EE22BTECH11049 - Shivansh Kirar

Question EC 31 2023
The signal-to-noise ratio (SNR) of an ADC with a full-scale sinusoidal input is given to be 61.96 dB. The resolution of the ADC is GATE EC 2023

Solution: The mean power in a sinusoidal input signal with unit amplitude is simply

$$P_s = \frac{1}{T}\int_0^T \sin^2(\omega t)\,dt \tag{1}$$

where T is the time period of the signal, and ω is the angular frequency.

Quantized to 2^n discrete levels, equivalent to n bits, one quantization step q has a size of

$$q = \frac{2}{2^n} = 2^{-(n-1)} \tag{2}$$

We know,quantization error has a maximum value of plus or minus half the step size, so

$$|e| \leq \frac{q}{2} \text{ and therefore, } |e| \leq 2^{-n} \tag{3}$$

For a large enough number of quantization steps, the probability density function of the quantization error tends toward being flat 1. Its value within the quantization error bounds is

$$p_e \approx \frac{1}{q} \tag{4}$$

So, we can calculate its mean power or variance as the 2nd moment of its distribution:

$$P_e = \int_{-\frac{q}{2}}^{\frac{q}{2}} p(e)e^2\,de \tag{5}$$

$$P_e = \frac{1}{q}\int_{-\frac{q}{2}}^{\frac{q}{2}} e^2\,de = \frac{1}{q}\cdot\frac{1}{3}\left(\left(\frac{q}{2}\right)^3 - \left(-\frac{q}{2}\right)^3\right) = \frac{1}{3q}\cdot\frac{q^3}{4} = \frac{q^2}{12} \tag{6}$$

On putting $q = 2^{-(n-1)}$, we have:

$$P_e \approx \frac{2^{-2n}}{3} \tag{7}$$

Thus, an ideal ADC would have a signal-to-noise ratio

$$SNR = \frac{P_s}{P_e} = 1.5\cdot 2^{2n} \tag{8}$$

or, expressed in decibels,

$$SNR = 1.76 + 6.02n \tag{9}$$

In this formula:

- SNR is the signal-to-noise ratio in dB, which is given as 61.96 dB in our case.
- 1.76 dB is a constant that accounts for quantization noise.
- 6.02 dB is the noise bandwidth factor for a sinusoidal input.

Plug in the values:

$$\text{Resolution (in bits)} = \frac{61.96 \text{ dB} - 1.76 \text{ dB}}{6.02 \text{ dB}} \approx 10 \text{ bits} \tag{10}$$

So, the resolution of the ADC is approximately 10 bits.