Visualizing electric fields

Physics 1051 Winter 2006

Department of Physics & Physical Oceanography
Memorial University of Newfoundland
Prof. Kris Poduska

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Introduction

An electric field exists in the space around a charged object. When another charged object is placed in this electric field, an electric force acts on it. This tutorial is designed to help you explore the effects of these electric forces and how they vary throughout the space surrounding charged objects. In it, you will work with a variety of different visual representations of electric fields, as well as the mathematics on which they are based.

There are three layers of the tutorial:

Layer 1. Seeing the bare basics. Observe the visual representations.
Layer 2. Probing a bit deeper. Adjust numerical values to change the visual representations you've just observed.
Layer 3. Working towards mastery. Learn to create visual representations on your own.

Each successive layer will require more thought and effort on your part, but the reward will be a much more thorough understanding of electric fields.

The physics described here is based on the topics covered in recent lectures, and in Sections 23.1-23.4 & 23.6 of Serway and Beichner's "Physics for Scientists and Engineers".

If you have questions about *Mathematica* (the program which runs this tutorial), stop in to one of the tutorial sessions in C2039 and talk to Dr. Poduska or a lab instructor (Kelly Shorlin or John Wells).

Layer 1: The Bare Basics

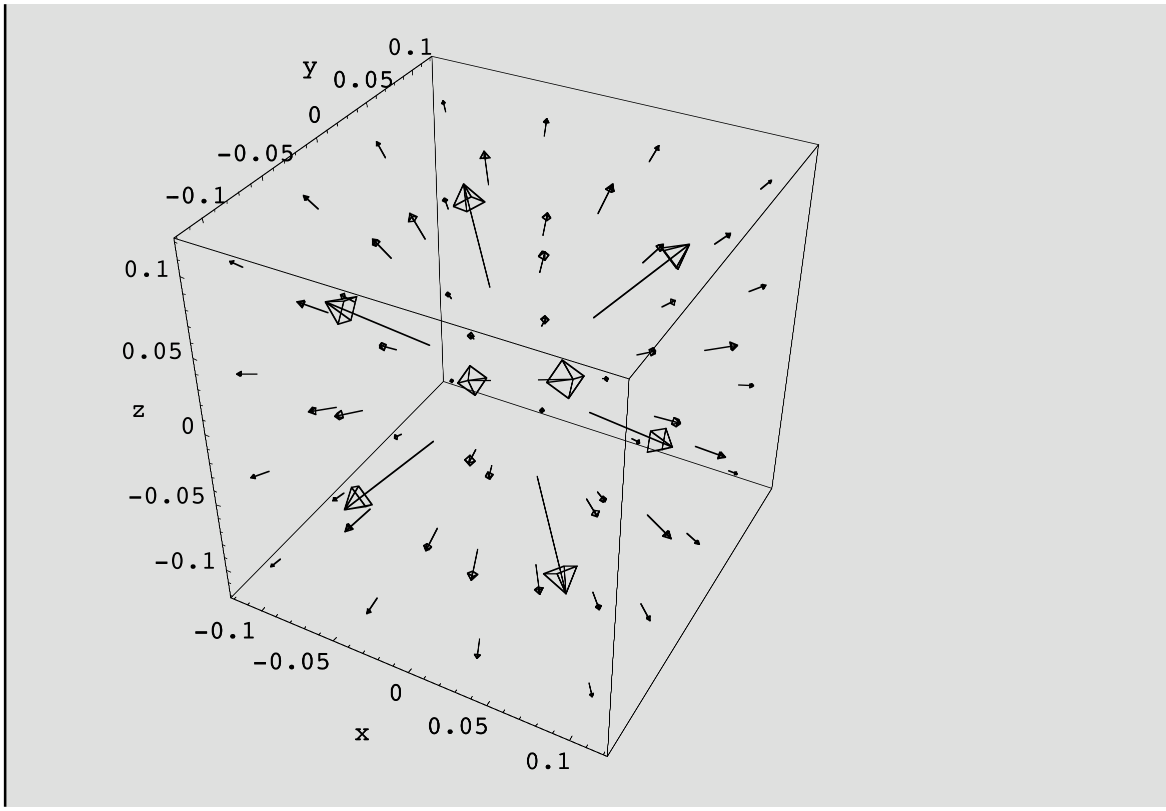
The simplest example: the electric field due to point source charge

Vector fields

The electric field at a point in space is defined as the electric force (F_e) acting on a positive test charge (q) placed at that point, divided by the magnitude of the test charge.

$$\vec{E} = \frac{\vec{F}_e}{q} \tag{1}$$

One way to picture the electric field due to a point source charge is with a vector field:



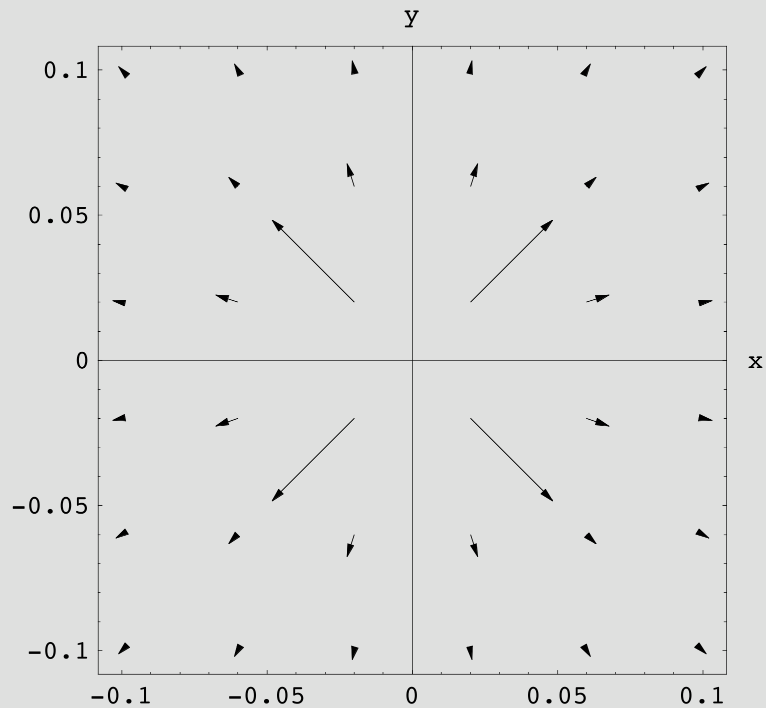
The vector field shown above is a three-dimensional representation of the electric field originating from a point source charge at the center of the cube. Each arrow in the vector field shows the magnitude and direction of the electric force that would act on a positive test charge positioned at its tail. In this example, you can see that all arrows are pointing away from the point source charge (meaning that the source charge is a positively charged object). It's also true -- but a bit harder to see -- that the arrows that start closer to the source charge are much longer than those that start further away (meaning that the magnitude of the electric field -- and hence the magnitude of the electric force that would act on another charged object at that point in space -- is much larger close to the source charge).

This picture of an electric vector field is consistent with the mathematical expression for the electric field due to a point source charge that we've seen in class:

$$\vec{E} = k\,\frac{q}{r^2}\,\hat{r} \tag{2}$$

In this equation, E represents the electric field, *k* is Coulomb's constant, *q* is the magnitude of the source charge that generates the field, and *r* is the radial distance from the point charge. Remember that electric field is a vector quantity; $\hat{r}$ means that the electric field of a point source charge is always directed radially inward or outward.

