Assignment 3

(i) Identify the random variables in the statements below , and write each of the following statements using symbols for random variables, logical connectives where necessary, and conditional probability notation.

Ans.

Random variables:

(a) Have traveled - T
(b) Have caught corona - C
(c) Have caught other disease - O
(d) Severely sick - S
(e) Person died - D
(f) Mildly sick - M

Now, we will write all the statements using the symbols for random variables.

(a) $T \land (C \lor O)$
Conditional Probability notation: $P(C|T) + P(O|T) = 0.825$
(b) $P(M \land C|T) = 0.15$
$P(S \land C|T) = 0.22$
(c) $P(O|T)) = 0.485$
(d) $P(D \land {\sim}C \land T) = 0.24$
(e) $P({\sim}T \land S \land C) = 0.025$
(f) $P(S|{\sim}T) = 0.457$
(g) $P(D \land C) = 0.059$
(h) $P(S \lor M) = 0.7$
(i) $P(T|S) = 0.8$
(j) $P(C) = 0.5$

(ii) Verify that these propositions create a valid probability distribution. List the set of axioms that they satisfy.

Ans.

The axioms that we need to satisfy are:
(i) The probability values should lie between $0 =< p =< 1$
(ii) The sum of all probabilities across all possible events should be 1

To verify that these propositions create a valid probability distribution, the propositions need to satisfy these 2 axioms.
For that we can see that in each of the statements, the probability values are between 0 and 1. Therefore, the first axiom is being satisfied.
For the second proposition, we need to that all the probability expressions are part of the total probability.
Let us take one for example. In the (j) part, the probability for a person to have corona whether they travelled or not is 0.5. That means that there is a probability that they person may have

some other disease of not be sick at all irrespective of whether they travelled or not. And this probability will have a value of 1-0.5 = 0.5.
Let us take one more example. In the (h) part, the probability of a person having a mild or severe sickness or any disease is 0.7. From here, we can infer that the probabilty of a person not being sick is 0.3. If we take the probability we just saw of a person may have some other disease of not be sick at all irrespective of whether they traveled or not having value 0.5, we can infer that the probability of being sick with some other disease will be 0.5-0.3 = 0.2.

(iii) Populate the full joint probability distribution table.
Ans.

(iv) Use the joint distribution table and check for conditional independence between all the random variables that you have identified.
Ans.

2.
(a) Should you switch your choice to the other unopened door to maximize your chance of winning the key?
Ans.
Yes, we should switch our choice to the other unopened door to maximize your chance of winning the key since by sticking to your original choice, the chance of winning the key would remain same, i.e. ⅓, but if we switch our choice, the chances of winning the keys gets increased to ⅔.

(b) However, if he occasionally makes a mistake, he reveals the loss of life with a probability of 1/3 and correctly reveals the key with a probability of 2/3. In this scenario, should you switch your choice to maximize your chances of winning the key?
Ans.
If he occasionally makes a mistake, he reveals the loss of life with a probability of 1/3 and correctly reveals the key with a probability of ⅔, then switching our choice would maximize the chances of winning the key. If the person occasionally makes a mistake, and if your original choice was the correct one, then switching makes the chances reduced to 0, but if your original choice was incorrect, which has a probability of ⅔ of happening, then switching your choice would increase the chances of winning the key to ⅔. Thus, ⅔ of the cases would increase the chance of winning the key when we switch. Also, if he reveals the door with the key by mistake, then your original choice was correct, and switching would reduce the chances to 0. Hence, in both scenarios, if he makes a mistake, and he reveals the loss of life with a probability of ⅓, then switching the choice increases the chance of winning the key to ⅔.

(c) If you choose to switch, what is the conditional probability that you win the key if the man has mistakenly revealed the door that shows life lost?

Ans.
If the man has mistakenly revealed the door that shows life lost, that means he must have made a mistake between 2 doors, one having a key and the other losing a life. That means the door I chose first loses a life. Now the situation is that I initially chose a door which a life loss, and the man has revealed the other door which looses a life. Thus only one door remains which certainly contains the key.
Thus, if I choose to switch, the conditional probability that you win the key if the man has mistakenly revealed the door that shows life lost is 1.

(d) Additionally, what is the conditional expectation of your prize (key/life lost) based on your choice to switch or stick, considering both possible scenar-ios? Would you choose to switch or stick based on the conditional expectation?
Ans.
The conditional expectation of your prize is based on your choice to switch or stick, considering both possible scenarios:
We are given here that there is a loss of life. Now, we need to determine the probability of getting a key if there is a switch and the probability of getting a key if there is no switch.
P(winning key |action = switch, loss of life) = 1 (as done in previous problem)
P(winning key | action = no switch, loss of life) = 0 (conclusive)

Therefore, the conditional expectation values dictate that we should make a switch.

Computational

The answers for the computation part are in the code file (.ipynb file). The file is well commented with explanations for every step using markdown. This has been done to make it easy to see the code while checking the results.