12/8/19

Chapter - 7

Alternating Current & Alternating Emf

A current / Emf whose magnitude change continously and direction ~~change~~ reverse periodically is know as Alternating Current / Alternating EMF

$E = E_0 \sin\omega t$ & $E = E_0 \cos\omega t$

$I = I_0 \sin\omega t$ or $I = I_0 \cos\omega t$

$$I = I_0 \sin\omega t$$

Phase

Time

angular frequency

$\omega = 2\pi\nu$

frequency

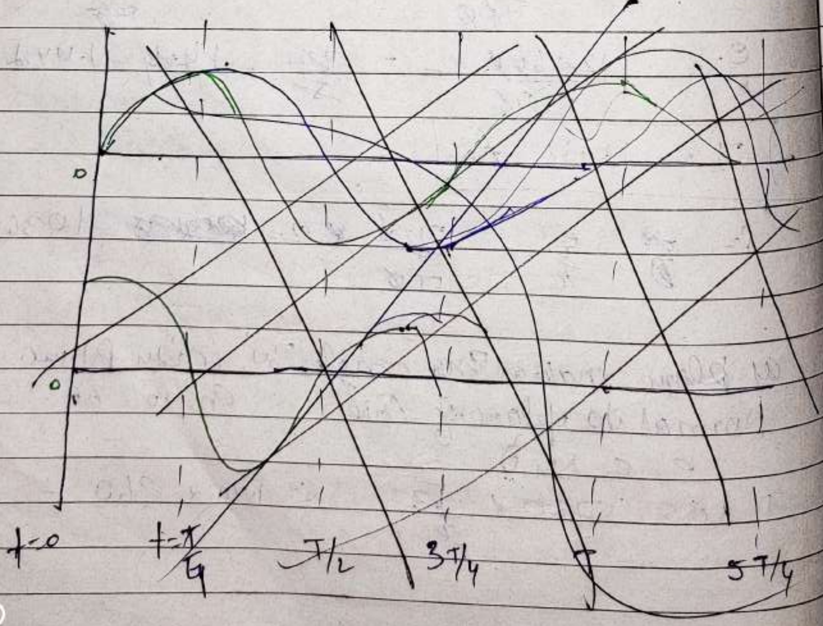
inst. value of current

Peak value

Current Amplitude

(Current displacement)

$\omega = \frac{2\pi}{T}$ → time period



Q. In what time a 50Hz Alternating Current Becomes $\frac{1}{\sqrt{2}}$ times of it maxi. Value starting from 0 zero.

~~$I = I_0 \sin\omega t$~~

~~$\cdot 50 = I_0 \sin\frac{2\pi}{4.8}$~~

sol =

Mean or Average Value of Ac

It is that steady current which when allowed to pass through a circuit or a resistor for a time $t = \frac{T}{2}$ half time peroid of Ac will send same amount of charge as done by Ac in the same circuit or resistance within the same interval of time. It is denoted by (I_m) or (I_a)

Relation b/w I_0 and I_m

Let ~~instanti~~ inst. value of A.C is given by

$I = I_0 \sin\omega t$ —(1)

Small charge sent by this current

$$dq = I\,dt$$

$$dq = I_0 \sin\omega t\,dt$$

$$q = I_0 \int_0^{T/2} \sin\omega t$$

$$q = I_0 \left[\frac{-\cos\omega t}{\omega}\right]_0^{T/2}$$

$$q = -\frac{I_0}{\omega}\left[\cos\frac{2\pi}{T}\times\frac{T}{2} - \cos 0\right]$$

$$q = -\frac{I_0}{\omega}\left[\cos\pi - \cos 0\right]$$

$$q = -\frac{I_0}{\omega_0}\left[-1-1\right]$$

$$\boxed{q = \frac{2I_0}{\omega}}$$

Now the steady current flow I in T/2

$$q' = I_m \cdot T/2 \quad -(2)$$

Acc. to ~~defection~~ definisation of mean value of Ac

$$q' = q$$

$$I_m \, T/2 = \frac{2I_0}{\omega}$$

$$I_m \times \frac{T}{2} = \frac{2 \times I_0 T}{2\pi}$$

$$\boxed{I_m = \frac{2}{\pi} I_0} \quad -(V)$$

or $\boxed{I_m = 0.636 I_0}$ -(vi)

$\boxed{I_m = 63.6\% \text{ of } I_0}$ -(vii)

Similarly

We will prove that

$$E_m = \frac{2}{\pi} E_0$$

$$E_m = 0.636 E_0$$

$$E_m = 63.6\% \text{ of } E_0$$

REDMI NOTE 10

$V = 2R$

$E = E_0 \sin\omega t$

$dq = I_0 \sin\omega t \, dt$

$dq = \frac{E_0}{R} \sin\omega t \, dt$

$q = \int_0^{T/2} \frac{E_0}{R} \sin\omega t \, dt$

$q = \frac{E_0}{R} \int_0^{T/2} \sin\omega t \, dt$

$q = \frac{E_0}{R} \left[-\frac{\cos\omega t}{\omega} \right]_0^{T/2}$

$q = \frac{-E_0}{\omega R} \left[\cos\omega t \right]_0^{T/2}$

$q = \frac{-E_0}{\omega R} \left[\cos\frac{2\pi}{T} \times \frac{T}{2} - \cos 0 \right]$

$q = \frac{-E_0}{R\omega} \left[-1 \quad -1 \right]$

$q = \frac{-E_0}{R\omega} \left[-2 \right] = \frac{2E_0}{R\omega}$

$\boxed{q = \frac{2E_0}{R\omega}}$

Now the steady current. Now $t = T/2$

$q' = \frac{E_m}{R} \cdot T/2$

$q' = q$

$\frac{E_m}{R} T/2 = \frac{2E_0}{\omega R} \Rightarrow \frac{E_m}{R} \cdot \frac{T}{2} = \frac{2E_0}{2\pi R} \times T$

$\boxed{E_m = \frac{2}{\pi} E_0}$ $\boxed{E_m = .636 E_0}$ A.

