EE22BTECH11049 - Shivansh Kirar

Question EC 31 2023

The signal-to-noise ratio (SNR) of an ADC with a full-scale sinusoidal input is given to be 61.96 dB. The resolution of the ADC is GATE EC 2023

Solution:

Variable	Defination
T	Time Period
f	Frequency
A	Amplitude
P_S	Signal Power
P_N	Noise Power
q	Quantization step size
n	No of bits
Y	Quantization Error

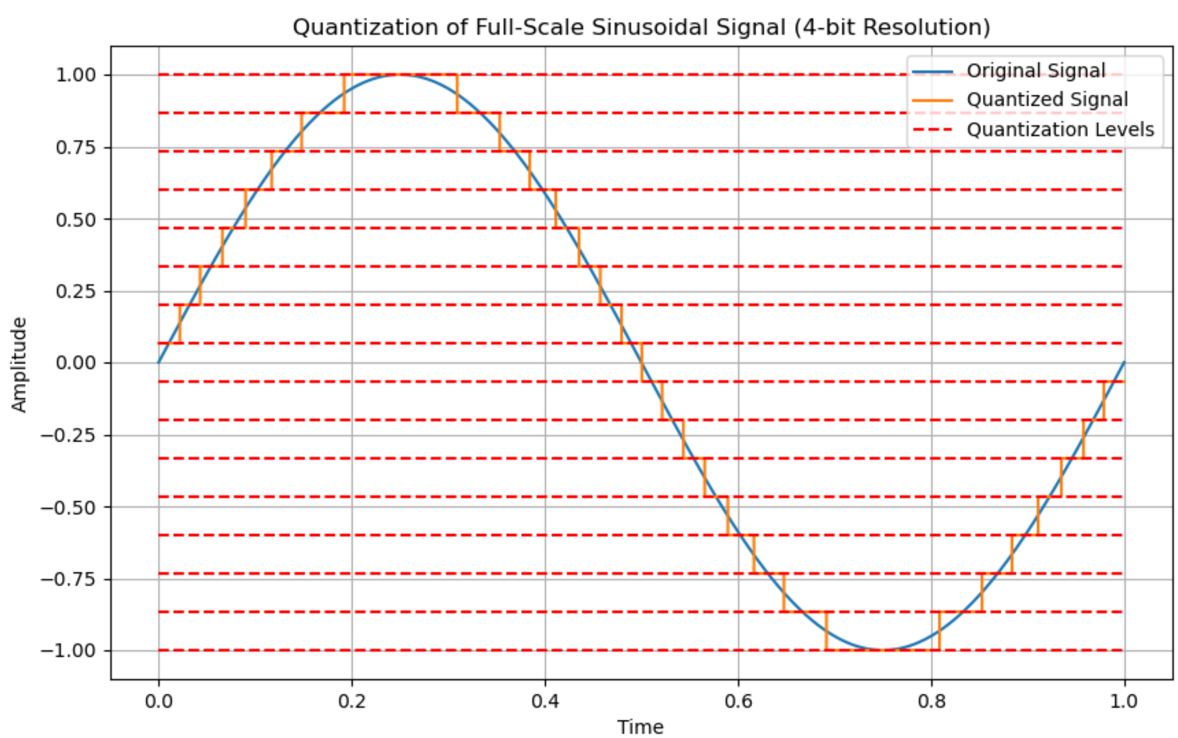

Fig. 1: Quantization of Sinusoidal Signal

1) Signal Power:
   The power of a continuous-time signal is defined as the average value of the square of the signal over a certain time interval. For a sinusoidal signal $x(t) = A\sin(2\pi ft + \phi)$, the power (P_S) is calculated as:

$$P_S = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt \tag{1}$$

where ϕ is the phase of the signal.

$$P_S = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |A\sin(2\pi f t + \phi)|^2 dt \tag{2}$$

$$P_S = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \sin^2(2\pi f t + \phi) dt \tag{3}$$

$$P_S = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \cdot \frac{1-\cos(4\pi f t + 2\phi)}{2} dt \tag{4}$$

$$P_S = \frac{1}{2} \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 dt - \frac{1}{2} \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \cos(4\pi f t + 2\phi) dt \tag{5}$$

$$P_S = \frac{1}{2} \cdot A^2 - 0 \tag{6}$$

$$P_S = \frac{A^2}{2} \tag{7}$$

$$\tag{8}$$

Here, A= 1, so:

$$P_S = \frac{1}{2} \tag{9}$$

2) Noise Power:
No of Quantization levels is given by 2^n, Where n is resolution or no of bits.
Distance between any two Quantization levels = Quantization step
(No of Quantization levels)*(Quantization step) = Peak Distance (Refer to Fig 1)

$$q = \frac{\text{Peak distance}}{2^n} = \frac{1-(-1)}{2^n} = \frac{2}{2^n} = 2^{-(n-1)} \tag{10}$$

