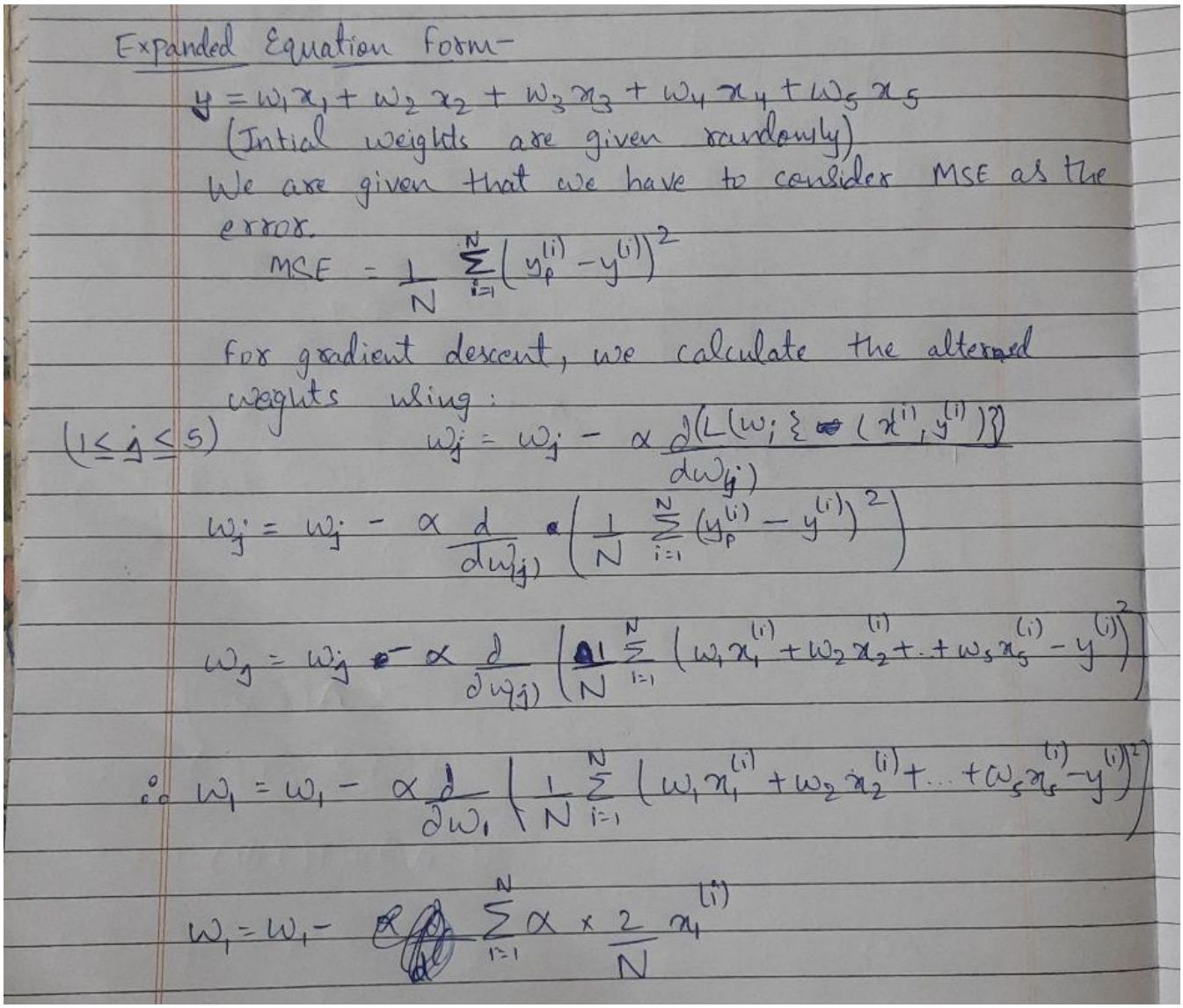
Assignment 4

Q1(a).



Expanded Equation Form-

$y = w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + w_5 x_5$

(Intial weights are given randomly)

We are given that we have to consider MSE as the error.

$$MSE = \frac{1}{N} \sum_{i=1}^{N} \left(y_p^{(i)} - y^{(i)} \right)^2$$

For gradient descent, we calculate the altered weights using:

$(1 \le j \le 5)$ $$w_j = w_j - \alpha \frac{\partial (L(w_j \{ (x^{(i)}, y^{(i)}) \})}{\partial w_j)}$$

$$w_j = w_j - \alpha \frac{d}{dw_j)} \left(\frac{1}{N} \sum_{i=1}^{N} (y_p^{(i)} - y^{(i)})^2 \right)$$

$$w_j = w_j - \alpha \frac{d}{\partial w_j)} \left(\frac{1}{N} \sum_{i=1}^{N} \left(w_1 x_1^{(i)} + w_2 x_2^{(i)} + \ldots + w_5 x_5^{(i)} - y^{(i)} \right)^2 \right)$$

$$\therefore w_1 = w_1 - \alpha \frac{d}{\partial w_1} \left(\frac{1}{N} \sum_{i=1}^{N} \left(w_1 x_1^{(i)} + w_2 x_2^{(i)} + \ldots + w_5 x_5^{(i)} - y^{(i)} \right)^2 \right)$$

$$w_1 = w_1 - \sum_{i=1}^{N} \alpha \times \frac{2}{N} x_1^{(i)}$$

$$\therefore w_1 = w_1 - \sum_{i=1}^{N} \frac{2\alpha x_1^{(i)}}{N}$$

Similarly, $$w_2 = w_2 - \sum_{i=1}^{N} \frac{2\alpha x_2^{(i)}}{N}$$

$$w_3 = w_3 - \sum_{i=1}^{N} \frac{2\alpha x_3^{(i)}}{N}$$

$$w_4 = w_4 - \sum_{i=1}^{N} \frac{2\alpha x_4^{(i)}}{N}$$

$$w_5 = w_5 - \sum_{i=1}^{N} \frac{2\alpha x_5^{(i)}}{N}$$

⇒ Vector form:

$$y = w^T x + b$$

For gradient descent, we calculate the altered weights using:

$$MSE = \frac{1}{N} \sum_{i=1}^{N} \left(w^T x^{(i)} - y^{(i)} \right)^2$$

$$\Rightarrow w_1 = w_1 - \alpha \frac{\partial}{\partial w_1} \left(\frac{1}{N} \sum_{i=1}^{N} \left(y_p^{(i)} - y^{(i)} \right)^2 \right)$$

$$w_1 = w_1 - \alpha \frac{\partial}{\partial w_1} \left(\frac{1}{N} \sum_{i=1}^{N} \left(w^T x^{(i)} - y^{(i)} \right)^2 \right)$$

$$w_1 = w_1 - \sum_{i=1}^{N} \frac{2\alpha x^{(i)}}{N}$$

Similarly other gradients can be calculated.

So In vector form, the gradient w will be-

$$w = \left[w_1 - \sum_{i=1}^{N} \frac{2\alpha x_1^{(i)}}{N}, \; w_2 - \sum_{i=1}^{N} \frac{2\alpha x_2^{(i)}}{N}, \; w_3 - \sum_{i=1}^{N} \frac{2\alpha x_3^{(i)}}{N}, \right.$$

$$\left. w_4 - \sum_{i=1}^{N} \frac{2\alpha x_4^{(i)}}{N}, \; w_5 - \sum_{i=1}^{N} \frac{2\alpha x_5^{(i)}}{N} \right]$$