Q. Why mean value of Ac is define over its half cycle

A. Because Average value of Ac over its complete cycle is always zero

$I = I_0 \sin \omega t$

small current pass thro. w.r.t

$dq = I\,dt$

$dq = I_0 \sin\omega t\, dt$

$\int_0^T dq = \int_0^T I_0 \sin\omega t\, dt$

$q = I_0 \int_0^T I_0 \sin\omega t\, dt$

$q = I_0 \left[-\frac{\cos\omega t}{\omega} \right]_0^T$

$q = -\frac{I_0}{\omega} \left[\cos\frac{2\pi}{T}\times T - \cos 0 \right]$

$q = \frac{-I_0}{\omega} \left[+1 - 1 \right]$

$\boxed{q = 0}$

~~Related~~ ~~defn~~: Charge sent by ~~Stead~~ Steady

$q' = I_m \times T$

REDMI NOTE 10

Acc. to defination the mean value of AC

$q' = q$

$I_m \times T = 0$ $\boxed{I_m = 0}$

RMS value of AC / [Root Mean Square] / effective value of AC / Virtual value of AC

[RMS] [I_{eff}] [I_v]

It is that steady current which when allow to pass throw the resistance for a given time, will produce a same amount of Heat as produce by AC in the same resistance with in a same interval of time

Relation b/w I_0 and I_{rms}

Let the inst. value of alt. current is given by

$$I = I_0 \sin \omega t \quad -(i)$$

Small amount of heat produced by this current in a resistor in time T

$$dH = I^2 R \, dt \quad -(ii)$$

Total heat produced in time T

$$H = \int_0^T I^2 R \, dt$$

$$H = \int_0^T I_0^2 \sin^2 \omega t \, R \, dt \quad \text{- from (i)}$$

$$H = I_0^2 R \int_0^T \sin^2 \omega t \, dt$$

$$H = I_0^2 R \left(\frac{1-\cos 2\omega t}{2}\right) dt$$

$$H = \frac{I_0^2 R}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t \, dt\right]$$

$$H = \frac{I_0^2 R}{2}\left\{[t]_0^T - \left[\frac{\sin 2\omega t}{2\omega}\right]_0^T\right\}$$

$$H = \frac{I_0^2 R}{2}\left[T - \frac{1}{2\omega}\left[\underbrace{\sin\frac{4\pi}{T}T}_{0} - \underbrace{\sin\frac{4\pi}{T}\times 0}_{0}\right]\right]$$

$$H = \frac{I_0^2 R}{2}[T]$$

$$H = \frac{I_0^2 RT}{2} \quad \text{--- (iii)}$$

Now Heat produce in steady current I_{rms} in same resistance for same time [T]

$$H' = I_{rms}^2 RT \quad \text{--- (iv)}$$

acc. to defination of I_{rms}

$$H' = H$$

$$I_{rms}^2 \cancel{RT} = \frac{I_0^2 \cancel{RT}}{2}$$

$$\boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$$

$$\boxed{\begin{aligned} I_{rms} &= 0.707\, I_0 \\ I_{rms} &= 70.7\% \text{ of } I_0 \end{aligned}}$$

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* we will prove that $E_{rms} = \frac{E_0}{\sqrt{2}}$

$$E = E_0 \sin\omega t$$

$$dH = \frac{E^2}{R}\, dt$$

$$dH = \frac{E_0^2}{R} \sin^2\omega t$$

$$H = \int_0^T \frac{E_0^2}{R} \sin^2\omega t \, dt$$

$$H = \frac{E_0^2}{R} \int_0^T \left[\frac{1-\cos 2\omega t}{2}\right] dt$$

$$H = \frac{E_0^2}{2R} \int_0^T dt - \int_0^T \cos 2\omega t \, dt$$

$$H = \frac{E_0^2}{2R}\left[[T] - \left[\frac{\sin 2\omega t}{2\omega}\right]_0^T\right]$$

$$H = \frac{E_0^2}{2R}\left[[T] - \frac{1}{2\omega}\left[\underbrace{\sin \frac{4\pi}{T} \times T}_{0} - \underbrace{\sin \frac{4\pi}{T} \times 0}_{0}\right]\right]$$

$$H = \frac{E_0^2}{2R}\left[T - \frac{1}{2\omega}[0]\right]$$

$$H = \frac{E_0^2}{2R} T = \qquad H = \frac{E_0^2 T}{2R} \quad \text{---(i)}$$

* Now Heat produce in steady current I_{rms} in same resistance for same time [T]

$$H' = \frac{E_m^2}{R} T$$

Acc. to defination of I_{rms} $\quad H' = H$

$$\frac{E_m^2 \cancel{T}}{\cancel{R}} = \frac{E_0^2 \cancel{T}}{2\cancel{R}}$$

$$\boxed{E_m = \frac{E_0}{\sqrt{2}}}$$

Short cut

$$I_{rms} = \sqrt{\frac{\int_0^T I_0^2 R \sin^2 \omega t \, dt}{\int_0^T dt}}$$

$$I_{rms} = \frac{I_0^2 R}{T} \int_0^T \sin^2 \omega t \, dt$$

$$\frac{I_0^2 R}{T} \int_0^T \frac{1 - \cos 2\omega t}{2} \, dt]$$

$$\frac{I_0^2 R}{2T} \int_0^T dt - \int_0^T \cos 2\omega t] dt$$

$$\frac{I_0^2 R}{2T} \left[[t]_0^T - \int_0 \frac{\sin 2\omega t}{2\omega} \right]_0^T$$

$$\frac{I_0^2 R}{2T} \left[T - \frac{1}{2\omega} \left[\sin \frac{4\pi}{T} \times T - \sin \frac{4\pi}{T} \times 0 \right] \right.$$

$$\frac{I_0^2 R}{2T} \left[T - \frac{1}{2\omega} [\, 0 \,] \right.$$

$$\frac{I_0^2 R}{2T} \times T = \frac{I_0^2 R}{2}$$

REDMI NOTE 10

AC Instruments Vs DC instrument

	Ac instrument	DC Instruments
1-	Works on Heating effect of electric current	It works Magnetic effect of electric current
2-	They read Rms value.	They read average Value
3-	Can be used in DC circuits	Can not be used in AC circuits as they read average value for complete cycle Avg. value of AC = 0
4-	The division on the scale are not equally spaced	The division on the scale are equally spaced
5-		

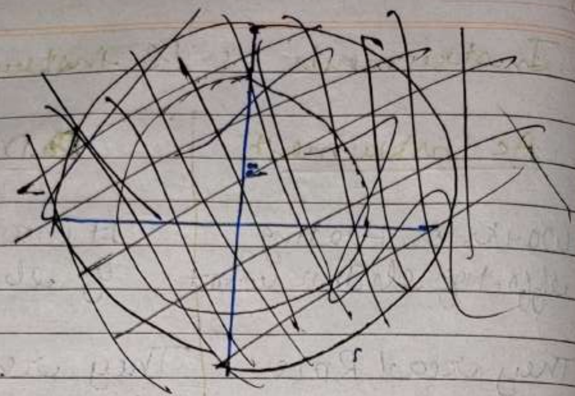
★ Phasor and Phasor dig.

