The distribution of each x_i is

$$p(x_i) = \begin{cases} 0, & x_i \leq a \text{ or } x_i \geq b \\ \frac{1}{b-a}, & a \leq x_i \leq b \end{cases}$$

Let X_{max} & X_{min} are the estimators for 'b' & 'a' respectively. where

$$X_{max} = \max(x_1, x_2, \ldots x_n)$$

$$X_{min} = \min(x_1, x_2, \ldots x_n)$$

Let F denote the CDF of each x_i

$$F(x_i) = P(x_i < x) = 0, \quad x_i \leq a$$

$$= \frac{x-a}{b-a}, \quad a \leq x_i \leq b$$

$$= 1, \quad x_i \geq b$$

$$\because F(x_i) = \int_{x_1}^{x_2} p(x_i)\,dx = \int_{x_1}^{x_2} \frac{1}{b-a}\,dx$$

$$= \left(\frac{x}{b-a}\right)\Big|_{x_1}^{x_2}$$

Now, consider the CDF of X_{max},

$$F(X_{max}) = P(X_{max} < x) = P(X_i < x, \forall i)$$

$$= \prod_{i=1}^{n} P(X_i < x)$$

$$= 1, \quad x \geq b$$

$$= \left(\frac{x-a}{b-a}\right)^n, \quad a \leq x \leq b$$

$$= 0, \quad x \leq a$$

PDF of X_{max} is derivative of its CDF,

$$p(X_{max}) = \begin{cases} 0, & x \geq b \\ \cancel{\frac{n}{(b-a)^n}(x-a)} \; \frac{n(x-a)^{n-1}}{(b-a)^n}, & a \leq x \leq b \\ 0, & x \leq a. \end{cases}$$

$$E(X_{max}) = \int_{-\infty}^{\infty} x \, p(X_{max}) \, dx$$

$$= \int_a^b \frac{n}{(b-a)^n} x(x-a)^{n-1} dx$$

Let $x - a = t$

$$= \frac{n}{(b-a)^n} \int_0^{b-a} (a+t) t^{n-1} dt$$

$$= \frac{n}{(b-a)^n} \left[a \frac{(b-a)^n}{n} + \frac{(b-a)^{n+1}}{n+1} \right]$$

$$= a + \frac{n}{n+1}(b-a) = \frac{nb+a}{n+1}$$

Now, let us proceed to calculate $E(X_{min})$, Note that

$$p(x_i > x) = 1 - p(x_i < x) = \begin{cases} 1, & x_i \leq a \\ \frac{b-x}{b-a}, & a \leq x_i \leq b \\ 0, & x_i \geq b. \end{cases}$$

$$P(X_{min} \leq x) = 1 - P(X_{min} \geq x)$$

$$= 1 - \prod P(x_i \geq x)$$

$$= 1 - \left(\frac{b-x}{b-a}\right)^n,$$

[Considering for x in $[a, b]$ ~~β~~, since for outside these intervals, $P(X_{min} \leq x)$ is a constant so its derivative will be zero].

PDF of X_{min} is derivative of above

$$P(X_{min}) = \frac{n}{(b-a)^n}(b-x)^{n-1}$$

$$E(X_{min}) = \int_a^b \frac{x\,n}{(b-a)^n}(b-x)^{n-1}\,dx$$

Let $t = b-x$,

$$= \frac{n}{(b-a)^n}\int_{b-a}^{0} -(b-t)\,t^{n-1}\,dt$$

$$= \frac{n}{(b-a)^n}\left[\frac{t^{n+1}}{n+1} - \frac{b\,t^n}{n}\right]_{b-a}^{0}$$

$$= \frac{n}{(b-a)^n}\left[\frac{b(b-a)^n}{n} - \frac{(b-a)^{n+1}}{n+1}\right]$$

$$= b - \frac{n}{n+1}(b-a)$$

$$= \frac{nb+b-nb+na}{n+1} = \frac{na+b}{n+1}$$

So we have

$$E(X_{max}) = \frac{nb+a}{n+1}, \quad E(X_{min}) = \frac{na+b}{n+1}$$

$$\underset{\text{for } b}{\text{Bias}(X_{max})} = E(X_{max}) - b$$

$$= \frac{nb+a}{n+1} - b = \frac{a-b}{n+1}$$

$$\underset{\text{for } a}{\text{Bias}(X_{min})} = \frac{na+b}{n+1} - a = \frac{b-a}{n+1}$$

So, X_{max} & X_{min} are biased estimators.

Let $A = \frac{n X_{max} - X_{min}}{n-1}$ be an estimator for b.

$$\text{Bias}(A) = E\left(\frac{n X_{max} - X_{min}}{n-1}\right) - b$$

$$= \frac{1}{n-1}\left[n\left(\frac{nb+a}{n+1}\right) - \frac{na+b}{n+1}\right] - b$$

$$= \frac{n^2\cancel{b} + \cancel{n}a - \cancel{n}a - \cancel{b} - \cancel{n^2}b + \cancel{b}}{n^2-1}$$

$$= 0$$

So, A is an unbiased estimator for b.

Let $B = \frac{n X_{min} - X_{max}}{n-1}$ be an estimator for a.

$$\text{Bias}(B) = E\left(\frac{n X_{min} - X_{max}}{n-1}\right) - a.$$

$$= \frac{1}{n-1}\left[n\left(\frac{na+b}{n+1}\right) - \frac{nb+a}{n+1}\right] - a$$

$$= \frac{\cancel{n^2a} + \cancel{nb} - \cancel{nb} - \cancel{a} - \cancel{n^2a} + \cancel{a}}{n^2-1}$$

$$= 0$$

Hence, $\frac{nX_{min} - X_{max}}{n-1}$ & $\frac{nX_{max} - X_{min}}{n-1}$

are unbiased estimators for 'a' & 'b' respectively.