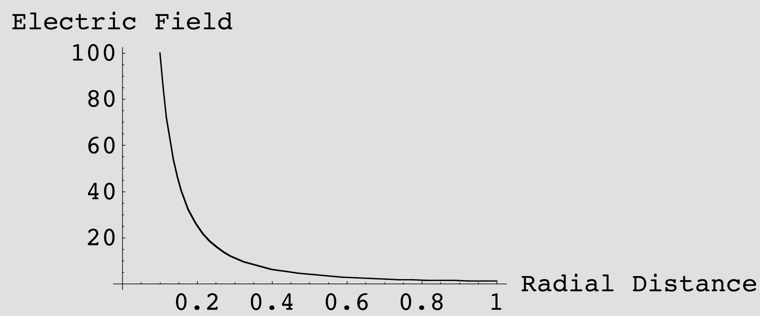
Although an electric field is a three-dimensional (3D) entity, at a practical level, it is easier to look at the electric vector field in a two-dimensional (2D) slice that contains the source charge.



Each arrow in the vector field shows the magnitude and direction of the electric force that would act on a positive test charge positioned at its tail. In this two-dimensional represenation, it is much easier to see that all arrows are pointing away from the point source charge (meaning that the source charge is a positively charged object), and that the arrows that start closer to the source charge are much longer than those that start further away (meaning that the magnitude of the electric field -- and hence the magnitude of the electric force that would act on another charged object at that point in space -- is much larger close to the source charge).

The vector field shown above is a two-dimensional representation of the electric field, and it shows only those points that lie in a plane that contains the source charge. It doesn't matter which plane we pick, as long as it contains the source charge; Equation 2 assures us that the electric field only depends on the radial distance, in any direction away from the source charge.

This discussion of dimensions brings up another possible way to picture the electric field around a point source charge. Since the magnitude of the electric field changes with the radial distance *r* from the source charge, we can also represent this change throughout space with a one-dimensional plot.

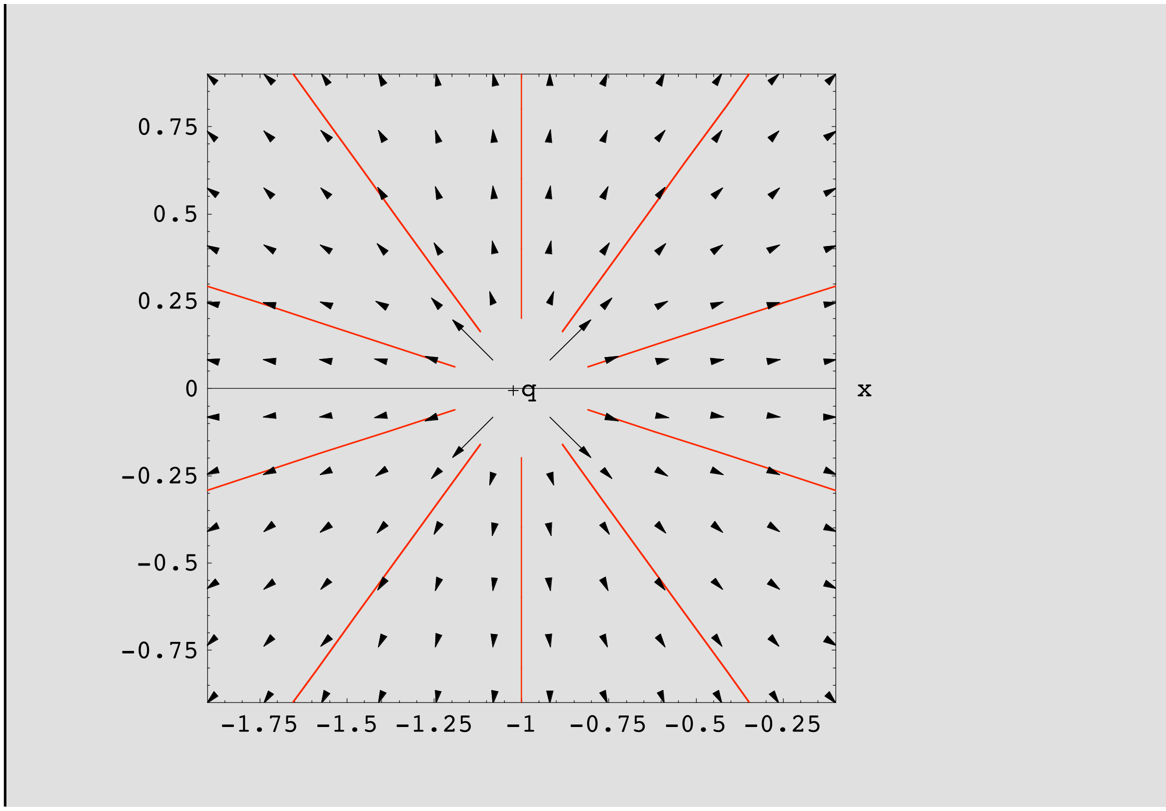


Just as the vector field showed us that the magnitude of the electric field (i.e. the length of the vector) is much larger closer to the source charge than further away, so too does this plot of electric field versus radial distance.

One thing that's easy to see with this plot of electric field versus distance is precisely how quickly the magnitude of the electric field decreases as the distance from the source charge increases. At a radius of 0.2 on the plot above, the magnitude of the electric field is 20; at a radius of 0.4, the magnitude of the electric field has decreased to 5. This agrees with the mathematical expression above (Equation 1), which tells us that if we move twice as far from the source charge, the magnitude of the electric field is only one fourth as large (that's a decrease of 75%).

■ Electric field lines

The vector field we've just seen is only one way to describe how the electric field varies throughout space. Another way of visualizing electric field patterns is to draw lines that follow the same direction as the electric field vector.

