

Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \quad (1)$$

TABLE 1: Vectors.

Parameters	Values	Description
$\mathbf{m}_1$	$\begin{pmatrix} -5 \\ -2 \end{pmatrix}$	$AB$
$\mathbf{m}_2$	$\begin{pmatrix} -1 \\ -4 \end{pmatrix}$	$BC$
$\mathbf{m}_3$	$\begin{pmatrix} 6 \\ -6 \end{pmatrix}$	$CA$
$\ A - B\ $	5.38	Length of $AB$
$\ B - C\ $	4.12	Length of $BC$
$\ C - A\ $	8.48	Length of $CA$
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	Non-collinear
$\mathbf{n}_1$	$\begin{pmatrix} -2 \\ 5 \end{pmatrix}$	$AB$
$c_1$	-14	
$\mathbf{n}_2$	$\begin{pmatrix} -4 \\ 1 \end{pmatrix}$	$BC$
$c_2$	0	
$\mathbf{n}_3$	$\begin{pmatrix} 6 \\ -6 \end{pmatrix}$	$CA$
$c_3$	21	
Area	9.0	Area of triangle
$\angle A$	23.19°	Angles
$\angle B$	125.83°	
$\angle C$	30.96°	

TABLE 2: Median.

Parameters	Values	Description
$\mathbf{D}$	$\begin{pmatrix} -1.5 \\ -2 \end{pmatrix}$	Midpoint of $BC$
$\mathbf{E}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	Midpoint of $CA$
$\mathbf{F}$	$\begin{pmatrix} 1.5 \\ 1 \end{pmatrix}$	Midpoint of $AB$
$\mathbf{m}_4$	$\begin{pmatrix} -5.5 \\ -4 \end{pmatrix}$	$AD$
$\mathbf{n}_4$	$\begin{pmatrix} -4 \\ 5.5 \end{pmatrix}$	
$c_4$	-5.0	
$\mathbf{m}_5$	$\begin{pmatrix} 2 \\ -1 \end{pmatrix}$	$BE$
$\mathbf{n}_5$	$\begin{pmatrix} -1 \\ -2 \end{pmatrix}$	
$c_5$	1	
$\mathbf{m}_6$	$\begin{pmatrix} 3.5 \\ 5 \end{pmatrix}$	$CF$
$\mathbf{n}_6$	$\begin{pmatrix} 5 \\ -3.5 \end{pmatrix}$	
$c_6$	4	
$\mathbf{G}$	$\begin{pmatrix} 0.33 \\ -0.67 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by $\mathbf{G}$
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	Collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

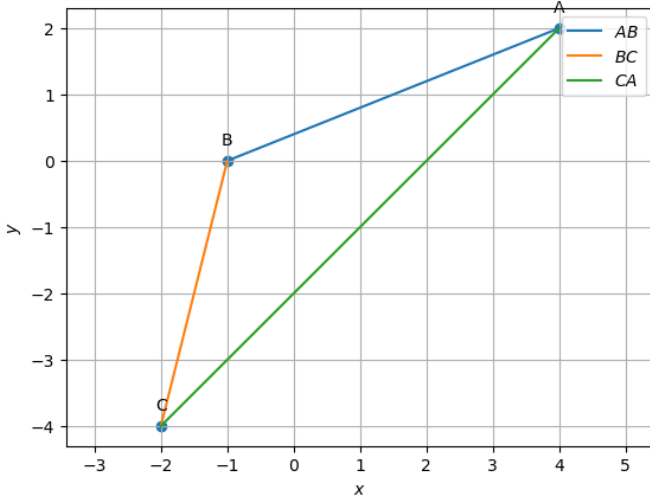


Fig. 1: Triangle plotted using python

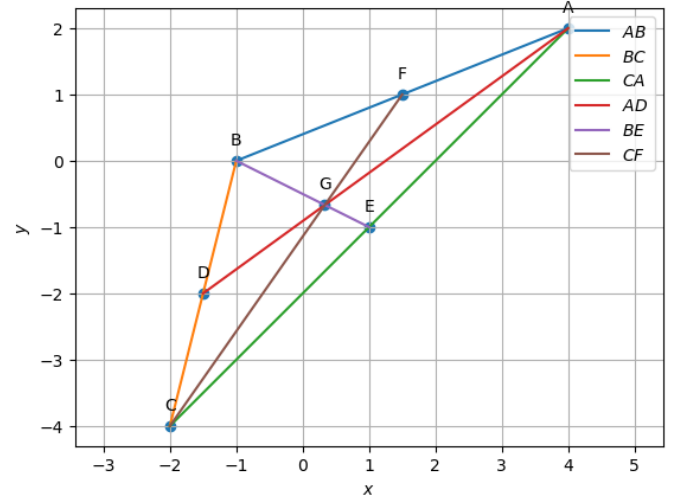


Fig. 2: Medians plotted using python

TABLE 3: Altitude.

