Let $X_1, X_2, --- X_n$ iids are normally distributed with σ variance & μ mean, Each one has pdf f & cdf F where

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2}}$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2}} dx$$

Density of median of $\{X_1, X_2, -- X_n\}$ is

$$P(X \in [x, x+\epsilon])$$

$= P[$one of the X's $\in [x, x+\epsilon]$ & and exactly $\frac{n-1}{2}$ of the others $< x)$

$= \sum_{i=1}^{n} P[X_i \in [x, x+\epsilon]$ and exactly $\frac{n-1}{2}$ of others $< x)$

$= n\,P[X_1 \in [x, x+\epsilon])\cdot P[$exactly $\frac{n-1}{2}$ of others $< x)$

$$= n\,P[X_1 \in [x, x+\epsilon]) \cdot P(\binom{n-1}{\frac{n-1}{2}} P[X < x)^{\frac{n-1}{2}} P(X > x)^{\frac{n-1}{2}}$$

$$g(x) = n\,f(x) \cdot \binom{n-1}{\frac{n-1}{2}} F(x)^{\frac{n-1}{2}} (1 - F(x))^{\frac{n-1}{2}}$$

$g(x)$ is distribution of median.

$$E[g(x)] = \int_{-\infty}^{\infty} x\, n f(x) \binom{n-1}{\frac{n-1}{2}} (F(x))^{\frac{n-1}{2}} (1-F(x))^{\frac{n-1}{2}}\, dx$$

Source: www2.stat.duke.edu/courses/Spring12/sta104.1/Lectures/Lec15.pdf