Exemplar - 10.13.6.16

EE22BTECH11049 - Shivansh Kirar*

There are three urns containing 2 white and 3 black balls, 3 white and 2 black balls, and 4 white and 1 black balls, respectively. There is an equal probability of each urn being chosen. A ball is drawn at random from the chosen urn and it is found to be white. Find the probability that the ball drawn was from the second urn.

Solution:
Consider the random variable U, which represents the selection of an urn.

Values	Description
U=0	Urn I is chosen
U=1	Urn II is chosen
U=2	Urn III is chosen

Consider the random variable X, which represents the color of ball.

Values	Description
X=0	Ball is White
X=1	Ball is Black

$$P(U = k) = \begin{cases} \frac{1}{3} & \text{if } k \in \{0, 1, 2\} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$\tag{2}$$

Required Probability $= P(U = 1|X = 0)$

$$= \frac{P(U = 1|X = 0)P(U = 1)}{P(X = 0)}$$

$$P(X = 0) = P(U = 0|X = 0)P(U = 0) + P(U = 1|X = 0)P(U = 1) + P(U = 2|X = 0)P(U = 2)$$

$$= \frac{1}{3}(\frac{2}{5} + \frac{3}{5} + \frac{4}{5})$$

$$= \frac{3}{5}$$

$$P(U = 1|X = 0) = \frac{\frac{3}{5}}{\frac{3}{5}} \cdot \frac{1}{3}$$

$$= \frac{1}{3}$$