We know,quantization error has a maximum value of plus or minus half the step size, so

$$|Y| \leq \frac{q}{2} \text{ and therefore, } |Y| \leq 2^{-n} \tag{11}$$

For a large enough number of quantization steps, the probability density function of the quantization error tends toward being flat 1. Pdf of error (Y) of quantization is defined as

$$p_Y(y) = \begin{cases} \frac{1}{q}, & \text{if } -\frac{q}{2} \leq y \leq \frac{q}{2} \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

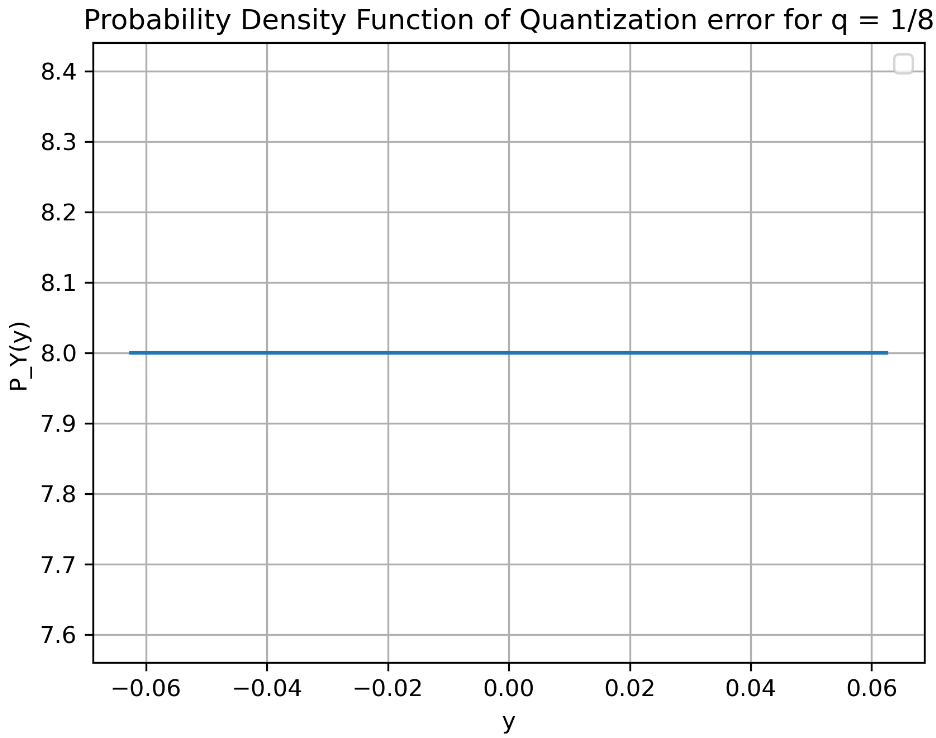

Fig. 2: plot of pdf of Quantization Error

So, we can calculate its mean power or variance as the 2nd moment of its distribution. Since, the distribution of error is uniform hence E[Y]=0.

$$
\begin{aligned}
E(Y) &= \int_{-\frac{q}{2}}^{\frac{q}{2}} p_Y(y) y \, de && (13)\\
&= \frac{1}{q} \cdot \frac{1}{2} \left(\left(\frac{q}{2}\right)^2 - \left(-\frac{q}{2}\right)^2 \right) 10 && (14)\\
&= \frac{1}{q} \cdot \frac{1}{2} (0 - 0) && (15)\\
&= 0 && (16)
\end{aligned}
$$

$$
\begin{aligned}
E[Y^2] &= \int_{-\frac{q}{2}}^{\frac{q}{2}} p_Y(y) y^2 \, de && (17)\\
&= \frac{1}{q} \cdot \frac{1}{3} \left(\left(\frac{q}{2}\right)^3 - \left(-\frac{q}{2}\right)^3 \right) 10 && (18)\\
&= \frac{1}{3q} \cdot \frac{q^3}{4} && (19)\\
&= \frac{q^2}{12} && (20)\\
& && (21)
\end{aligned}
$$

On putting $q = 2^{-(n-1)}$, we have:

$$E[Y^2] \approx \frac{2^{-2n}}{3} 10 \tag{22}$$

$$P_Y \approx \frac{2^{-2n}}{3} \tag{23}$$

$$P_N \approx \frac{2^{-2n}}{3} \tag{24}$$

$$\tag{25}$$

Thus, an ideal ADC would have a signal-to-noise ratio

$$SNR = \frac{P_S}{P_N} = 1.5 \cdot 2^{2n} \tag{26}$$

or, expressed in decibels,

$$SNR = 10 \left(\log_{10}(1.5 \cdot 2^n)\right) \tag{27}$$

$$SNR = 10 \left(\log_{10}(1.5) + \log_{10}(2^{2n})\right) \tag{28}$$

$$SNR = 10 \left(0.176 + 2n \cdot 0.3010\right) \tag{29}$$

$$SNR = 1.76 + 6.02n \tag{30}$$

Plug in the values:

$$n = \frac{61.96 \text{ dB} - 1.76 \text{ dB}}{6.02 \text{ dB}} \approx 10 \text{ bits} \tag{31}$$

So, the resolution of the ADC is approximately 10 bits.