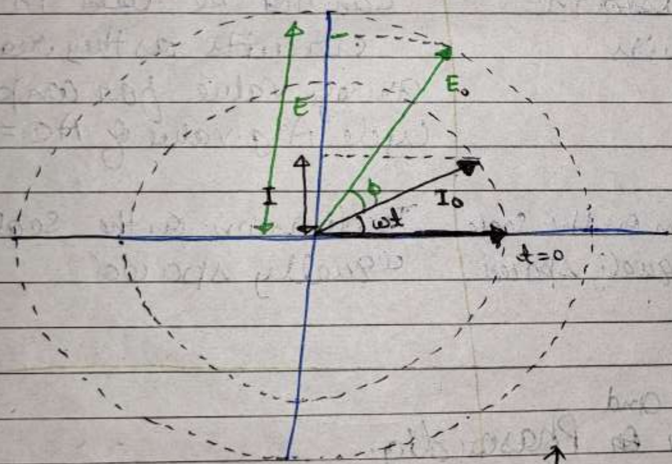
A Quanity which varies sinusoidally [S = Sine function] with time and which can be represented as the projection of rotating vector is known as phasor.

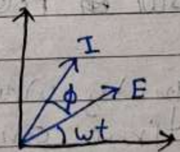
Gn:- Alternating Current & Emf

A diag. in which phasors are shown along with phase relation b/w that is know as phasor diagram.

REDMI NOTE 10







$E = E_0 \sin \omega t$

$I = I_0 \sin(\omega t + \phi)$

Ac thr

Consider

- ~~####~~ (Ze

source

ωt E

I

~~If ins~~

$E =$

÷ by R

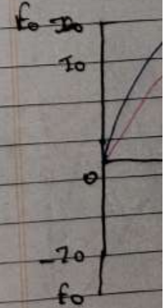
$\frac{E}{R} =$

$\boxed{I =}$

Compare

when a.

E & I



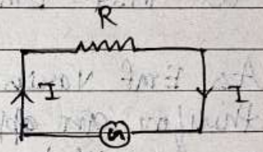
Ac through A Resistar

Consider a pure Resistar it means ~~zero~~ (Zero inductance) is connected to a Ac source

Let E is a inst. Emf

I is a inst. current

~~If~~ Let inst. given by $E = E_0 \sin\omega t$

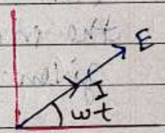


$$E = E_0 \sin\omega t \quad -(i)$$

$\div$ by R

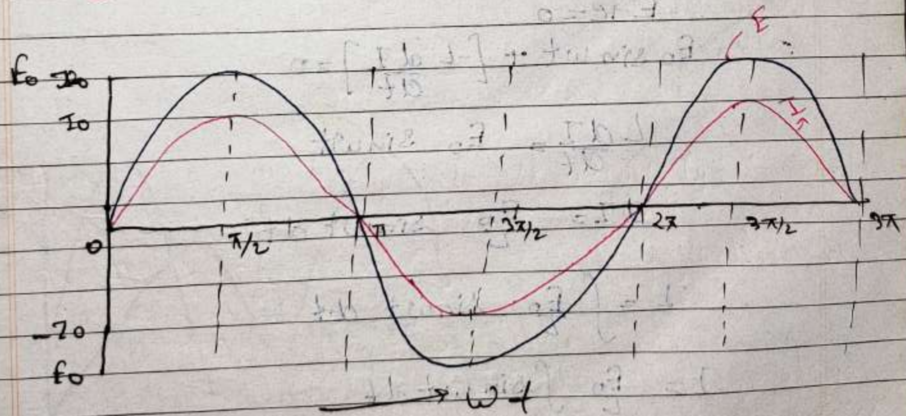
$$\frac{E}{R} = \frac{E_0}{R} \sin\omega t$$

$$\boxed{I = I_0 \sin\omega t} \quad -(ii)$$



Compare eq (ii) with eq (i)

When a.c passes through a pure resistor E & I are in same phase i.e $\phi = 0$



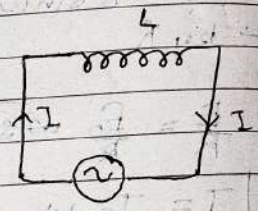
Ac. through Inductor

Consider a Pure Inductor of in [Zero Ohm Resistance) of inductance L is connected to a Ac source. let

let E inst. Emf

I inst. current

let inst. Emf is given by : $E = E_0 \sin\omega t$ —①



As Emf varies with time therefor an opposing induce emf will be produce in the inductor which is given by

$$e = -L\frac{dI}{dt} \quad \text{—②}$$

Total inst. Emf = E + e

but Total inst. Emf = 0 [Acc. KVL law]

$$E + e = 0$$

$$E_0 \sin\omega t + \left[-L\frac{dI}{dt}\right] = 0$$

$$L\frac{dI}{dt} = E_0 \sin\omega t$$

$$dI = \frac{E_0}{L}\sin\omega t\, dt$$

$$I = \int \frac{E_0}{L}\sin\omega t\, dt$$

$$I = \frac{E_0}{L}\int \sin\omega t\, dt$$

$$I = \frac{E_0}{L}\left[-\frac{\cos\omega t}{\omega}\right]$$

at $t=0 \rightarrow$ Open circuit

$t=\infty \rightarrow$ Wire

$$I = -\frac{E_0}{\omega L}\left[\sin(\pi/2 - \omega t)\right]$$

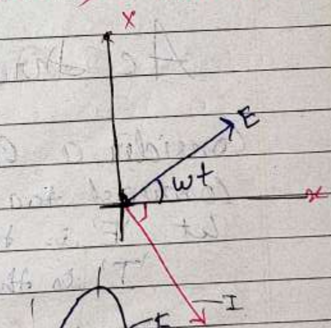
$$\boxed{I = \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)} \quad \text{(iii)}$$