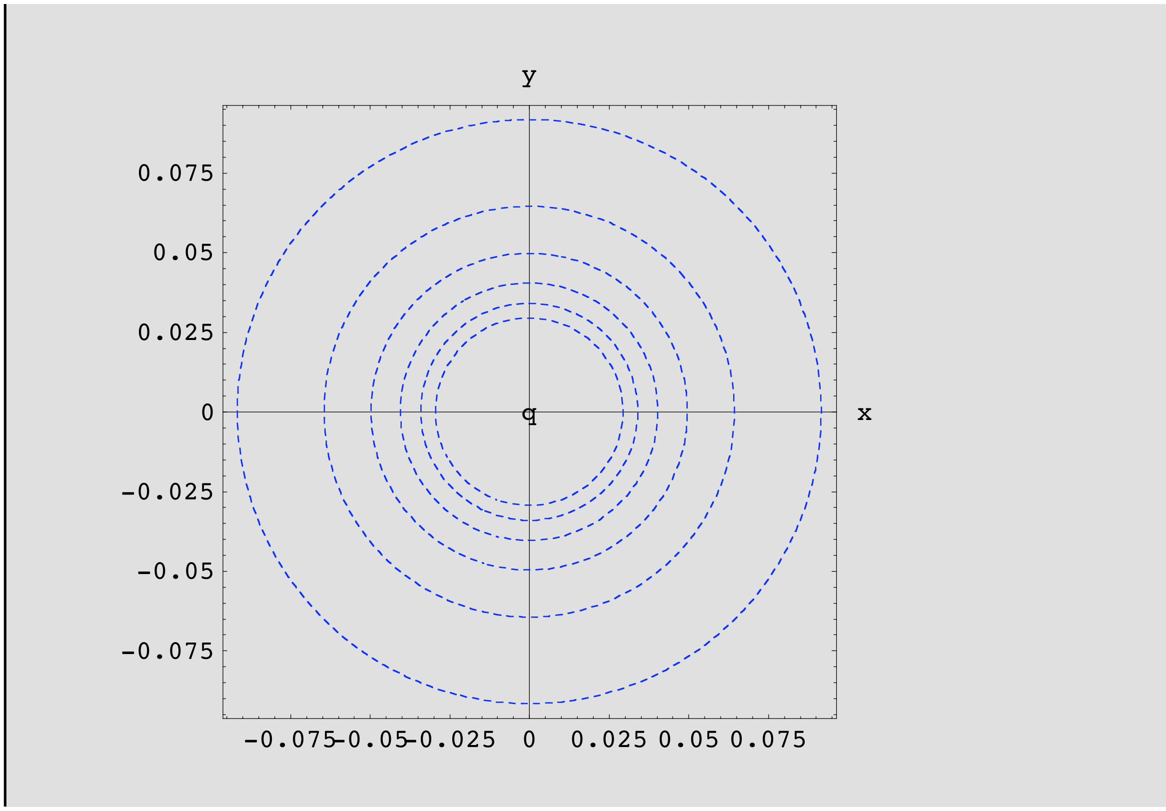


How can one determine the magnitude and direction of the electric field at any one point in space from a plot of electric field lines? Here are the two key points to remember. First, the electric field direction at any one point in space is given by the direction tangent to the electric field line at that point. Second, the closer together the lines are in a given region of space, the larger the magnitude of the electric field in that region.

In this case, you can see that all lines are directed away from the point source charge (meaning that the source charge is a positively charged object), and that the lines are more densely arranged closer to the source charge than those further away (meaning that the magnitude of the electric field -- and hence the magnitude of the electric force that would act on another charged object at that point in space -- is much larger close to the source charge).

■ Lines of constant electric field magnitude

There is yet another kind of plot which is useful for depicting how an electric field varies throughout space: a contour plot. This kind of representation is like a topographical map which shows lines of constant altitude; in this case, the lines correspond to lines of constant electric field magnitude.



In this case, you can see that all lines of constant electric field magnitude are circles centred on the point source charge (meaning that the magnitude of the electric field depends on the radial distance from the source charge), and that the lines are more densely spaced closer to the source charge than those further away (meaning that the magnitude of the electric field -- and hence the magnitude of the electric force that would act on another charged object at that point in space -- increases rapidly close to the source charge).

Two things about the electric field are difficult to see from this plot. First, is the source charge is a positively charged or negatively charged object? The only way to tell is to look at the labels on the contour lines to see whether they are positive electric fields (indicating a positive source charge) or if the electric fields are negative (indicating a negative point source charge). Second, the plot itself doesn't show us the direction of the electric field directly. However, we can reason through the direction without much trouble. We saw in the vector field and field line plots that the electric field is always directed radially inward or outward from the point source charge. This means that the electric field lines and vectors are always perpendicular to the lines of constant electric field magnitude, pointing radially inward (negative source charge) or outward (positive source charge).

Why go to the trouble of a contour plot if electric field lines or vector fields are easier to decode? I show this kind of plot to you now because you'll soon see it again when we talk about electric potential, and we will be relating it back to the other kinds of plots you've seen in this tutorial. For now, though, we'll spend the rest of this tutorial with vector fields and field lines.

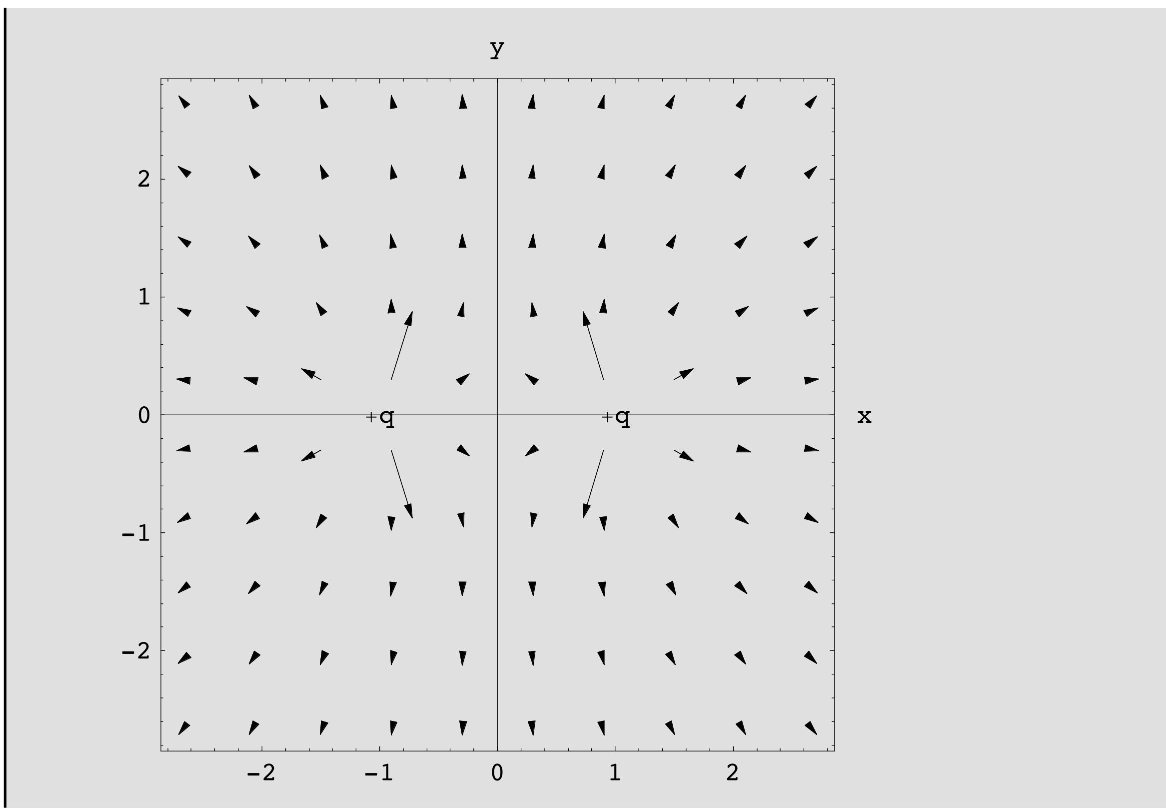
Electric fields due to multiple point source charges

Now that we've seen a few different ways to visualize electric fields, let's apply these representations to electric fields that result from a collection of two or more point source charges.

