

## EE22BTECH11049 - Shivansh Kirar

**Question EC 31 2023**

The signal-to-noise ratio (SNR) of an ADC with a full-scale sinusoidal input is given to be 61.96 dB.  
The resolution of the ADC is GATE EC 2023

**Solution:**

Variable	Defination
$T$	Time Period
$f$	Frequency
$A$	Amplitude
$P_S$	Signal Power
$P_N$	Noise Power
$q$	Quantization step size
$n$	No of bits
$Y$	Quantization Error

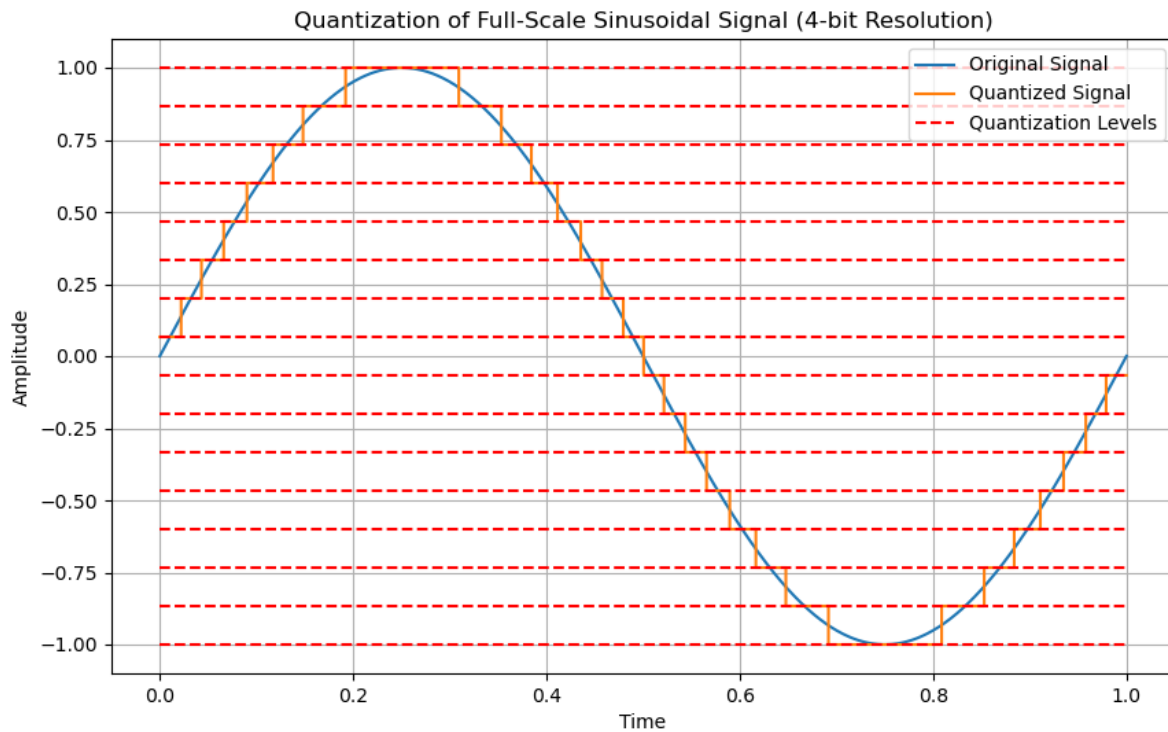


Fig. 1: Quantization of Sinusoidal Signal

1) Signal Power:

The power of a continuous-time signal is defined as the average value of the square of the signal over a certain time interval. For a sinusoidal signal  $x(t) = A \sin(2\pi ft + \phi)$ , the power ( $P_S$ ) is calculated as:

$$P_S = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt \quad (1)$$

where  $\phi$  is the phase of the signal.

$$P_S = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |A \sin(2\pi ft + \phi)|^2 dt \quad (2)$$

$$P_S = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \sin^2(2\pi ft + \phi) dt \quad (3)$$

$$P_S = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \cdot \frac{1 - \cos(4\pi ft + 2\phi)}{2} dt \quad (4)$$

$$P_S = \frac{1}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 dt - \frac{1}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \cos(4\pi ft + 2\phi) dt \quad (5)$$

$$P_S = \frac{1}{2} \cdot A^2 - 0 \quad (6)$$

$$P_S = \frac{A^2}{2} \quad (7)$$

$$(8)$$

Here,  $A = 1$ , so:

$$P_S = \frac{1}{2} \quad (9)$$

## 2) Noise Power:

No of Quantization levels is given by  $2^n$ , Where  $n$  is resolution or no of bits.