Parameters	Values	Description
$D_1$	$\begin{pmatrix} -0.23 \\ 3.05 \end{pmatrix}$	Foot of altitude from $A$
$E_1$	$\begin{pmatrix} 0.5 \\ -1.5 \end{pmatrix}$	Foot of altitude from $B$
$F_1$	$\begin{pmatrix} -3.24 \\ -0.89 \end{pmatrix}$	Foot of altitude from $C$
$m_7$	$\begin{pmatrix} -4.23 \\ 1.05 \end{pmatrix}$	$AD_1$
$n_7$	$\begin{pmatrix} 1.05 \\ 4.23 \end{pmatrix}$	
$c_7$	12.70	
$m_8$	$\begin{pmatrix} 1.5 \\ -1.5 \end{pmatrix}$	$BE_1$
$n_8$	$\begin{pmatrix} -1.5 \\ -1.5 \end{pmatrix}$	
$c_8$	1.5	
$m_9$	$\begin{pmatrix} -1.24 \\ 3.10 \end{pmatrix}$	$CF_1$
$n_9$	$\begin{pmatrix} 3.10 \\ 1.24 \end{pmatrix}$	
$c_9$	-11.17	
$H$	$\begin{pmatrix} -5.33 \\ 4.33 \end{pmatrix}$	Orthocentre

TABLE 4: Perpendicular Bisector.

Parameters	Values	Description
$m_{10}$	$\begin{pmatrix} -4 \\ 1 \end{pmatrix}$	$AD_2$
$n_{10}$	$\begin{pmatrix} -1 \\ -4 \end{pmatrix}$	
$c_{10}$	9.5	
$m_{11}$	$\begin{pmatrix} -6 \\ 6 \end{pmatrix}$	$BE_2$
$n_{11}$	$\begin{pmatrix} -6 \\ -6 \end{pmatrix}$	
$c_{11}$	0	
$m_{12}$	$\begin{pmatrix} 2 \\ -5 \end{pmatrix}$	$CF_2$
$n_{12}$	$\begin{pmatrix} 5 \\ 2 \end{pmatrix}$	
$c_{12}$	9.5	
$O$	$\begin{pmatrix} 3.16 \\ -3.16 \end{pmatrix}$	Circumcentre
$\ O - A\ $	5.23	$OA = OB = OC = R$
$\ O - B\ $	5.23	
$\ O - C\ $	5.23	
$R$	5.23	
$\angle BOC$	$46.39^\circ$	$\angle BOC = 2\angle BAC$
$\angle BAC$	$23.19^\circ$	
$\angle AOC$	$108.32^\circ$	$\angle AOC = 2\angle ABC$
$\angle ABC$	$125.83^\circ$	
$\angle AOB$	$298.07^\circ$	$\angle AOB = 2\angle BCA$
$\angle BCA$	$30.96^\circ$	