In class, we've seen that at any point in space, the total electric field due to a collection of point source charges is equal to the vector sum of the electric fields of the individual charges. In other words, we can use Equation 1 to determine the electric field due to the each of the point source charges individually, and then just add the results together to get the total electric field. Remember that the electric field due to a single point source charge is still a vector quantity, and that's why we have to make sure that we use vectors throughout to keep track of direction, as well as magnitude of the electric field at each point in space. We can write the vector sum in a compact format

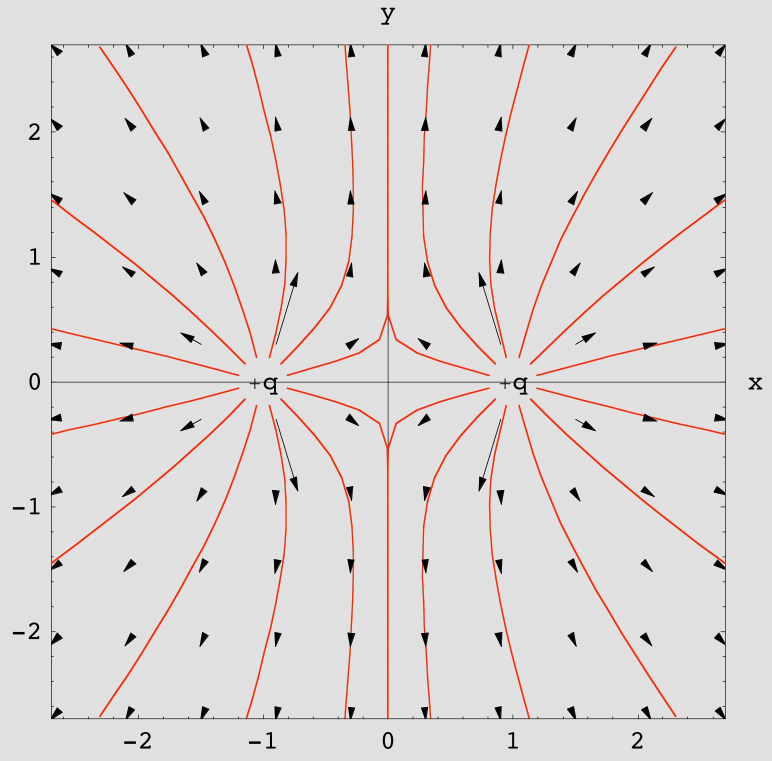
$$E = k \sum_{i} \frac{q_i}{r_i^2} \hat{r} \qquad (3)$$

Let's look at the electric vector field for two point source charges with the same magnitude of charge and the same sign.



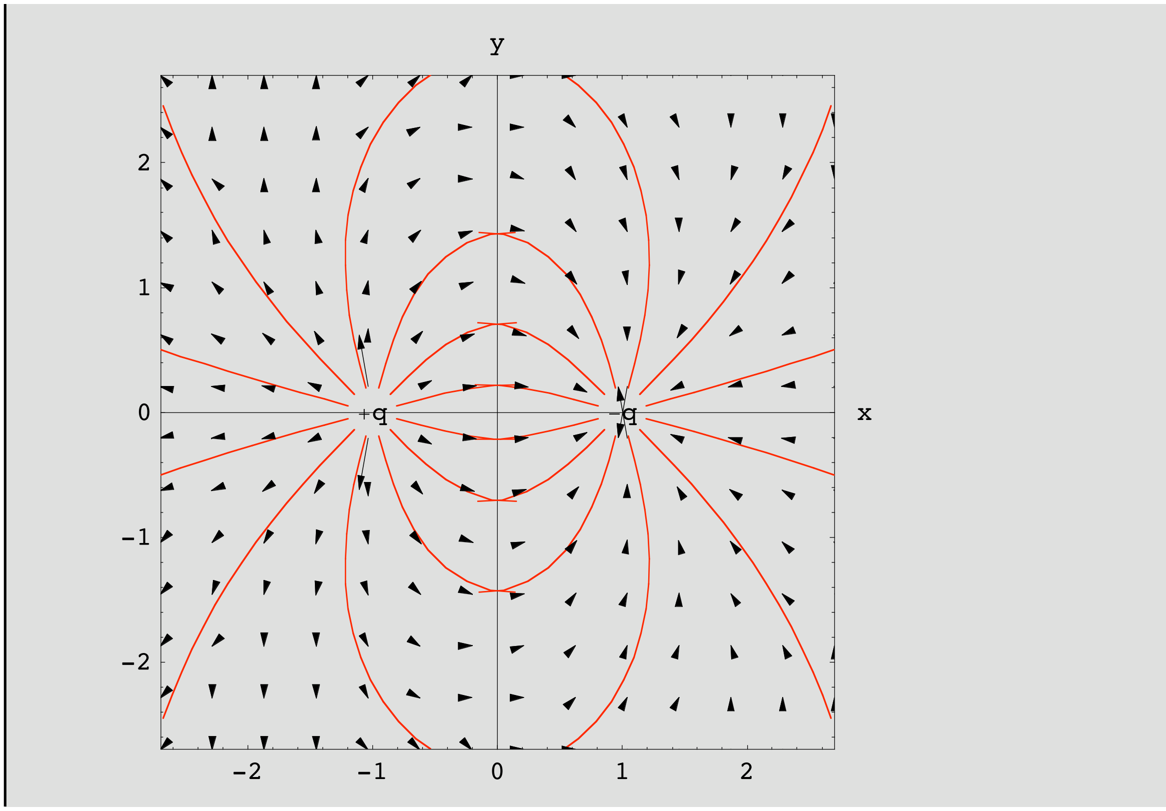
There are a few features of this plot worth mentioning. First, notice that the electric field vectors are no longer the same length for the same radial distance around one of the source charges. For example, the electric field is larger in the region directly between the two source charges than it is on the far sides of each source charge.

Compare this electric vector field with electric field lines:



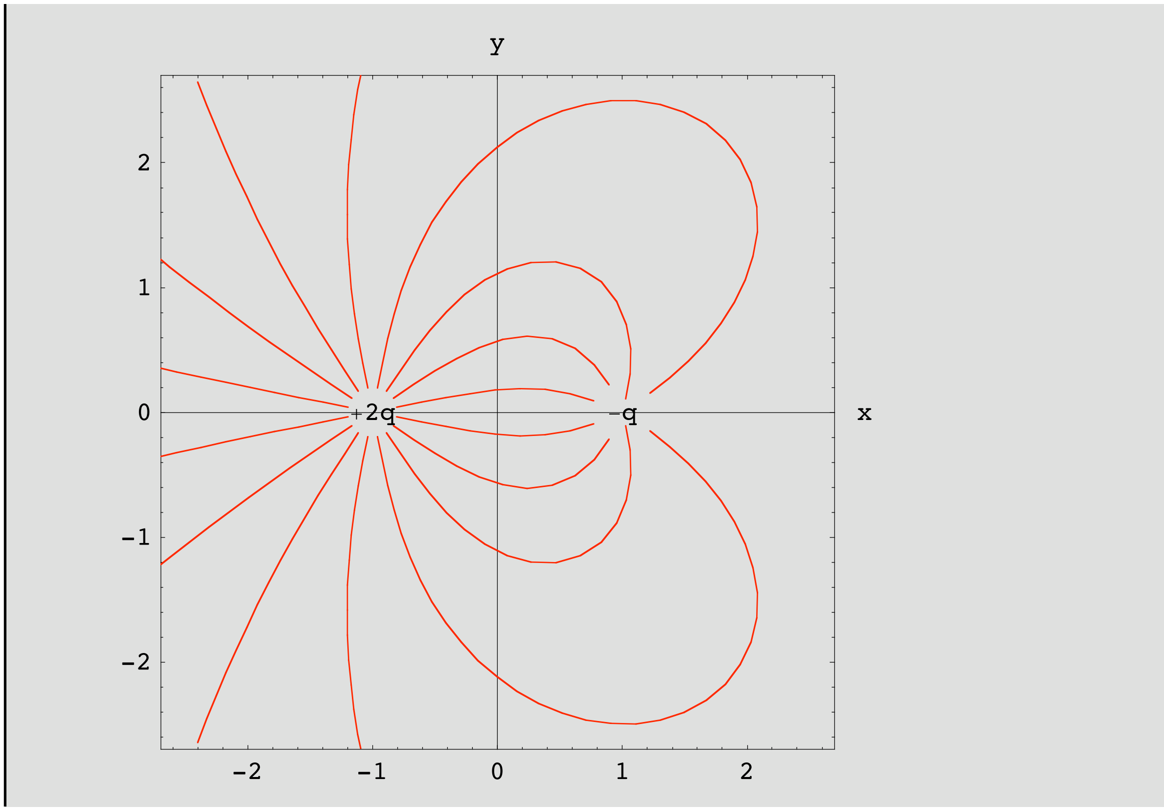
Within a given region of space, the more closely spaced the lines, the greater the magnitude of the electric field.

