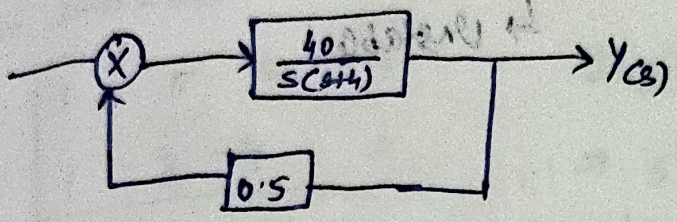
Assignment - 3

A1)



Closed loop T/F $\Rightarrow$ $T(s) = \dfrac{G(s)}{1 + G(s)H(s)}$

$$T(s) = \frac{\frac{40}{s(s+4)}}{1 + \frac{20}{s(s+4)}} = \frac{40}{s^2+4s+20}$$

- Sensitivity wrt forward path transfer function

$$S_G = \frac{1}{1+G(s)H(s)} = \frac{1}{1 + \frac{40}{s(s+4)} \times 0.5} = \frac{s^2+4s}{s^2+4s+20}$$

Substitute $s = j1.3$

$$S_G = \frac{(j1.3)^2 + 4(j1.3)}{(j1.3)^2 + 4(j1.3) + 20} = \frac{-1.69 + j5.2}{18.31 + j5.2}$$

$\boxed{|S_G| \approx 0.287}$

- Sensitivity wrt feedback path transfer function

$$S_H = \frac{-G(s)}{1+G(s)H(s)} = \frac{\frac{-40}{s(s+4)}}{1 + \frac{40}{s(s+4)} \times 0.5}$$

$$S_H = \frac{-40}{s(s+4)+20}$$ Substituting $s = j1.3$ we get

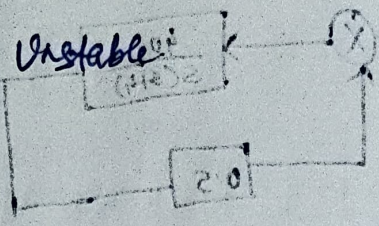
$\boxed{|S_H| \approx 2.10}$

A2) (i) $s^3+4.5s^2+3.5s+16=0$

s^3	1	7/2
s^2	9/2	16
s^1	-1/18	0
s^0	16	

Sign changed twice

↳ Unstable



(ii) $s^5+1.5s^4+2s^3+4s^2+5s+10=0$

s^5	1	2	5
s^4	3/2	4	10
s^3	-2/3	-5/3	0
s^2	-9/4	10	
s^1	-25/9	0	
s^0	10		

Sign Changed twice

↳ Unstable

(iii) $s^5+s^4+2s^3+2s^2+11s+10=0$

s^5	1	2	11
s^4	1	2	10
s^3	e	1	0
s^2	-11	10	
s^1	1	0	
s^0	10		

Sign changed twice

↳ Unstable.

(iv) $s^6 + s^5 + 5s^4 + 3s^3 + 2s^2 - 4s - 8 = 0$

All the coefficients are not +ve, so the system is unstable

s^6	1	5	2	−8
s^5	1	3	−4	0
s^4	1	3	−4	
s^3	2	3		
s^2	$3/2$	−4		
s^1	$25/3$	0		
s^0	−4			

Auxiliary eq^n

$\Rightarrow s^4 + 3s^2 - 4 = 0$

$\frac{dA}{ds} \hookrightarrow 4s^3 + 6s$

(v). $\Delta(s) = 1 + G(s)H(s)$

$= s(s+2) + e^{-sT}$

$= s(s+2) + \frac{1 + sT}{1 + sT}$

$= s(s+2)(1+sT) + (1 - sT)$

$= (s^2 + 2s)(1+sT) + (1 - sT)$

$= s^2 + 2s + s^3T + 2s^2T + 1 - sT$

$= s^3T + s^2[2T + 1 - \cdot] + [2 - T]s + 1$

s^3	T	$2 - T$
s^2	$2T + 1$	1
s^1	$\frac{2T + T + 1}{2T + 1}$	0
s^0	1	

System is stable when $T > 0$

(vii) $s^3+10s^2+18s+k=0$ about $s=-1$

$s'=s'-1$

$\Rightarrow (s'-1)^3+10(s'-1)^2+18(s'-1)+k=0$

$\Rightarrow (s')^3-3(s')^2+3s'-1+10[(s')^2-2s'+1]$

$+18[s'-1]+k=0$

$\Rightarrow (s')^3+7(s')^2+s'+(k+27)=0$

s^3	1	1
s^2	7	$k+27$
s^1	$\frac{-k-20}{7}$	0
s^0	$k+27$	

System is stable when $-27<k<-20$