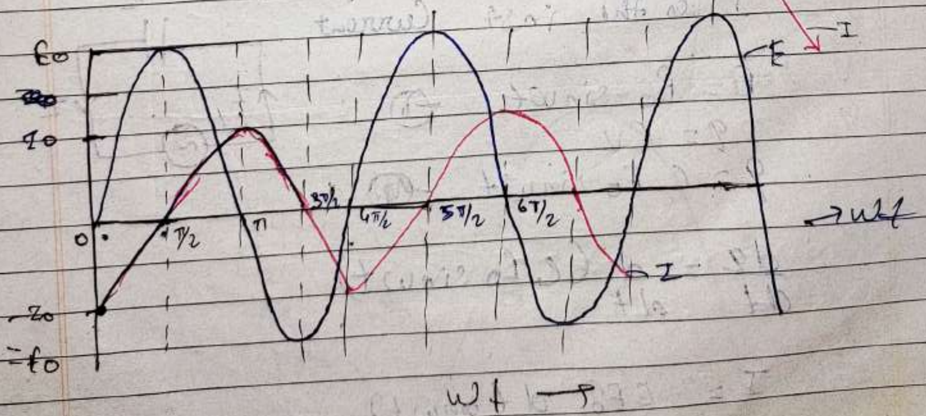
Here $\omega L = X_L$ is the Resistance offered by the inductor in the path of AC Know as inductive Reactants of the inductor

eq (iii) become

$$I = \frac{E_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{(iv)}$$

Comparing eq(iv) & eq (1) it is clear that when AC Passes throw a pure inductor, Currents lags or (Emf leads current) by phase angle by $(\pi/2)$





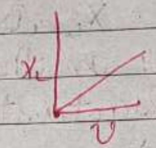
Note

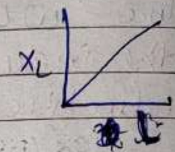
① Units & dimension of X_L are same as that of 'R'

② $X_L = \omega L = 2\pi \nu L$

$X_L \propto \nu$

$X_L \propto L$





③ as $X_L = 2\pi \nu L$

For DC $\nu = 0$

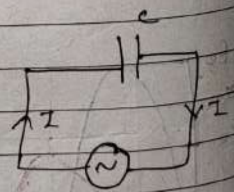
$\boxed{X_L = 0}$ An inductor offers zero resistance in Flow/Path of DC

Ac through a Capacitor :-

Consider a Capacitor of Capacitance 'C' is connected to a AC source

let 'E' is the inst. Emf

'I' is the inst Current



$E = E_0 \sin \omega t$ — ①

$q = CV$

$q = C E_0 \sin \omega t$ — ②

$\frac{dq}{dt} = \frac{d}{dt}(C E_0 \sin \omega t)$

$I = C E_0 \frac{d}{dt}(\sin \omega t)$

$I = CE_0 \cdot (\cos\omega t \cdot \omega)$

$I = \omega CE_0 \cos\omega t$

$$I = \frac{E_0}{1/\omega C} \sin\left(\frac{\pi}{2} + \omega t\right)$$

$$I = \frac{E_0}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{--- (2)}$$

here $1/\omega C = X_C$ is the Resistance offered by capacitor in the path of AC, known as capacitive reactance

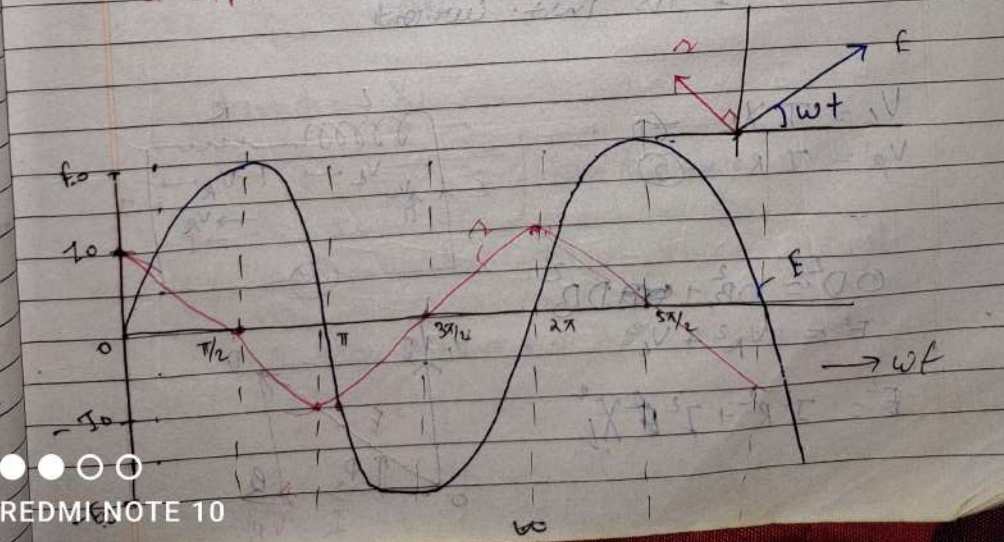
from eq (2)

$$I = \frac{E_0}{1/\omega C} \sin(\omega t + \pi/2)$$

$$I = I_0 \sin(\omega t + \pi/2) \quad \text{--- (3)}$$

Compare eq (1) with eq (3)

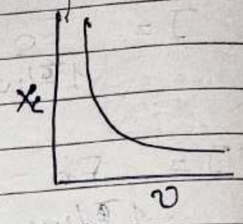
It is clear that when a AC passes through a capacitor I leads E (or E lags behind I by $\pi/2$)



Note

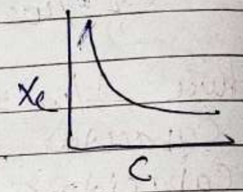
Unit of X_C is same as 'R'

$X_C = \frac{1}{\omega C}$



$X_C \propto \frac{1}{\omega C} \Rightarrow X_C \propto \frac{1}{2\pi \nu C}$

$\therefore X_C \propto \frac{1}{\nu} \quad \therefore X_C \propto \frac{1}{C}$



For DC $\quad X_C = \frac{1}{2\pi f C} \quad \boxed{f = 0}$

$\boxed{X_C = \infty}$

- AC through L.R. Series Circuit

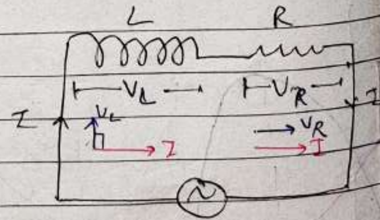
Consider of inductor of inductance L and resistance of resistor R are connected in series to an AC source

E is inst. emf

I is inst. current

$V_L = I X_L$ — (1)

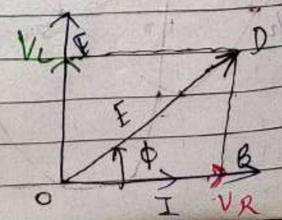
$V_R = I R$ — (2)