What happens if the charges have the same magnitude, but opposite signs?



In this representation, the vectors point away from the positive source charge and towards the negative source charge, consistent with what we've seen with electric vector fields for single point source charges. Also, notice that electric field lines always originate from positive source charges and terminate at negative source charges.

Just one more example, a bit more complicated this time: two point source charges, with different magnitudes and opposite charges. This time, we'll just look at the electric field lines.



The same features apply, but now notice that the number of lines that begin (or end) at a given source is proportional to the relative magnitude of that particular source charge.

Moving to three or more point source charges isn't any more difficult. The total electric field at any point in space is still just the vector sum of the electric fields due to each point source charge, and the electric field can still be represented in terms of vector fields or electric field lines.

Layer 2: Probing a bit deeper

A simple example: the electric field due to point source charge

■ Changing the sign of the source charge

A point source charge produces an electric field that is directed radially inward or outward from the source charge, and its magnitude depends on the distance from the source charge (r) and the magnitude of the source charge itself (q). Here's an equation we saw earlier:

$$\vec{E} = k \frac{q}{r^2} \hat{r} \quad (4)$$

How does the appearance of the vector field change we change the sign of the source charge? Try it yourself! The input cell below provides a way to change the value of q. Start by clicking on the input box, set q = -1 on the first line, and then press "Enter" (not "return") to see the affect this has on the vector field. (Note: The commands in the rest of the input box set up the equation and plotting parameters. You can see the value of q that you set will be used in an expression very similar to Equation 4. You won't need to change anything other than the value of q during this part of the tutorial. However, if you're curious and would like to know more, talk to one of your lab instructors.)

```
(* First, we need to start with a clean slate and
  clear all previous definitions for the variable q *)

ClearAll["Global`*"]

(* NEXT, ENTER YOUR VALUE FOR THE SOURCE CHARGE BELOW *)

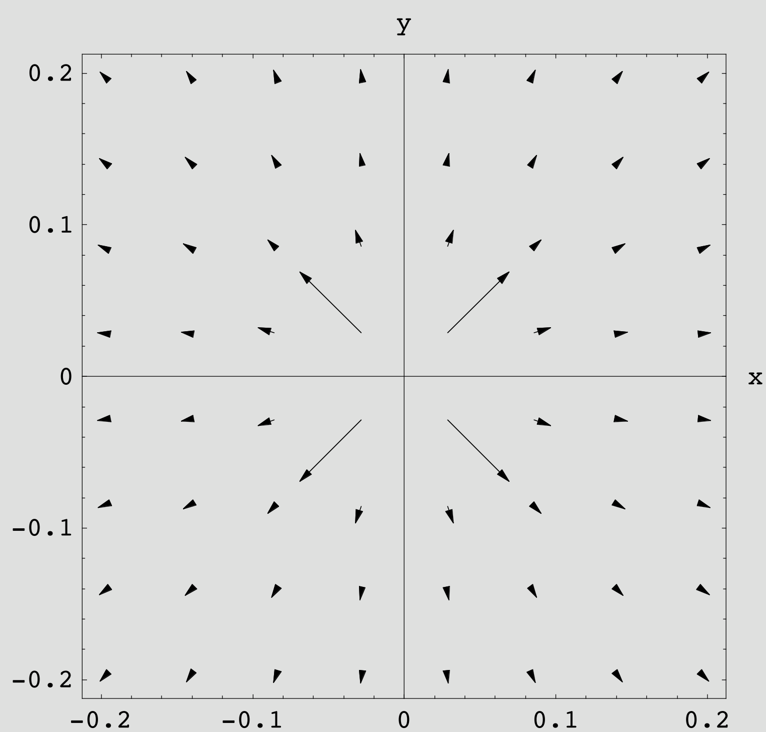
q = 1;

(* Finally, the rest of the commands use your value
  of q in Equation 4 to find the value of the electric
  field at different points, and then plots the result *)

k = 1;
PlotGradientField[
```

$$-\left(\frac{k\,q}{\sqrt{(x)\,\hat{}\,2 + (y)\,\hat{}\,2}}\right),$$

```
  {x, -.2, .2}, {y, -.2, .2}, PlotPoints → 8,
  Axes → True, Frame → True, AxesLabel → {"x", "y"}];
```



For our example above, you see that the arrows nearest the source charge cross each other when you set q = -1. This looks a bit funny, but there's no problem with our simulation. As we saw before, each arrow in the vector field shows the magnitude and direction of the electric force that would act on a positive test charge positioned at its *tail*. Arrows that start closer to the source charge are longer relative those that start further away, in direct proportion to the magnitude of the electric field. We could change our picture to avoid these arrows crossing each other by choosing to make *all* of the arrow proportionally shorter, but it wouldn't change the actual magnitude of the electric field at any particular point.

We've also seen that the magnitude of the electric field changes with the radial distance r from the source charge, so we can also represent this change throughout space with a one-dimensional plot. How does the appearance of the vector field change we change the sign of the source charge? Try it yourself! The input cell below provides a way to change the value of q. Start by clicking on the input box, set q = -1, and then press "Enter" (not "return") to see the affect this has on the electric field as a function of radial distance.

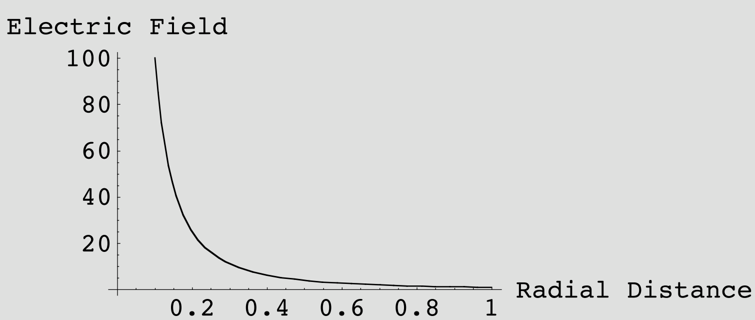
```
(* First, we need to start with a clean slate and
  clear all previous definitions for the variable q *)

ClearAll["Global`*"]

(* NEXT, ENTER YOUR VALUE FOR THE SOURCE CHARGE BELOW *)

q = 1;

(* Finally, the rest of the commands use your value of q to
  calculate the how the electric field varies with the radial
  distance from the source charge,and then plots the result *)
k = 1;
epoint[r_] = k q / r^2;
Plot[epoint[r], {r, 0.1, 1},
  AxesLabel → {"Radial Distance" , "Electric Field" }];
```



By changing to a negative source charge, you see that the electric field has a negative value, just as Equation 4 says it should. You can also try changing the magnitude of the source charge (for example, let q = -2). What happens to the shape of the curve? What happens to the values of the electric field? These observations are also consistent with Equation 4. If you don't understand why, seek out help from one of the lab instructors before proceeding.