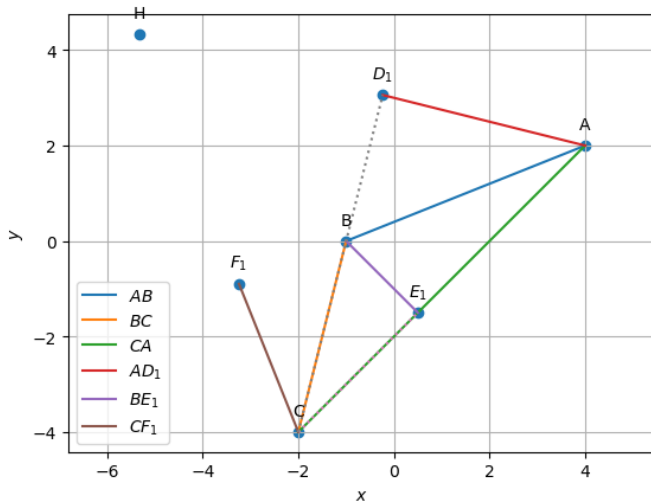


Fig. 3: Altitudes plotted using python

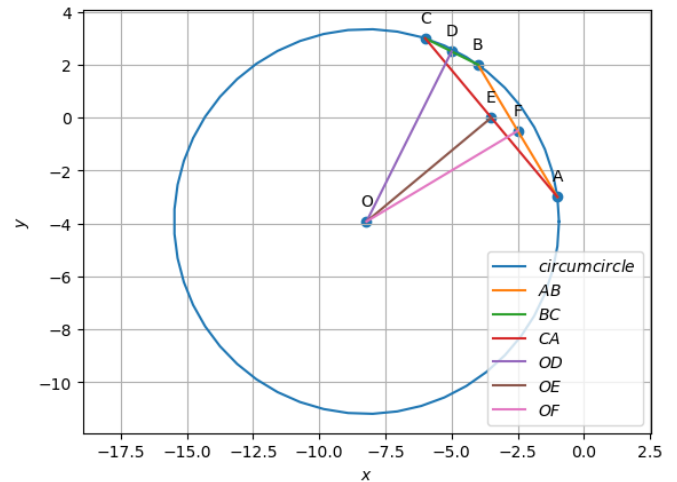


Fig. 4: Perpendicular bisectors plotted using python

TABLE 5: Angular Bisector.

Parameters	Values	Description
$\mathbf{m}_{13}$	$\begin{pmatrix} 1.63 \\ 1.07 \end{pmatrix}$	$AI$
$\mathbf{n}_{13}$	$\begin{pmatrix} 1.07 \\ -1.63 \end{pmatrix}$	
$c_{13}$	1.04	
$\mathbf{m}_{14}$	$\begin{pmatrix} 0.68 \\ -0.59 \end{pmatrix}$	$BI$
$\mathbf{n}_{14}$	$\begin{pmatrix} 0.59 \\ 0.68 \end{pmatrix}$	
$c_{14}$	-0.59	
$\mathbf{m}_{15}$	$\begin{pmatrix} -0.94 \\ -1.67 \end{pmatrix}$	$CI$
$\mathbf{n}_{15}$	$\begin{pmatrix} 1.67 \\ -0.94 \end{pmatrix}$	
$c_{15}$	0.44	
$\mathbf{I}$	$\begin{pmatrix} -0.15 \\ -0.73 \end{pmatrix}$	Incentre
$\mathbf{D}_3$	$\begin{pmatrix} -1.12 \\ -0.49 \end{pmatrix}$	Point of contact with $BC$
$\mathbf{E}_3$	$\begin{pmatrix} -0.55 \\ -1.44 \end{pmatrix}$	Point of contact with $AC$
$\mathbf{F}_3$	$\begin{pmatrix} -0.52 \\ 1.89 \end{pmatrix}$	Point of contact with $AB$
$\ \mathbf{I} - \mathbf{D}_3\ $	1.00	$ID_3 = IE_3 = IF_3 = r$
$\ \mathbf{I} - \mathbf{E}_3\ $	1.00	
$\ \mathbf{I} - \mathbf{F}_3\ $	1.00	
$r$	1.00	
$\angle BAI$	$11.59^\circ$	$\angle BAI = \angle CAI$
$\angle CAI$	$11.59^\circ$	
$\angle ABI$	$62.91^\circ$	$\angle ABI = \angle CBI$
$\angle CBI$	$62.91^\circ$	
$\angle ACI$	$15.48^\circ$	$\angle ACI = \angle BCI$
$\angle BCI$	$15.48^\circ$	

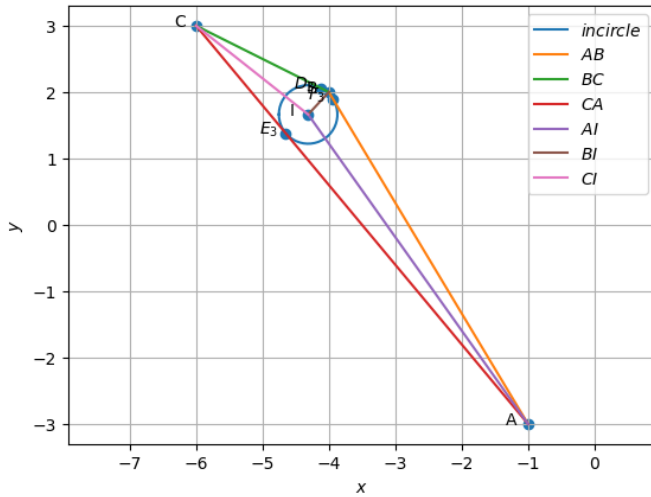


Fig. 5: Angle bisectors plotted using python