$OD^2 = OB^2 + DB^2$

$E^2 = V_R^2 + V_L^2$

$E^2 = I^2 R^2 + I^2 X_L^2$



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~~$E = I^2(L^2+R^2)$~~ $\quad E = I^2(R^2 + X_L^2)$

$$E = I\sqrt{R^2 + X_L^2} \quad \text{—(iii)}$$

$$\boxed{I = \frac{E}{\sqrt{R^2 + X_L^2}}} \quad \text{—(iv)}$$

* $\sqrt{R^2 + X_L^2} = Z$ Resistance ~~of~~ offered by L.R series circuit in the path of ~~DC~~ AC know as IMPEDANCE of L.R series circuit

$$\sqrt{R^2 + X_L^2} = Z$$

$$\boxed{I = \frac{E}{Z}} \quad \text{—(vi)}$$

Phase difference b/w E & I

$$\tan\phi = \frac{BD}{OB} = \frac{OE}{OB} = \frac{V_L}{V_R} = \frac{IX_L}{IR}$$

$$\tan\phi = \frac{X_L}{R}$$

$$\boxed{\tan\phi = \frac{\omega L}{R}} \quad \text{—(viii)}$$

For phase dig. it is clear that when AC pass throught LR series circuit, E leads I (or I lags behind E) by phase angle

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\boxed{0 < \phi < \pi/2}$$

Ac through CR Series Circuit

Consider an Capacitor of Capacitance 'C' & a resistor of Resistance 'R' are connected in Series to any A.C source let

E is the int. Emf

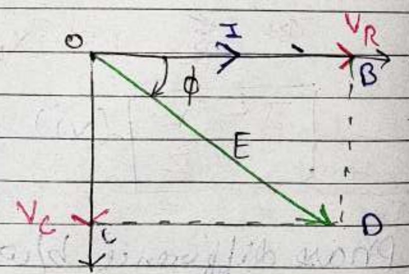
I is the inst. Current

V_C = Voltage across Capacitor

V_R = Voltage across Resistance

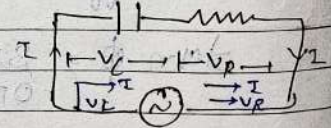
$V_C = I \times X_C$

$V_R = I R$



$OD^2 = OB^2 + BD^2$

$E^2 = V_R^2 + V_C^2$



$E^2 = I^2R^2 + I^2X_C^2$

$E^2 = I^2(R^2 + X_C^2)$

$E = I\sqrt{R^2 + X_C^2}$

$$\boxed{I = \frac{E}{\sqrt{R^2 + X_C^2}}} \quad \text{(iv)}$$

Here $\sqrt{R^2 + X_C^2} = Z$ Resistance offered in C.R Series circuit in the path of AC Know as IMPEDANCE of C.R Series circuit

Re $\sqrt{R^2 + X_L^2} = Z$

eq (iv) become = $\boxed{I = \frac{E}{Z}}$ ---(vi)

Phase diff. b/w E & I

$\tan\phi = \frac{BD}{OB} = \frac{DC}{OB} = \frac{V_C}{V_R}$

$\tan\phi = \frac{\cancel{I} X_C}{\cancel{I} R}$

$\boxed{\tan\phi = \frac{X_C}{R}}$ ---(vii)

For phasor dig it is clear that when AC passes through CR series circuit I leads E (or E lags behind I) by phase angle $\phi = \tan^{-1}\left[\frac{1/\omega C}{R}\right]$

$-\frac{\pi}{2} < \phi < 0$

Q The eq of AC of a circuit is given by $I = 50\cos 100\pi t$ find (i) f of AC, (ii) mean value of current during + half cycle (iii) Virtual value of the current (iv) the value of current at $t = \frac{1}{300}$ sec

(i) $I_0 = 50$ $\omega t = 100\pi t$

$\frac{2\pi}{T} = 100\pi$ $f = \frac{1}{T} = 50$

Mean = $.636 \times 50 = 31.8$

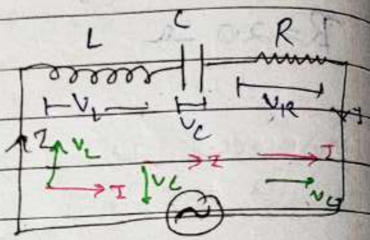
Rms = $.707 \times 50 = 35.35$

$I = 50\cos 100\pi \times \frac{1}{300}$ $= 50\cos\frac{\pi}{3}$

$\boxed{I = 25A}$

LCR Series Circuit

Consider of inductance L capacitor of capacitance C & Resistor of Resistance R are connected in series to an AC source



let E inst. Emf

I inst current

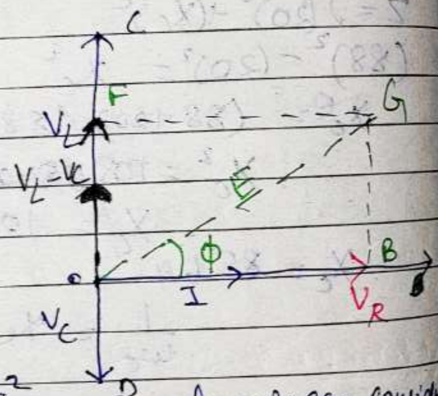
let V_C, V_L, V_R are voltage across capacitor, Inductor, Resistance

$V_L = I X_L$ —(1)

$V_C = I X_C$ —(2)

$V_R = I R$ —(3)

Various phase diag V_C, V_L, V_R



here we can consider $V_L > V_C$

$(OG)^2 = (OB)^2 + (GB)^2$

$(OG)^2 = (V_R)^2 + (V_L - V_C)^2$

$E^2 = I^2R^2 + [I X_L - I X_C]^2$

$E^2 = I^2 R^2 + I^2 [X_L - X_C]^2$

$E^2 = I^2 [R^2 + (X_L - X_C)^2]$

$E = I \sqrt{R^2 + (X_L - X_C)^2}$

$I = \dfrac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$ —(IV)

here $\sqrt{R^2+(X_L-X_C)^2} = Z$ is the resistance ~~offered~~ by LCR series circuit in the path of ac known as ~~impe~~ IMPEDENCE of LCR series circuit

$$Z = \sqrt{R^2+(X_L-X_C)^2} \quad \text{—(v)}$$

Eq (v)

$$\boxed{I = \frac{E}{Z}} \quad \text{—(vi)}$$

Phase Relation b/w E & I

$$\tan\phi = \frac{BG}{OB} = \frac{OF}{OB} = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

$$= \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} \quad \text{—(vii)}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)} \quad \text{—(viii)}$$