Electric fields due to multiple point source charges

Recall that the total electric field due to a collection of point source charges is equal to the vector sum of the electric fields of the individual charges. Remember that the electric field due to a single point source charge is still a vector quantity, and that's why we have to make sure that we use vectors throughout to keep track of direction, as well as magnitude of the electric field at each point in space.

$$E = k \sum_{i} \frac{q_i}{r_i^{2}} \hat{r} \tag{5}$$

Let's look at the electric vector field for three point source charges each with different charge magnitudes and signs. Try it yourself! The input cell below provides a way to change the value of q. Start by clicking on the input box, set *q1* = 1, *q2* = 2, and *q3* = -1, then press "Enter" (not "return") to see the affect this has on the vector field.

```
(* First, we need to start with a clean slate and
  clear all previous definitions for the variable q *)

ClearAll["Global`*"]

(* NEXT, ENTER YOUR VALUE FOR THE THREE SOURCE CHARGES BELOW *)

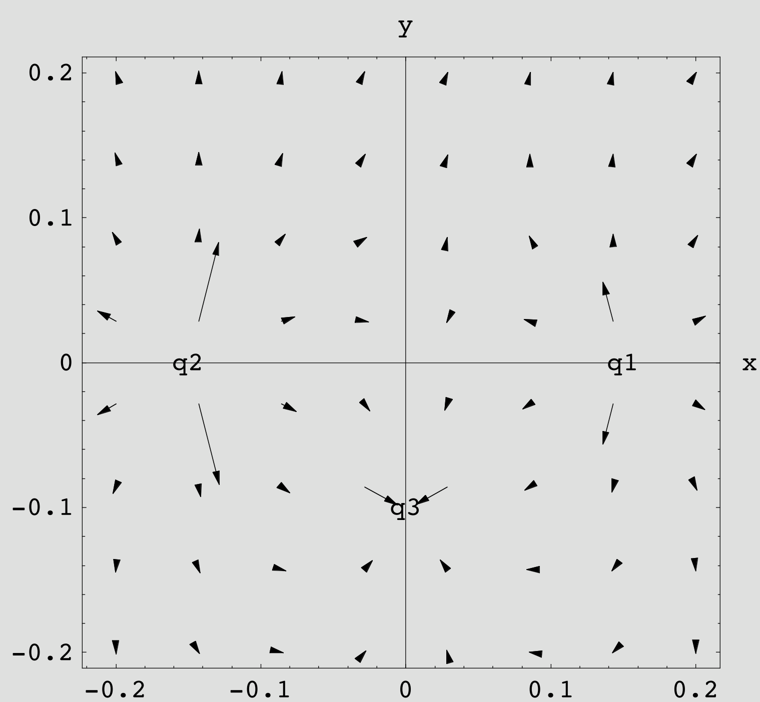
q1 = 1;
q2 = 2;
q3 = -1;

(* Finally, the rest of the commands use your values for q1,
 q2, and q3 in Equation 5, and then plots the result *)

k = 1;
Show[PlotGradientField[

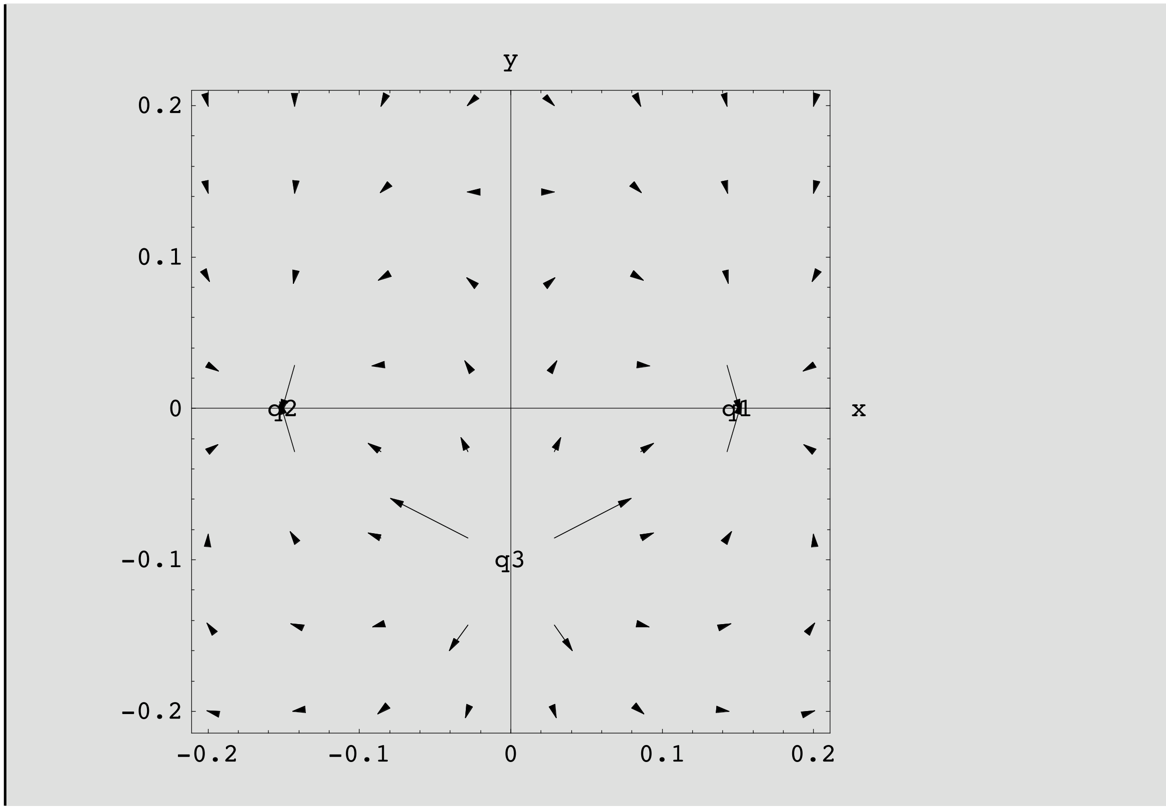
   -(k q1 / Sqrt[(x - 0.15)^2 + (y)^2] +
      k q2 / Sqrt[(x + 0.15)^2 + (y)^2] + k q3 / Sqrt[(x)^2 + (y + 0.1)^2]),

   {x, -.2, .2}, {y, -.2, .2}, PlotPoints → 8,
   Axes → True, Frame → True, AxesLabel → {"x", "y"}],
  Graphics[{Text["q1", {.15, 0}]}],
  Graphics[{Text["q2", {-.15, 0}]}],
  Graphics[{Text["q3", {0, -.1}]}]];
```



Because the vectors point away from positive source charges and towards the negative source charges, it is relatively easy to determine which of the three charges are positive. Since the length of the vectors is proportional to the magnitude of the total electric field at that point, in this example it is also easy to tell which charges are largest and smallest.

