady State Output Data

- Experimental observations for output analysis should be made at steady state, i.e. after a transient state of the stochastic simulation run.

Transient State

For a discrete-time stochastic process (e.g., Markov chain), the transient distribution at time t is a function of both the state i and the initial condition I_0, and it varies with time until the process possibly reaches a steady-state. The transient distribution is denoted as:

Note that the transient distribution can be different for each state i.

Steady State

If all outputs y and the initial condition I_i are given, then the state distribution $F_i(y)$ converges to a steady state distribution $F_i^{\infty}(y)$ as $i \longrightarrow \infty$.

Graphically we can show that as:

Random Nature of Simulation Output

- The model output consists of one or more random variables because the model is an input-output transformation, and the input variables are random variables.

Bias

The bias (or bias function) of an estimator is the difference between this estimator's expected value and the true value of the parameter being estimated. An estimator or decision rule with zero bias is called unbiased.

Suppose we have our measurement data paramterized by the variable A and we want to estimate $\hat{A}$. Given that our data has some unknown distribution $P(x|A)$, we define the bias of $\hat{A}$ relative to A as

$$\text{Bias}(\hat{A}, A) = E[\hat{A} - A]$$

with $E(\cdot)$ denoting the expectation over the distribution. Subsequently, an estimator is said to be unbiased iff

$$E[\hat{A}] = A$$

For illustration, see the two PDFs shown below:

For any good probabilistic estimator, the desirable characteristic is No bias.

Performance Measures for Stochastic Simulations

Conisder the estimation of a parameter θ for a simulated system. For

- Discrete time data:
- Continuous time data:

Single Point Estimation for Discrete Data

The estimation of the parameter θ given discrete data Y_i is given as:

In case the estimator is unbiased, one should get :

Single Point Estimation for Continuous Data

The estimation of the parameter θ given discrete data $Y(t)$ is given as:

In case the estimator is unbiased, one should get:

What?

Confidence interval estimation is a statistical technique used to estimate the range within which a parameter (like a mean or proportion) is likely to lie, with a certain level of confidence.

A confidence interval is typically of the form:

The margin of error is then computed as

If the population standard deviation is known

The critical z-value, $z^{\star}$ corresponds to the number of standard deviations away from the mean needed to capture a certain confidence level in a standard normal distribution (mean = 0, std. dev. = 1).

It is computed as

If the population standard deviation is unknown

The critical t-value, $t^{\star}$ corresponds to the number of standard deviations away from the mean needed to capture a certain confidence level in a standard t-distribution with degrees of freedom: n-1.

It is computed as

Performance Measures: Confidence Interval Estimation

Critical values of *t* for two-tailed tests

Significance level (α)

Degrees of freedom (*df*)	.2	.15	.1	.05	.025	.01	.005	.001
1	3.078	4.165	6.314	12.706	25.452	63.657	127.321	636.619
2	1.886	2.282	2.920	4.303	6.205	9.925	14.089	31.599
3	1.638	1.924	2.353	3.182	4.177	5.841	7.453	12.924
4	1.533	1.778	2.132	2.776	3.495	4.604	5.598	8.610
5	1.476	1.699	2.015	2.571	3.163	4.032	4.773	6.869
6	1.440	1.650	1.943	2.447	2.969	3.707	4.317	5.959
7	1.415	1.617	1.895	2.365	2.841	3.499	4.029	5.408
8	1.397	1.592	1.860	2.306	2.752	3.355	3.833	5.041
9	1.383	1.574	1.833	2.262	2.685	3.250	3.690	4.781
10	1.372	1.559	1.812	2.228	2.634	3.169	3.581	4.587
11	1.363	1.548	1.796	2.201	2.593	3.106	3.497	4.437
12	1.356	1.538	1.782	2.179	2.560	3.055	3.428	4.318
13	1.350	1.530	1.771	2.160	2.533	3.012	3.372	4.221
14	1.345	1.523	1.761	2.145	2.510	2.977	3.326	4.140
15	1.341	1.517	1.753	2.131	2.490	2.947	3.286	4.073
16	1.337	1.512	1.746	2.120	2.473	2.921	3.252	4.015
17	1.333	1.508	1.740	2.110	2.458	2.898	3.222	3.965
18	1.330	1.504	1.734	2.101	2.445	2.878	3.197	3.922
19	1.328	1.500	1.729	2.093	2.433	2.861	3.174	3.883
20	1.325	1.497	1.725	2.086	2.423	2.845	3.153	3.850
21	1.323	1.494	1.721	2.080	2.414	2.831	3.135	3.819
22	1.321	1.492	1.717	2.074	2.405	2.819	3.119	3.792

Considering that the population mean is not known, first a t-statistic is computed to check if assumed sample mean can be used. The t-statistic is given for dof = n-1 is given as:

Then,

- Compare this t-value with $t^{\star}$
  1. if $|t| > t^{\star}$, **reject** H_0
  2. Else fail to reject H_0.

Ex: A sample of 16 students has an average test score of 78, with a standard deviation of 10. Test if this differs significantly from a hypothesized population mean of 75 with CI 95%.

Ex Consider a call center system simulation where the average waiting time in minutes is computed as an output observation. After one simulation run, the waiting time is given as follows:

Customer	**Waiting Time (min)**
1	5.1
2	6.2
3	4.8
4	5.9
5	6.3
6	4.5
7	5.7
8	5.8
9	6.1
10	5.6

Compute a 95% Confidence Interval (CI) for the mean waiting time, assuming the population standard deviation is known.

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