* Three cases arise :-

① $V_L > V_C$

$IX_L > IX_C$

$\omega L > \frac{1}{\omega C}$

from eq (vii) $\tan\phi$ is (+ve) $\therefore$ ϕ is (+ve)

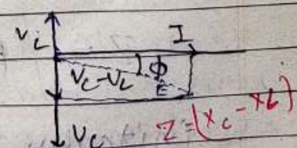
E leads I by phase angle ϕ

② $V_L < V_C$

$IX_L < IX_C$

$\omega L < 1/\omega C$

from (vii) $\tan\phi$ is (−ve) $\therefore$ ϕ is (−ve)



E lags behind I by phase angle ϕ

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

(3) $\boxed{V_L = V_C}$

$I X_L = I X_C$

$\boxed{\omega L = 1/\omega C}$

for Eq(vii) $\tan\phi = 0$ 'zero'

ϕ is zero

E & I are in same phase

Resonance Circuit

Note Case(III) is the condition of Resonance in LCR Series Circuit. Therefor at Resonance in LCR Series Circuit

$V_L = V_C$

I & E are in same phase

Circuit behave as R- circuit

Series Resonance Circuit / Resonance in LCR series Circuit

Consider a series LCR Circuit with a variable frequency AC Source

impedance of LCR Series circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$Z = \sqrt{R^2 + \left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)^2}$$

initially ν is very low, then $2\pi\nu L$ is very low & $1/2\pi\nu C$ is very large

REDMI NOTE 10

diff $\left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)$ is very large hence Z is very large

(·) I is very small

* If ν is gradually increased, $2\pi\nu L$ is increase & $1/2\pi\nu C$ is decrease

* I increase, at one stage, say when $\omega = \omega_0$ (or $\nu = \nu_0$) $\boxed{2\pi\nu L = \frac{1}{2\pi\nu C}}$ - I maxi

$\therefore \nu_0^2 = \frac{1}{4\pi^2 LC}$

$\boxed{\nu_0 = \frac{1}{2\pi\sqrt{LC}}}$ - (ii) or $2\pi\nu_0 = \frac{1}{\sqrt{LC}}$

$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$ - (iii)

* in such case
⇒ Z is minium
⇒ hence I is maxi

× A Series LCR circuit which admits a maxi current at a particular value of the frequency of Ac source [called resonance frequency] is ~~call~~ know as Series Resonance circuit

The frequency of Ac source at which resonance [I maxi & Z = min] occurs in LCR series circuit is Resonance frequency (ω_0) in corresponding Angular frequency is called Resonance frequency. (ω_0)

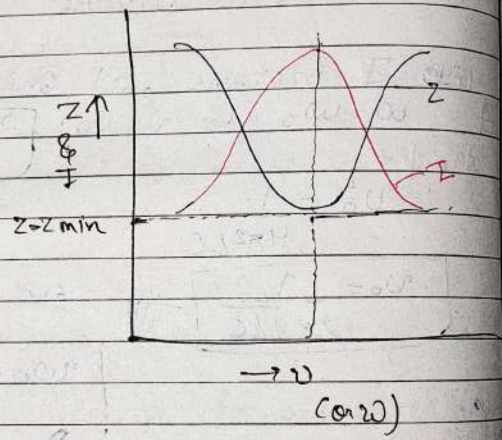
If ν increase further

diff. $\left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)$ starts increasing

again $\therefore$ Z also starts increasing

$\therefore$ I starts increasing

$$\overset{R+}{\begin{cases} \omega_0 > \omega & X_C > X_L \\ \omega = \omega_0 & X_C = X_L \\ \omega_0 < \omega & X_L > X_C \end{cases}}$$



Cal the value of inductance which should be ~~connected~~ connect in series with $C = 5\mu F$, $R = 10\Omega$, $AC = 50 Hz$ so that the current & EMF are same phase

ATQ : E & I are in same phase

this will drive the LCR circuit at resonance

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}} = 2\pi\sqrt{LC} = \frac{1}{50}$$

$$4\pi^2 LC = \frac{1}{2500} \qquad L = \frac{1}{2500\ 4\pi^2\ 5\times10^{-6}}$$

Q The Ac Voltage is given. $V = 310 \cos 314t$ this is applied to a series combination of 25 μF ·1 H & ·24 Ω Cal. the following

(i) Peak Voltage (iv) Reactance of a circuit
(ii) RMS voltage (v) Impedance of circuit
(iii) & ν (vi) Phase Angle of Current w.r.t.

$V = 310 \cos 314t$

$\boxed{V_0 = 310 V}$

$\omega t = 314t$

$\omega = 314$

$2\pi\nu = 314$

$\boxed{\nu = \frac{314}{2\pi}}$ $\nu = 50\ Hz$

Rms = $V = \frac{V_0}{\sqrt{2}}$

$\boxed{V_{Rms} = \cdot 707 \times 310}$

$X_L = \omega L = 314 \times \cdot 1 = 31 \cdot 4$

$X_C = \frac{1}{\omega C} = \frac{10^6}{314 \times 25} = \frac{4 \times 10^4}{314 \times 25}$

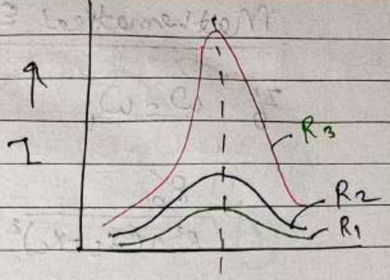
Sharpness of Resonance in Series Resonance circuit & Quality factor

Consider an LCR Series circuit having a variable frequency Ac source. connected with it. let the value L & c are fixed and R is variable consider 3 different value of R say R_1, R_2, R_3 ($R_1 > R_2 > R_3$) as ω_0 is independent of R therefore ω_0 will be same for each of the 3 case the corresponding I ω. graph is shown in below

$\omega_c = \frac{1}{\sqrt{LC}}$

$R =$ LCR $X_C - X_L$

It is clear that the I, ω curve is Quite flat for large value of R (R_1) and is more sharpe for least value of (R_3)



further when are is low corresponding to the small chang in ω there is a large change in I in comperasion case When R is large