Now, try putting in other combinations of charge magnitudes and signs in the input cell above to see if you can reproduce the figure below.



If you have difficulties with this exercise, talk to one of the lab instructors before proceeding.

Layer 3: Working towards mastery

You've seen how electric vector fields change with the magnitude and signs of the source charges. Now you'll have the opportunity to change the positions of the source charges as well. The input cell below provides a way to change the values of charges (*q1, q2, q3*) and positions ({*x1,y1*}, {*x2,y2*}, {*x3,y3*}). Start by clicking on the input box to set the position of source charge 1 to be (*x1* = 0.15, *y1* = 0), source charge 2 at (0,0) and source charge 3 at (0,-0.15) and then press "Enter" (not "return") to see the affect this has on the vector field.

```
(* First, we need to start with a clean slate and
  clear all previous definitions for the variable q *)

ClearAll["Global`*"]

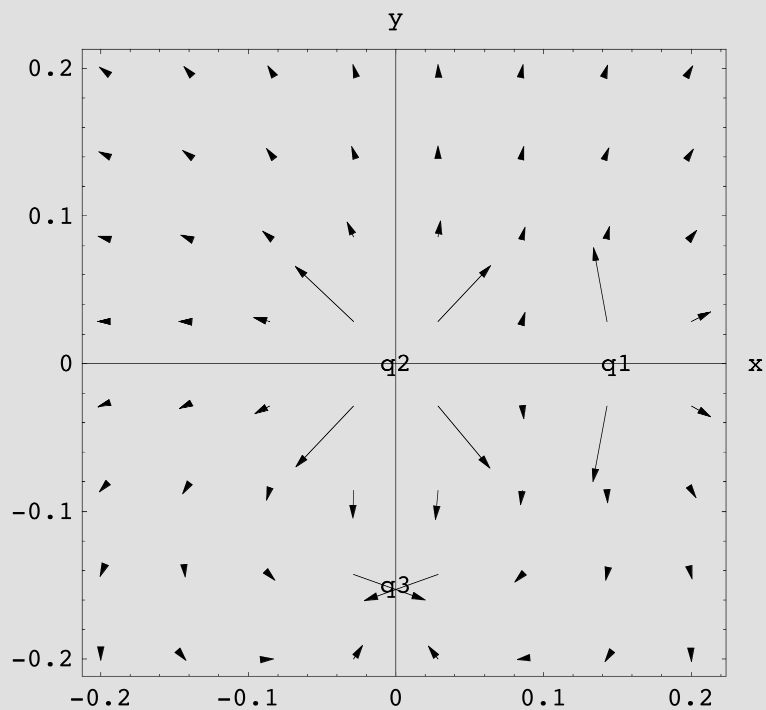
(* NEXT, ENTER YOUR VALUE FOR THE THREE
  SOURCE CHARGES AND THEIR x and y COORDINATES BELOW *)
q1 = 1;
q2 = 2;
q3 = -1;
x1 = 0.15;
y1 = 0;
x2 = -0;
y2 = 0;
x3 = 0;
y3 = -0.15;

(* Finally, the rest of the commands use your values for q1,
 q2, and q3 in Equation 5, and then plots the result *)

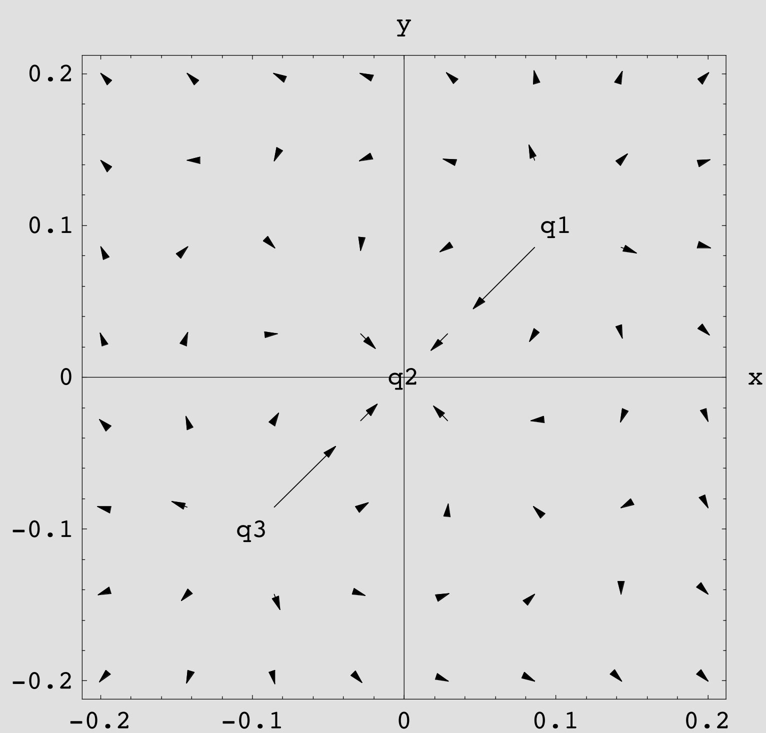
k = 1;
Show[PlotGradientField[

   -(k q1 / Sqrt[(x - x1) ^2 + (y - y1) ^2] +
      k q2 / Sqrt[(x - x2) ^2 + (y - y2) ^2] +
      k q3 / Sqrt[(x - x3) ^2 + (y - y3) ^2]),

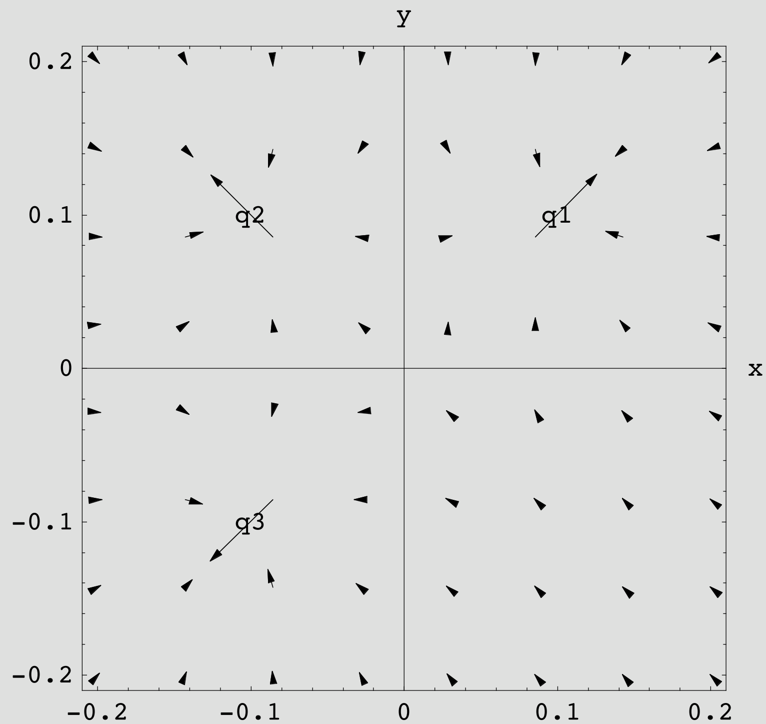
   {x, -.2, .2}, {y, -.2, .2}, PlotPoints → 8,
   Axes → True, Frame → True, AxesLabel → {"x", "y"}],
  Graphics[{Text["q1", {x1, y1}]}],
  Graphics[{Text["q2", {x2, y2}]}],
  Graphics[{Text["q3", {x3, y3}]}]] ;
```



Now, try putting in other combinations of charge magnitudes, signs, and positions in the input cell above to see if you can reproduce the figure below.



