

EE22BTECH11049 - Shivansh Kirar

Question EC 31 2023

The signal-to-noise ratio (SNR) of an ADC with a full-scale sinusoidal input is given to be 61.96 dB.
The resolution of the ADC is GATE EC 2023

Solution:

Variable	Defination
T	Time Period
f	Frequency
A	Amplitude
P_S	Signal Power
P_N	Noise Power
q	Quantization step size
n	No of bits
Y	Quantization Error

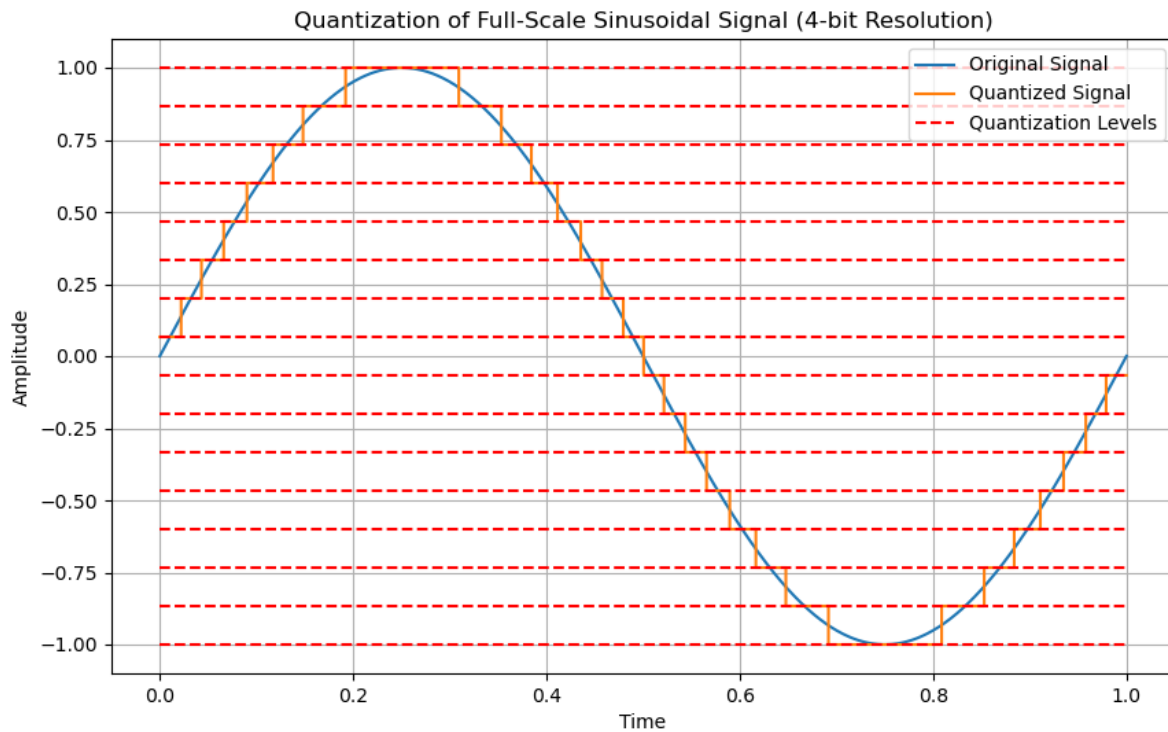


Fig. 1: Quantization of Sinusoidal Signal

1) Signal Power:

The power of a continuous-time signal is defined as the average value of the square of the signal over a certain time interval. For a sinusoidal signal $x(t) = A \sin(2\pi ft + \phi)$, the power (P_S) is calculated as:

$$P_S = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt \quad (1)$$

where ϕ is the phase of the signal.

$$P_S = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |A \sin(2\pi ft + \phi)|^2 dt \quad (2)$$

$$P_S = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \sin^2(2\pi ft + \phi) dt \quad (3)$$

$$P_S = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \cdot \frac{1 - \cos(4\pi ft + 2\phi)}{2} dt \quad (4)$$

$$P_S = \frac{1}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 dt - \frac{1}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \cos(4\pi ft + 2\phi) dt \quad (5)$$

$$P_S = \frac{1}{2} \cdot A^2 - 0 \quad (6)$$

$$P_S = \frac{A^2}{2} \quad (7)$$

$$(8)$$

Here, $A = 1$, so:

$$P_S = \frac{1}{2} \quad (9)$$

2) Noise Power:

No of Quantization levels is given by 2^n , Where n is resolution or no of bits.

Distance between any two Quantization levels = Quantization step

(No of Quantization levels)*(Quantization step) = Peak Distance (Refer to Fig 1)

$$q = \frac{\text{Peak distance}}{2^n} = \frac{1 - (-1)}{2^n} = \frac{2}{2^n} = 2^{-(n-1)} \quad (10)$$

We know, quantization error has a maximum value of plus or minus half the step size, so

$$|Y| \leq \frac{q}{2} \text{ and therefore, } |Y| \leq 2^{-n} \quad (11)$$

For a large enough number of quantization steps, the probability density function of the quantization error tends toward being flat 1. Pdf of error (Y) of quantization is defined as

$$p_Y(y) = \begin{cases} \frac{1}{q}, & \text{if } -\frac{q}{2} \leq y \leq \frac{q}{2} \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