Distance between any two Quantization levels = Quantization step

(No of Quantization levels)\*(Quantization step) = Peak Distance (Refer to Fig 1)

$$q = \frac{\text{Peak distance}}{2^n} = \frac{1 - (-1)}{2^n} = \frac{2}{2^n} = 2^{-(n-1)} \quad (10)$$

We know, quantization error has a maximum value of plus or minus half the step size, so

$$|Y| \leq \frac{q}{2} \text{ and therefore, } |Y| \leq 2^{-n} \quad (11)$$

For a large enough number of quantization steps, the probability density function of the quantization error tends toward being flat 1. Pdf of error (Y) of quantization is defined as

$$p_Y(y) = \begin{cases} \frac{1}{q}, & \text{if } -\frac{q}{2} < y < \frac{q}{2} \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

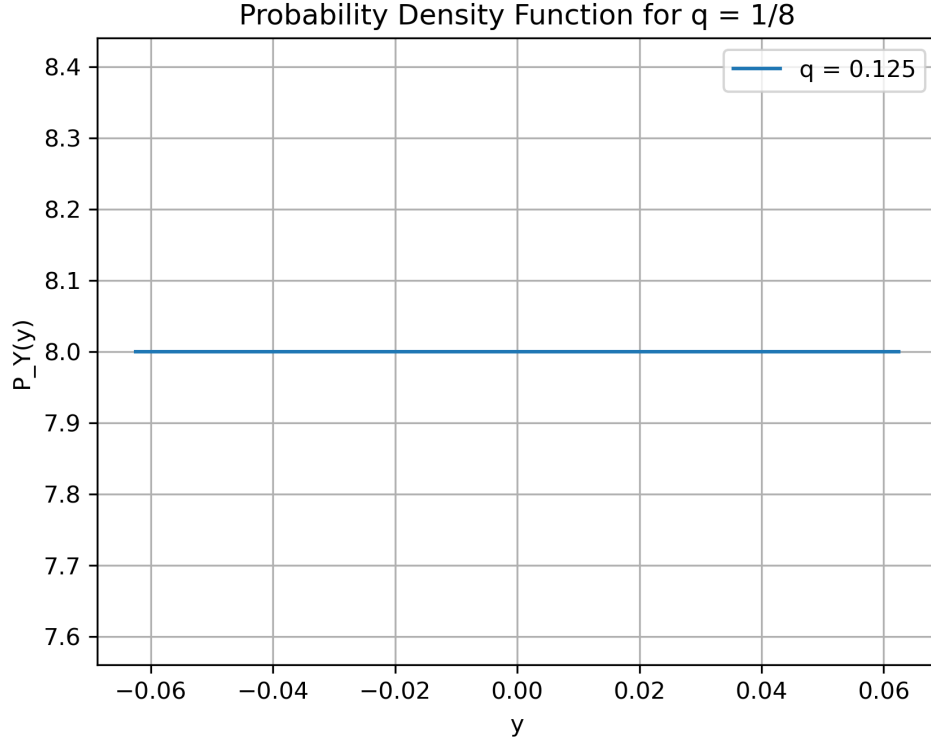


Fig. 2: plot of pdf of Quantization Error

So, we can calculate its mean power or variance as the 2nd moment of its distribution. Since, the distribution of error is uniform hence  $E[Y]=0$ .

$$E(Y) = \int_{-\frac{q}{2}}^{\frac{q}{2}} p_Y(y)y \, dy \quad (13)$$

$$= \frac{1}{q} \cdot \frac{1}{2} \left( \left( \frac{q}{2} \right)^2 - \left( -\frac{q}{2} \right)^2 \right) 10 \quad (14)$$

$$= \frac{1}{q} \cdot \frac{1}{2} (0 - 0) \quad (15)$$

$$= 0 \quad (16)$$

$$E[Y^2] = \int_{-\frac{q}{2}}^{\frac{q}{2}} p_Y(y)y^2 \, dy \quad (17)$$

$$= \frac{1}{q} \cdot \frac{1}{3} \left( \left( \frac{q}{2} \right)^3 - \left( -\frac{q}{2} \right)^3 \right) 10 \quad (18)$$

$$= \frac{1}{3q} \cdot \frac{q^3}{4} \quad (19)$$

$$= \frac{q^2}{12} \quad (20)$$

$$(21)$$

On putting  $q = 2^{-(n-1)}$ , we have:

$$E[Y^2] \approx \frac{2^{-2n}}{3} 10 \quad (22)$$

$$P_Y \approx \frac{2^{-2n}}{3} \quad (23)$$

$$P_N \approx \frac{2^{-2n}}{3} \quad (24)$$

$$(25)$$

Thus, an ideal ADC would have a signal-to-noise ratio

$$SNR = \frac{P_S}{P_N} = 1.5 \cdot 2^{2n} \quad (26)$$

or, expressed in decibels,

$$SNR = 1.76 + 6.02n \quad (27)$$

Plug in the values:

$$n = \frac{61.96 \text{ dB} - 1.76 \text{ dB}}{6.02 \text{ dB}} \approx 10 \text{ bits} \quad (28)$$

So, the resolution of the ADC is approximately 10 bits.