

Question EC 31 2023

The signal-to-noise ratio (SNR) of an ADC with a full-scale sinusoidal input is given to be 61.96 dB.
The resolution of the ADC is GATE EC 2023

Solution:

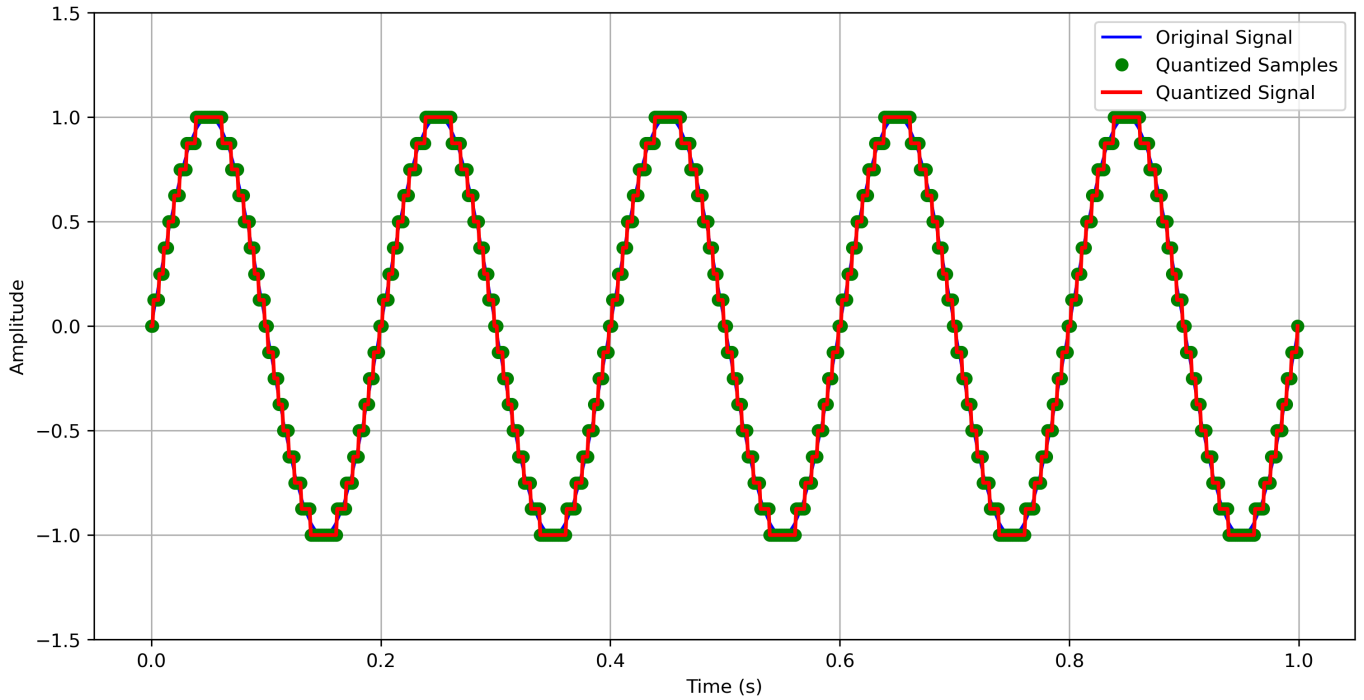


Fig. 1: Quantization of Sinusoidal Signal

The mean power in a sinusoidal input signal with unit amplitude is simply

$$P_s = \frac{1}{T} \int_0^T \sin^2(\omega t) dt \quad (1)$$

where T is the time period of the signal, and ω is the angular frequency.

Quantized to 2^n discrete levels, equivalent to n bits, one quantization step q has a size of

$$q = \frac{2}{2^n} = 2^{-(n-1)} \quad (2)$$

We know, quantization error has a maximum value of plus or minus half the step size, so

$$|e| \leq \frac{q}{2} \text{ and therefore, } |e| \leq 2^{-n} \quad (3)$$

For a large enough number of quantization steps, the probability density function of the quantization error tends toward being flat 1. Its value within the quantization error bounds is

$$p_e \approx \frac{1}{q} \quad (4)$$

So, we can calculate its mean power or variance as the 2nd moment of its distribution. Since, the distribution of error is uniform hence $E[e]=0$.

$$E[e^2] = \int_{-\frac{q}{2}}^{\frac{q}{2}} p(e)e^2 de = \frac{1}{q} \cdot \frac{1}{3} \left(\left(\frac{q}{2} \right)^3 - \left(-\frac{q}{2} \right)^3 \right) = \frac{1}{3q} \cdot \frac{q^3}{4} = \frac{q^2}{12} \quad (5)$$

On putting $q = 2^{-(n-1)}$, we have:

$$E[e^2] \approx \frac{2^{-2n}}{3} \quad (6)$$

$$P_e \approx \frac{2^{-2n}}{3} \quad (7)$$

Thus, an ideal ADC would have a signal-to-noise ratio

$$SNR = \frac{P_s}{P_e} = 1.5 \cdot 2^{2n} \quad (8)$$

or, expressed in decibels,

$$SNR = 1.76 + 6.02n \quad (9)$$

Plug in the values:

$$\text{Resolution (in bits)} = \frac{61.96 \text{ dB} - 1.76 \text{ dB}}{6.02 \text{ dB}} \approx 10 \text{ bits} \quad (10)$$

So, the resolution of the ADC is approximately 10 bits.