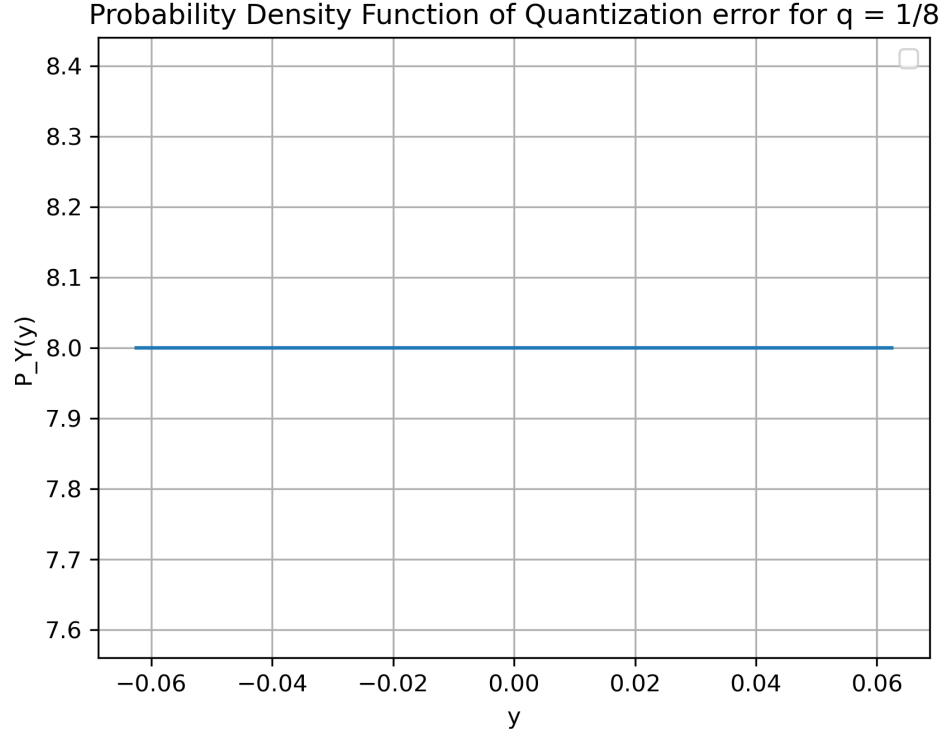


Fig. 2: plot of pdf of Quantization Error

So, we can calculate its mean power or variance as the 2nd moment of its distribution. Since, the distribution of error is uniform hence $E[Y]=0$.

$$E(Y) = \int_{-\frac{q}{2}}^{\frac{q}{2}} p_Y(y)y \, dy \quad (13)$$

$$= \frac{1}{q} \cdot \frac{1}{2} \left(\left(\frac{q}{2} \right)^2 - \left(-\frac{q}{2} \right)^2 \right) 10 \quad (14)$$

$$= \frac{1}{q} \cdot \frac{1}{2} (0 - 0) \quad (15)$$

$$= 0 \quad (16)$$

$$E[Y^2] = \int_{-\frac{q}{2}}^{\frac{q}{2}} p_Y(y)y^2 \, dy \quad (17)$$

$$= \frac{1}{q} \cdot \frac{1}{3} \left(\left(\frac{q}{2} \right)^3 - \left(-\frac{q}{2} \right)^3 \right) 10 \quad (18)$$

$$= \frac{1}{3q} \cdot \frac{q^3}{4} \quad (19)$$

$$= \frac{q^2}{12} \quad (20)$$

$$(21)$$

On putting $q = 2^{-(n-1)}$, we have:

$$E[Y^2] \approx \frac{2^{-2n}}{3} 10 \quad (22)$$

$$P_Y \approx \frac{2^{-2n}}{3} \quad (23)$$

$$P_N \approx \frac{2^{-2n}}{3} \quad (24)$$

$$(25)$$

Thus, an ideal ADC would have a signal-to-noise ratio

$$SNR = \frac{P_S}{P_N} = 1.5 \cdot 2^{2n} \quad (26)$$

or, expressed in decibels,

$$SNR = 10 (\log_{10}(1.5 \cdot 2^n)) \quad (27)$$

$$= 10 (\log_{10}(1.5) + \log_{10}(2^{2n})) \quad (28)$$

$$= 10 (0.176 + 2n \cdot 0.3010) \quad (29)$$

$$= 1.76 + 6.02n \quad (30)$$

Substituting value of SNR

$$61.96 = 1.76 + 6.02n \quad (31)$$

$$n = \frac{61.96 - 1.76}{6.02} \quad (32)$$

$$= \frac{60.2}{6.02} \quad (33)$$

$$= 10 \quad (34)$$

So, the resolution of the ADC is approximately 10 bits.

Simulation Steps :-

- Initialize the simulation parameters, including the given SNR in dB (which is 61.96 dB) and signal power (P_s) (which is 0.5), and create an array of quantization error values (q -values) to test.
- Generate random variables uniformly distributed over $-\frac{q}{2}$ to $\frac{q}{2}$. For each quantization error value, calculate mean error, variance error, and average power (which is equivalent to noise power), and then use these values to calculate the SNR.
- Compare the calculated SNR to the target SNR_dB, and when it's within a specified tolerance, print the corresponding number of bits required for quantization.
- The program breaks the loop and exits after finding the number of bits that achieves the target SNR_dB.