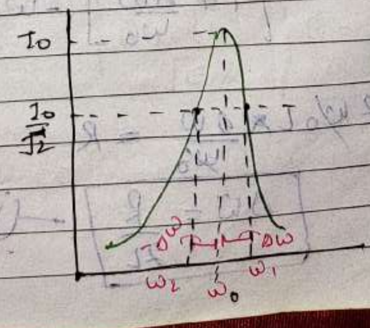
Such an LRC series circuit is said to be more selected and more sharp. of The sharpness of Resonance is determin by the factor know as Quality factor [Q-factor)

The Quality factor of a series Resonance circuit is the Ratio of resonance frequency to the differen in two frequency taken on either side of the resonance frequency at which the current amplitude is $\frac{1}{\sqrt{2}}$ times of its value at Resonance frequency

$$Q = \frac{\omega_0}{\omega_1 - \omega_2} = \frac{\omega_0}{2\Delta\omega} = \frac{\text{Resonance Angular frequency}}{\text{Band width}}$$

$\omega_1 = \omega_0 + \Delta\omega$

$\omega_2 = \omega_0 - \Delta\omega$



$\Delta\omega = \frac{R}{L}$

Mathematical Expression

If $\omega = \omega_1$ $\quad I = \frac{I_0}{\sqrt{2}}$

$$\therefore \frac{E_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{E_0}{\sqrt{2}R}$$ Sq. Both side & reverse the eq.

$$2R^2 = R^2 + (X_L - X_C)^2$$
$$R^2 = (X_L - X_C)^2$$
$$R = (X_L - X_C)$$

$$R = \left(\omega_1 L - \frac{1}{\omega_1 C}\right)$$

$$\Rightarrow R = (\omega_0 + \Delta\omega)L - \frac{1}{(\omega_0 + \Delta\omega)C}$$

$$R = L\omega_0\left(1 + \frac{\Delta\omega}{\omega_0}\right) - \frac{1}{\omega_0 C\left(1 + \frac{\Delta\omega}{\omega_0}\right)}$$

$$R = \omega_0 L\left[1 + \frac{\Delta\omega}{\omega_0}\right] - \frac{\omega_0 L}{\left[1 + \frac{\Delta\omega}{\omega_0}\right]} = R \quad \left[\because \omega_0 L = \frac{1}{\omega_0 C}\right.$$

$$R = \omega_0 L\left[1 + \frac{\Delta\omega}{\omega_0}\right] - \omega_0 L\left[1 + \frac{\Delta\omega}{\omega_0}\right]^{-1} = R$$

$$R = \omega_0 L\left[1 + \frac{\Delta\omega}{\omega_0}\right] - \omega_0 L\left[1 - \frac{\Delta\omega}{\omega_0}\right] = R \quad \left[\text{Using Bionimal}\right.$$

$$2\omega_0 L \times \frac{\Delta\omega}{\omega_0} = R$$

$$\boxed{\Delta\omega = \frac{R}{2L}} \quad \text{—(iv)}$$

$$\boxed{2\pi \nu \omega = \frac{R}{L}} \quad \text{-(v)}$$

$$Q = \frac{\omega_0}{R/L} \qquad \boxed{Q = \frac{\omega_0 L}{R}} \quad \text{-(vi)}$$

Imp (1)

as $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\boxed{Q = \frac{1}{\omega_0 RC}}$$

$$Q = \frac{1}{\sqrt{LC}} \times \frac{L}{R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}} \quad \text{-(vii)}$$

Q-Factor is define as the ratio of voltage across Inductor to the applied voltage

$$Q = \frac{V_L}{V_R} \quad \text{or} \quad \frac{V_C}{V_R}$$

$$Q = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega_0 L}{R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}} \quad \text{-viii}$$

(2)

$$\frac{V_C}{V_R}$$

$$Q = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{\omega_0 CR}$$

$$= \frac{1}{\frac{1}{\sqrt{LC}} CR}$$

$$Q = \frac{\sqrt{L}}{\sqrt{C} \cdot R} \qquad \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

LC oscillation

LC-Oscillations: ***When a charged capacitor is allowed to discharge through a pure inductor, electrical oscillations of constant amplitude and frequency are produced, known as LC-oscillations.*** The mechanism is explained below.

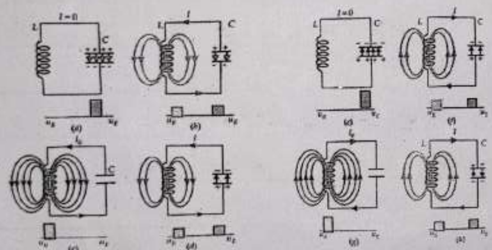
Fig. (a, b): Let a charged capacitor (charge q_0, and U_E = max. $= \frac{1}{2}\frac{q_0^2}{C}$) is connected to an ideal inductor($U_B = 0$). As soon as the circuit is closed, the capacitor begins to discharge through inductor. It sends a current I (anticlockwise) thereby producing a growing magnetic field in the inductor, which in turn produces an induced emf, opposing the growth of the current hence the capacitor takes some finite time to discharge completely. As I increases, a part of U_E is stored in the inductor in the form of U_B $(= \frac{1}{2} L I^2)$

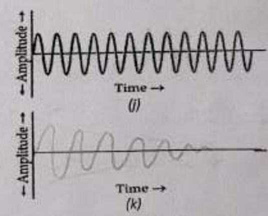
Fig. (c): The capacitor is fully discharged ($U_E = 0$), I reaches to its maximum value ($= I_0$) Thus $U_B = \max = \frac{1}{2} L I_0^2$.

Fig. (d, e): After the discharge of the capacitor is complete, ϕ_B linked with inductor decreases, inducing a current in the same direction as the earlier current. The current thus persists, though with decreasing magnitude, and charges the capacitor in the opposite direction. U_B begins to change into U_E until the capacitor is fully charged (with reverse polarity) hence U_E becomes maximum and $U_B = 0$.

Fig. (f, g): The capacitor begins to discharge again, sending the current in opposite direction. U_E once again starts converting into U_B until U_B becomes maximum and $U_E = 0$.

Fig. (h): The process repeats in the opposite direction and the circuit eventually returns to the initial state as shown in fig. (a).





Due to the repetition of such process the electro-magnetic oscillations are produced. If there is no resistance of the circuit no dissipation of energy will take place, and undamped oscillations are produced as shown in fig.(j). Practically due to some finite resistance of the circuit, the damped oscillations are produced as shown in fig.(k). The frequency of oscillations is given by $\nu = \frac{1}{2\pi\sqrt{LC}}$.

