Exemplar - 12.13.6.46

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There are three urns containing 2 white and 3 black balls, 3 white and 2 black balls, and 4 white and 1 black balls, respectively. There is an equal probability of each urn being chosen. A ball is drawn at random from the chosen urn and it is found to be white. Find the probability that the ball drawn was from the second urn.

Solution:

Consider the random variable U, which represents the selection of an urn.

Values	Description
U=0	Urn I is chosen
U=1	Urn II is chosen
U=2	Urn III is chosen

Consider the random variable X, which represents the color of ball.

Values	Description
X=0	Ball is White
X=1	Ball is Black

$$P(U = k) = \begin{cases} \frac{1}{3} & \text{if } k \in \{0, 1, 2\} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(2)

$$
\begin{aligned}
\text{Required Probability} &= P(U=1|X=0) \\
&= \frac{P(U=1|X=0)P(U=1)}{P(X=0)} \\
P(X=0) &= P(U=0|X=0)P(U=0) + P(U=1|X=0)P(U=1) + P(U=2|X=0)P(U=2) \\
&= \frac{1}{3}(\frac{2}{5} + \frac{3}{5} + \frac{4}{5}) \\
&= \frac{3}{5} \\
P(U=1|X=0) &= \frac{\frac{3}{5}}{\frac{3}{5}} \cdot \frac{1}{3} \\
&= \frac{1}{3}
\end{aligned}
$$