..and one more to try:



Finally, we'll extend these explorations to electric field lines. We've saved these for last because the commands that we need to draw field lines with *Mathematica* are quite complicated. Don't panic! You won't need to learn (or enter) these commands yourself; as before, you will just adjust the magnitudes and signs of the source charges. To make things even simpler, most of the gory details are hidden from view. (If you'd like to see them, ask a lab instructor to show you how to "open" the cells below.)

Here are a series of exercises for you to try. If you have questions, please ask a lab instructor for assistance.

(1) Let *q1* and *q2* be positive, but change their relative magnitudes. What happens to the number of electric field lines around each source charge?

(2) Compare the electric fields lines corresponding to the situation *q1* = *q2* = 1 with the electric field lines for *q1* = *q2* = 4. What happens to the number of electric field lines around each source charge? What does this tell you about how *Mathematica* is calculating the number of electric field lines to use?

(3) Now let *q1* = -1 and let *q2* = 1. What happens to the number of electric field lines around each source charge? How are the shapes of the field lines different than when *q1* = *q2* = 1?

(4) Finally, let *q1* be any negative number, and let *q2* be any positive number such that |*q2*| > |*q1*|. What happens to the number of electric field lines around each source charge? How are the shapes of the field lines affected?

```
(* First, we need to start with a clean slate and
  clear all previous definitions for the variable q *)

ClearAll["Global`*"]

(* NEXT, ENTER YOUR VALUES FOR THE SOURCE CHARGES *)

q1 = 1;
q2 = 1;

(* FINALLY,
 TO RUN THE COMMANDS TO CALCULATE AND PLOT THE ELECTRIC FIELD,
 CLICK ENTER IN THIS INPUT BOX, THEN CLICK ENTER ON THE "EMPTY"
  SPACE BELOW (BETWEEN THIS INPUT BOX AND THE PLOTTING BOX).*)
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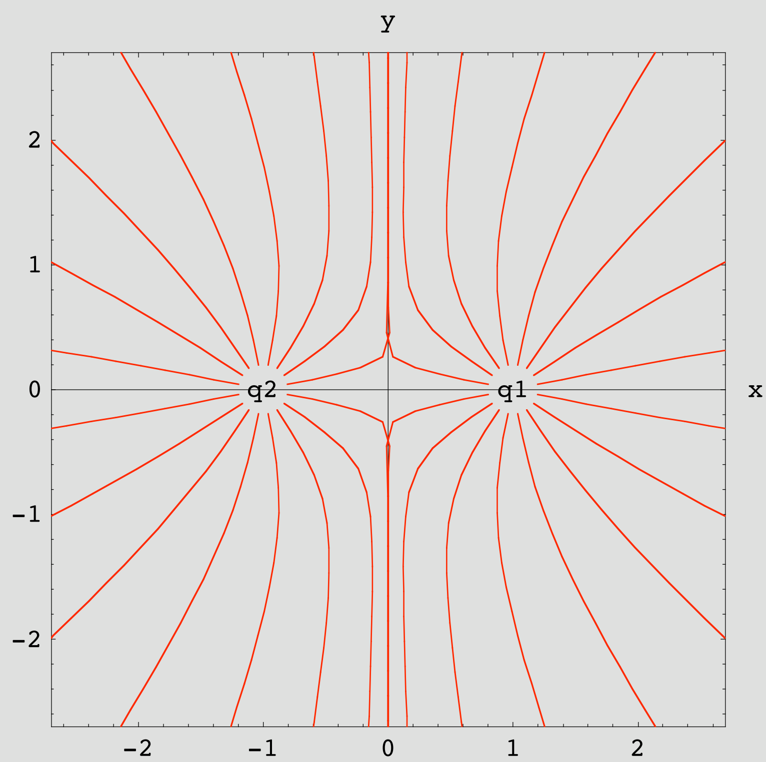
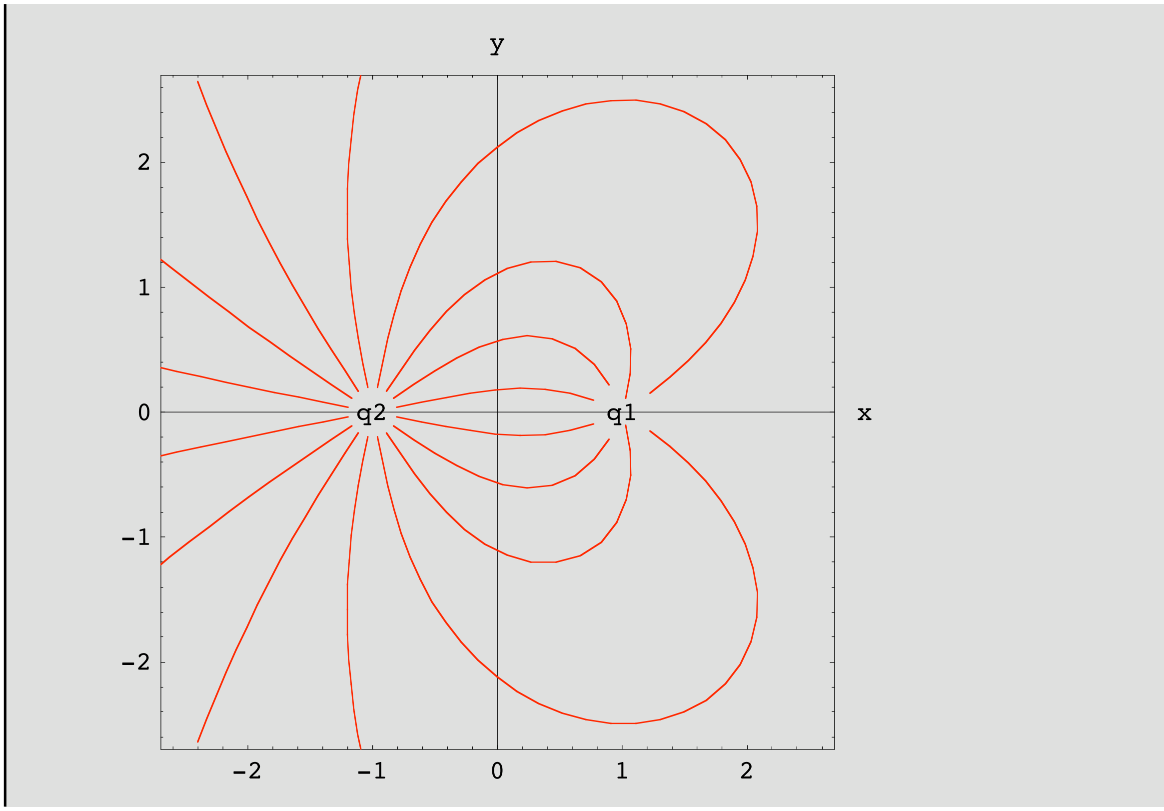


Here's one final exercise to try: if q1 = -4, what value must q2 have to yield the electric field lines below? (Check your answer by using the input cell above!)



End of Tutorial