Rishabh Kr Sharma

Imp Imp ★

Power in Ac circuit

Let the inst. value of Emf & Current in an ac circuit are given by E

$$E = E_0 \sin \omega t$$

$$I = I_0 \sin(\omega t + \phi)$$ → is a phase differen E & I

Let these value remain const for small time dt then small amount of energy consume dW

$$dW = E I dt$$

$$dW = (E_0 \sin \omega t)(I_0 \sin(\omega t + \phi)) dt$$

$$dW = E_0 I_0 \sin \omega t(\sin \omega t \cos\phi + \sin\phi \cos \omega t) dt$$

$$dW = E_0 I_0 [\sin^2 \omega t \cos\phi + \sin\phi \sin \omega t \cos \omega t] dt$$

Total energy consume in time '0' to 'T'

$$W = \int_0^T E_0 I_0 \left[\left(\frac{1 - \cos 2\omega t}{2}\right) \cos\phi + \frac{\sin 2\omega t}{2} \sin\phi \right] dt$$

$$W = \frac{E_0 I_0}{2} \int_0^T \left[\cos\phi \int_0^T dt - \cos\phi \underbrace{\int_0^T \cos 2\omega t \, dt}_{0} + \sin\phi \underbrace{\int_0^T \frac{\sin 2\omega t}{2} dt}_{0} \right]$$

$$W = \frac{E_0 I_0 \cos\phi \, T}{2}$$

$$W = \frac{E_0 I_0 T \cos\phi}{2} \quad \text{(iii)}$$

$$\frac{\text{Total Energy consume}}{\text{Time}} = \text{Power}$$

$$\text{Power} = \frac{E_0 I_0 \cancel{T} \cos\phi}{2 \cancel{T}} = \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi$$

★ $\boxed{\text{Power} = E_{rms} \cdot I_{rms} \cos\phi}$

REDMI NOTE 10

$\boxed{P_{avg} = \frac{V_{rms}^2}{Z} \cos\phi}$ $\boxed{\cos\phi = \frac{R}{Z}}$ → $\frac{V^2 R}{Z^2}$

Power of an AC circle

$P_{av} = E_{rms} \cdot I_{rms} \cos\phi$ —(iv)

av. Power   virtual Power   apparent Power   → Power factor

$$\boxed{\text{Power factor} = \frac{\text{Avg. Power}}{\text{App. Power}}}$$

Hence Power factor of an AC circuit may be define as the ratio of avg. Power to the apparent Power consume in a.c circuit

(a) For R Circuit $\phi = 0$

Power factor $\cos\phi = \cos 0 = 1$ (maxi)

avg Power $P_{av} = E_{rms} I_{rms} \cos 0$

$P_{av} = E_{rms} I_{rms}$ (vi)

avg Power = apparent Power

(b) For L circuit

For " " $\phi = -\pi/2$ eq (1) & (2)

Power factor $\phi = -\pi/2 = \cos(-\pi/2) = \cos\frac{\pi}{2} = 0$

avg. Power = $P_{av} = E_{rms} \cdot I_{rms} \cos(-\pi/2) = 0$

$\boxed{P_{av} = 0}$

(c) For C circuit

For " " $\phi = \pi/2$ eq (1) (2)

Power factor $\phi = \pi/2$ $\cos\frac{\pi}{2} = 0$

avg Power = $P_{av} = E_{rms} \cdot I_{rms} \cos\pi/2 = 0$

$\boxed{P_{av} = 0}$

$\frac{V^2 R}{Z^2}$ $\quad C\frac{\cancel{}}{F}$ $\quad R \rightarrow$ $\quad L\frac{}{}\;C\frac{}{}$

(d) For LR circuit

for " " $\tan\phi = \frac{\omega L}{R}$

Power factor $\cos\phi = \frac{R}{\sqrt{(\omega L)^2 + R^2}} = \frac{R}{Z}$

avg Power $P_{av} = E_{rms}\, I_{rms} \cos\phi$

$= E_{rms}\, I_{rms} \frac{R}{Z}$

$$\boxed{P_{av} = E_{rms}\, I_{rms} \frac{R}{\sqrt{R^2 + \omega^2 L^2}}}$$

(e) For CR circuit

$\tan\phi = \frac{1/\omega C}{R}$

Power factor $\cos\phi = \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}} = \frac{R}{Z}$

avg Power $= E_{rms}\, I_{rms} \cos\phi$

$E_{rms}\, I_{rms} \frac{R}{Z}$

$$\boxed{P_{av} = E_{rms}\, I_{rms} \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}}$$

(f) For LCR Circuit

$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$

Power factor $\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega^2 L^2 - \frac{1}{\omega^2 C^2}\right)^2}} = \frac{R}{Z}$

avg Power $P_{av} = E_{rms}\, I_{rms} \cos\phi$

$= E_{rms}\, I_{rms} \frac{R}{Z}$

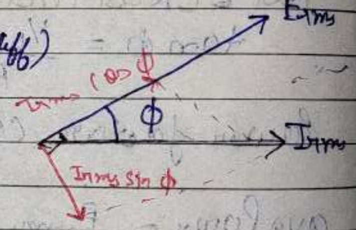
$$\boxed{P_{av} = E_{rms}\, I_{rms} \frac{R}{\sqrt{R^2 + \left(\omega^2 L^2 - \frac{1}{\omega^2 C^2}\right)^2}}}$$

WCC $I_{wc} = I_{rms} \sin\phi$

Wattless Currents

Consider an AC circuit in which 'ϕ' is the phase difference current and EMF (as shown in fig.) $I_{rms}\cos\phi$ and ~~...~~ $I_{rms}\sin\phi$ are the components of current along & '⊥' to E_{rms} (as shown)

That's calculate the Power consumption due to each of two components separately



$P_{av} = E_{rms} I_{rms} \cos\theta$ (phase diff)

$P_{av} = E_{rms} (I_{rms} \cos\phi) \cos 0$

$P_{av} = E_{rms} I_{rms} \cos\phi$ —(1)

(b) due to $I_{rms}\sin\phi$

$P_{av} = E_{rms} \cdot I_{rms} \cos$ (phase diff)

$P_{av} = E_{rms} [I_{rms} \sin\phi) \cos \pi/2$

$\boxed{P_{av} = 0}$

Hence This component doesn't consume power in A.C circuit

Therefore the component of current which doesn't contribute the power loss in AC circuit is known as Wattless / ~~Ideal~~ Idle component of current.