EE655: Computer Vision and Deep Learning

HOMEWORK - 3

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Date of submission: 11/02/2025

GitHub Repository of this Course:

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1 Proof that derivative of $\sigma(x)$ = $\sigma(x)$*(1- $\sigma(x)$)

Proof:

Let $\sigma(x)$ denote the sigmoid function, defined as:

$$\sigma(x) = \frac{1}{1+e^{-x}} = (1+e^{-x})^{-1}$$

We want to find the derivative of $\sigma(x)$ with respect to x, denoted as $\sigma'(x) = \frac{d}{dx}\sigma(x)$. Using the chain rule, we have:

$$\sigma'(x) = \frac{d}{dx}(1+e^{-x})^{-1} = -1 \cdot (1+e^{-x})^{-2} \cdot \frac{d}{dx}(1+e^{-x})$$

Now, we compute the derivative of $(1+e^{-x})$ with respect to x:

$$\frac{d}{dx}(1+e^{-x}) = \frac{d}{dx}(1) + \frac{d}{dx}(e^{-x}) = 0 + e^{-x} \cdot \frac{d}{dx}(-x) = e^{-x} \cdot (-1) = -e^{-x}$$

Substituting this back into the expression for $\sigma'(x)$:

$$\sigma'(x) = -(1+e^{-x})^{-2} \cdot (-e^{-x}) = \frac{e^{-x}}{(1+e^{-x})^2}$$

Now we need to show that this is equal to $\sigma(x)(1-\sigma(x))$. Let's compute $\sigma(x)(1-\sigma(x))$:

$$\sigma(x)(1-\sigma(x)) = \frac{1}{1+e^{-x}}\left(1 - \frac{1}{1+e^{-x}}\right)$$

To simplify the term in the parenthesis:

$$1 - \frac{1}{1+e^{-x}} = \frac{(1+e^{-x})-1}{1+e^{-x}} = \frac{e^{-x}}{1+e^{-x}}$$

Now, substitute this back into the expression for $\sigma(x)(1-\sigma(x))$:

$$\sigma(x)(1-\sigma(x)) = \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} = \frac{e^{-x}}{(1+e^{-x})^2}$$

Comparing the expressions for $\sigma'(x)$ and $\sigma(x)(1-\sigma(x))$, we see that they are identical:

$$\sigma'(x) = \frac{e^{-x}}{(1+e^{-x})^2} = \sigma(x)(1-\sigma(x))$$

Thus, we have proven that the derivative of sigmoid(x) is indeed sigmoid(x)*(1- sigmoid(x)).

2 Programmatic Plot of Sigmoid and its Derivative

The following Python code generates a plot of the sigmoid function and its derivative:

Listing 1: Python code for plotting sigmoid and its derivative

```python
import numpy as np
import matplotlib.pyplot as plt

def sigmoid(x):
    return 1 / (1 + np.exp(-x))

def sigmoid_derivative(x):
    return sigmoid(x) * (1 - sigmoid(x))

x = np.linspace(-10, 10, 400)
y_sigmoid = sigmoid(x)
y_derivative = sigmoid_derivative(x)

plt.figure(figsize=(10, 6))
plt.plot(x, y_sigmoid, label='Sigmoid(x)', color='blue')
plt.plot(x, y_derivative, label='Sigmoid Derivative', color='red', linestyle='--')
plt.title('Sigmoid Function and its Derivative')
plt.xlabel('x')
plt.ylabel('y')
plt.grid(True)
plt.legend()
plt.show()
```

The generated plot is displayed below:

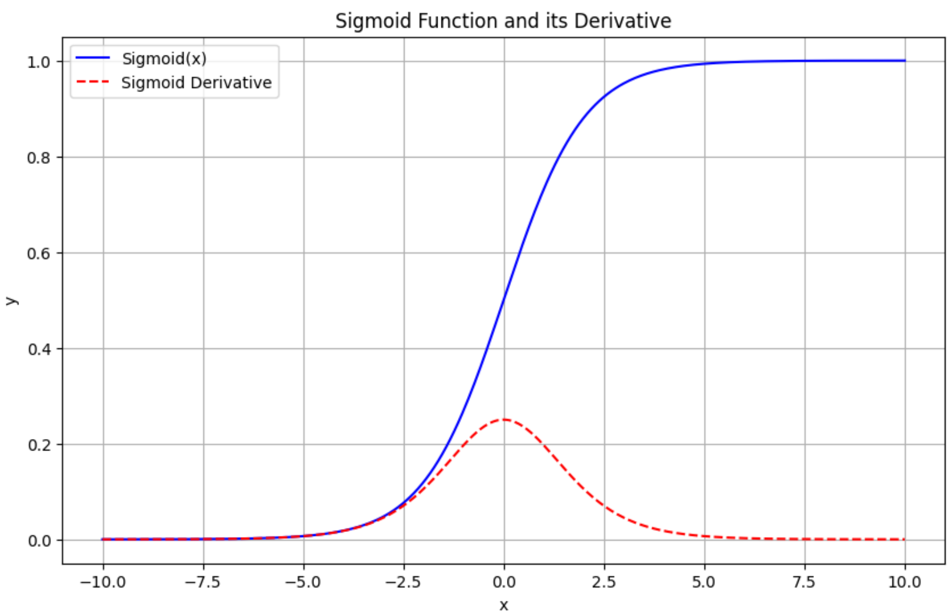

Figure 1: Plot of Sigmoid Function and its Derivative

The plot visually confirms that the sigmoid function has an S-shaped curve (blue) and its derivative forms a bell-shaped curve (red dashed line), supporting our theoretical proof.